

# 원추체의 스피닝 이론에 대한 연구

김 만 수

## A Study on a Theory of Spinning of Cones

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### Summary

The present paper deals with the analysis of spin forging process. The study must be regarded as new forming methods for producing axially symmetric parts, in which progressive forming is concentrated in a relatively small zone of the circumference of the part. A shear type of deformation is postulated, based on experimental evidence.

The displacement, velocity, strain rate, and stresses fields are computed for mises material.

The power consumed in the operation is computed from the strain rate and stress fields.

The numerical solution is compared with actual power and force measurement in experimented tests.

### Nomenclature

$F_t$  : tangential force component, Lb.  
 $F_p, F_q$  : force components in parallel and perpendicular directions, to the roller axis, respectively Lb.  
 $F_r, F_z$  : force components in the radial and axial directions, respectively Lb.  
 $N$  : Speed of rotation of the mandrel, rpm.  
 $R_i, R_o, R_m$  : Blank radii during operation.  
 $R, \theta, Z$  : cylindrical co-ordinates.  
 $X, Y, Z$  : rectangular co-ordinates.  
 $d_o$  : roller diameter, in.  
 $f$  : feed, ipr.  
 $n_o$  : number of passes of the element across the deformation zone during its deformation.  
 $r_o$  : corner radius of roller, in.  
 $\rho, \rho_n$  : radii of curvature, in.  
 $h$  : distance of a point from the neutral surface in the thickness direction, in.  
 $t_o, t_f$  : initial and final thickness, respectively, in.  
 $w$  : specific deformation energy, in-Lb/in<sup>2</sup>.  
 $p$  : normal stress on the contact surface between roller and cone, psi.  
 $\bar{\sigma}$  : effective stress, psi.  
 $d\bar{\epsilon}_n$  : increment of effective strain for one pass of an element across the deformation zone.

$\Delta\bar{\epsilon}_\alpha$  : increment of normal strain due to bending.  
 $\int d\bar{\epsilon}$  : total effective strain.  
 $\alpha$  : one half the cone angle, deg.  
 $\psi_o$  : roller position angle, deg.  
 $\theta_o, \bar{\theta}_o$  : angle for deformation zone and to average value, respectively radian.  
 $\theta_o', \bar{\theta}_o'$  : angle functionally related to  $\theta_o$  and  $\bar{\theta}_o$  respectively, radian.  
 $\gamma_1, \gamma_2$  : ratios of strains.

### Introduction

#### In the analysis

1. Mises material is used, which implies that:
  - (a) The material is homogeneous and isotropic.
  - (b) There is no elastic deformation and consequently no volumetric change
  - (c) There is no strain-hardening
2. The thickness of the blank is much smaller than the minimum radius of the cone
3. The metal deforms under the roller in pure shear
4. The frictional force under the roller is neglected.  
This is justified, because the relative velocity between the cone and the roller is very small.

### Materials and Methods

#### Deformation mode

The deformation mechanism of shear spinning of cones is shown schematically in Fig 1. The process is characterized by

the fact that the radial position of an element in the blank remains the same during deformation. A flat blank of thickness  $t_f$  is held by a tail stock against the apex of a mandrel of  $\alpha_0/2$  half angle. This is called sine-law spinning, but deviations from it are possible and are practical.

$$t_f = t_0 \sin \alpha_0 \quad (1)$$

Each element is strained plastically during transformation from A1, B1 to A2, B2 in a stepwise manner Fig. 2.

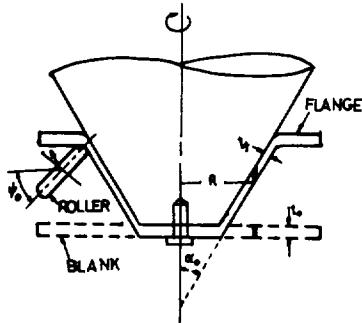


Fig. 1 Schematic diagram of shear spinning process

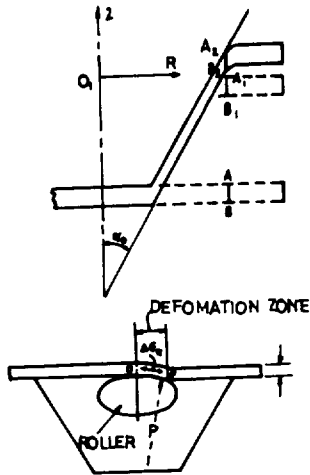


Fig. 2 Deformation in Shear spinning

Inspection of Fig. 2. reveals the fact that there R-plane (plane normal to the R-direction) as well as redundant straining may be assumed to be the result of a combination of shearing in two directions on the  $\theta$ -plane, or of bending and unbending about the R-direction.

In either case the results would show that the radial cross-sectional plane remains radial.

It is possible that bending not only about the R-direction, but also about the  $\theta$  - direction, may cause the rotation  $\theta_r$  by reducing the thickness of the element.

