Development of Microcomputer Package for Solving Bicriteria Linear Programming and Its Application*

Kang Duk-su , Mitsuo Gen and Kenichi Ida***

二目的線型計劃을 해결하기 위한 마이크로컴퓨터 패키지 開發과 그 應用. *

姜 德 壽, *Mitsuo Gen* and Kenichi Ida***

Abstrsct

Linear programming