In the analysis to follow the deformation mechanism will be idealized in that all minor straining will be ignored.

Deformation energy

The remainder of the done is strain-free at the instant. From a variety of roller shapes, as shown in Fig. 3, shape I was chosen to be analyzed. The roller is assumed to have its axis parallel to the side of the cone.

The choice of the co-ordinate axes XYZ with the origin at o is indicated in Fig. 4. Another set of cylindrical polar co-ordinates (R,  $\theta$ , Z) with the same origin and Z-axis in XYZ-co-ordinates was introduced.

In the analysis the deformation-energy theory was used which is indicated as follows :

$$dW = w \cdot dv \quad (2)$$

w: the specific work dv : the volume of the metal

$$F_t = t_0 \int \sin \alpha_0 w = t_0 \int \sin \alpha_0 \sigma d\bar{\epsilon} \quad (3)$$

where  $\bar{\sigma}$  and  $d\bar{\epsilon}$  are effective stress and infinitesimal effective strain, respectively.

since  $dv = t_0 f \sin \alpha_0 d\ell$ , where  $t_0$  is the initial blank thickness,

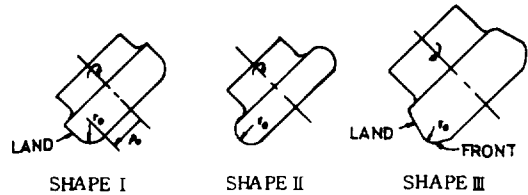


Fig. 3 Different shape of roller

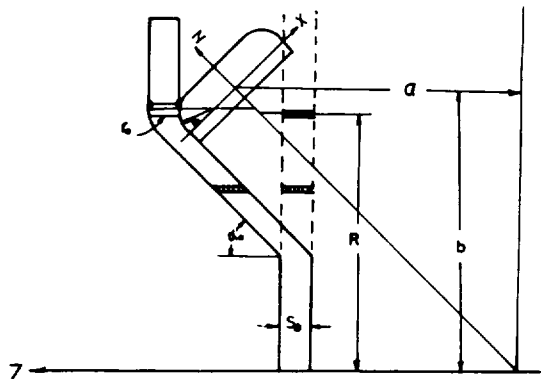


Fig. 4 Deformation in process

$f$  is the feed, and is one half the cone angle.

$$\Delta \gamma_{RZ} = \Delta \left( \frac{\partial U_Z}{\partial R} \right), \Delta \gamma_{\theta Z} = \frac{1}{R} \Delta \left( \frac{\partial U_Z}{\partial \theta} \right) \quad (4)$$

where  $\Delta$  is the symbol for the integrants of the infinitesimal strain during one pass of the element under the roller.

$$\Delta \bar{\epsilon}_n = \frac{Rz}{\sqrt{3}} [\gamma_1 + \sqrt{1+\gamma_1^2 + \gamma_2^2}] \quad (5)$$

where  $\gamma_1 = |\Delta \epsilon_\theta| / \frac{d\gamma_{RZ}}{2}$  and  $\gamma_2 = \Delta \gamma_{\theta Z} / \Delta \gamma_{RZ}$  the total effective strain of an element  $f d\bar{\epsilon}$ , therefore, can be obtained as

$$\int d\bar{\epsilon} = \frac{n_0}{1} \Delta \bar{\epsilon}_n \quad (6)$$

where  $n_0$  (an integer) is the number of passes of the element under the roller during its major straining.

The average value of the strain  $|\Delta \epsilon_\theta|$  of equation in the thickness direction is given by

$$|\Delta \epsilon_\theta| = \frac{t_0}{4\rho_n} \quad (7)$$

assuming that the neutral surface coincides with the central surface. Moreover, the strain  $\Delta \gamma_{\theta Z}$  of equation (4) can be written as

$$\Delta \gamma_{\theta Z} = \frac{1}{R_n} \Delta \left( \frac{\partial U_Z}{\partial \theta} \right) = \tan \delta' \quad (8)$$

substitution of equation (7) into equation (8) and (8) yields

$$|\Delta \epsilon_\theta| = \frac{t_0/2}{(Rn\theta_0)^2} Z n' \quad (9)$$

$$\Delta \gamma_{\theta Z} = \frac{2\sqrt{m}}{Rn\theta_0'} Z n'$$

the total effective strain  $\int d\bar{\epsilon}$  becomes

$$\int d\bar{\epsilon} = \sum_1^{n_0} \Delta \bar{\epsilon}_n = \frac{\cot \alpha_0}{\sqrt{3}} [\nu_1 + \sqrt{1+\nu_1^2 + \nu_2^2}] \quad (10)$$

$\bar{\theta}_0$  of equation (10) is an average value of  $\theta_0$  in the interval between  $R_n = R_0$  and  $R_n = R_i$  and is taken at the value of  $\theta_0$  for  $R_n = \frac{R_0+R_i}{2} = R_0 - \frac{\gamma_0 \cos \alpha_0}{2}$

The average value of the angle for the deformation Zone  $\bar{\theta}_0$  is given by

$$\bar{\theta}_0 = \cos^{-1} \left[ \frac{\frac{b^2}{a^2} - \nu_1 - \left(\frac{b}{R_0}\right)^2 \left(\frac{b^2}{a^2} - 1\right)}{\frac{b^2}{a^2} - 1} \right] \quad (11)$$

Force components.

The tangential component of force on the roller from equation (3) may be written in the form of

$$F_t = t_0 f \sin \alpha_0 \bar{\sigma}_m \int d\bar{\epsilon} \quad (12)$$

Where  $\bar{P}_m$  is the mean effective stress defined by

$$\bar{\sigma}_m = \int \bar{\sigma} d\bar{\epsilon} / \int d\bar{\epsilon} \quad (13)$$

and the total effective strain  $\int d\bar{\epsilon}$  is given by equation (10) These strain rates can be estimated from the equation,

$$\frac{\dot{\epsilon}}{\epsilon_{ave}} = \frac{\int d\bar{\epsilon}}{n_0} = \frac{\int d\bar{\epsilon}}{\theta_0} = \frac{\dot{\theta}_0}{2\pi N} \quad (14)$$

The strain rates in the present investigation were of the order of 10 to 10<sup>2</sup> m/in per sec and, consequently, are high, but not as high as these found in metal cutting.

The two force components  $F_r$  and  $F_z$ , too, are important from the point of view of machine design in spinning.

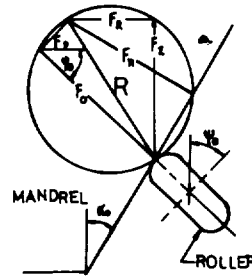


Fig.5 Force system

The force of the roller can be resolved in to the radial and axial directions of the mandrel.

The two components  $F_R$  and  $F_Z$  may be expressed as functions of  $F_p$  and  $F_Q$  which may be measured together with the tangential force component  $F_x$  by a three-component dynamometer, where  $F_p$  and  $F_Q$  are taken in the parallel and normal directions, respectively, relative to the roller axis.

$$\left. \begin{aligned} F_R &= F_\theta \cos \psi_0 - F_p \sin \psi_0 \\ F_Z &= F_\theta \sin \psi_0 + F_p \cos \psi_0 \end{aligned} \right\} \quad (15)$$

If it is assumed that a normal stress p acting on the contact surface between the roller and cone is uniformly distributed, then

$$\left. \begin{aligned} F_t &= p A_t \\ F_R &= p A_R \\ F_Z &= p A_Z \end{aligned} \right\} \quad (16)$$

$$F_R = F_t \frac{A_R}{A_t}, F_Z = F_t \frac{A_Z}{A_t} \quad (17)$$

$$F_R = F_t \frac{R_0 \bar{\theta}_0}{f \cos \alpha_0} \cdot \frac{1}{\sqrt{m}} \left[ \left( \frac{1 - \sin \alpha_0}{\cos \alpha_0} \right) + \frac{f}{2\gamma_0} \right] \quad (18)$$

$$F_Z = F_t \frac{R_0 \bar{\theta}_0}{f \cos \alpha_0} \cdot \frac{1}{\sqrt{m}}$$

$$F_t = t_0 f \sin \alpha_0 \bar{\sigma}_m \frac{\cot \alpha_0}{\sqrt{3}} \quad (10)$$

### Result and Discussion

The experimental work was out on a modified lathe and a Universal milling machine. The roller forces were measured with a three-component dynamometer and were recorded autographically. It is necessary to obtain the instantaneous cone radius corresponding to each force reading, since the force components as well as the cone radius change during a single spinning test.

After the blank was spun into a cone, its radii at the

marked positions were determined, and the final thickness was checked according to equation (1).

The assumption was that the strain rates in spinning would be low enough not to alter the stress-strain properties significantly. In the analysis of shear spinning, the dynamic stress-strain relationship for lead given were assumed to be applicable. The tangential force components are plotted as functions of  $t_0 f \sin \alpha_0$  where  $t_0$  is the initial disk thickness,  $f$  is the feed, and  $\alpha_0$  is one half the cone angle.

The two solid curves are theoretical solutions given by equation (3). The stress-strain relationship at strain-rates of the order of  $10$  to  $10^2$  in/in per sec must be applied.

It is seen that for lead equation (3) for  $m=1$  is in good agreement with the experimental results for roller of 1 in and 2 in diameter, while it appears that  $m=0.5$  for a 3-in diameter.

The theoretical values is given by equation (19). Because, the stress acting on the overlapped portion would have an opposite component in the radial direction.

Thus, the actual radial force component may deviate from the theoretical values given by equation (18).

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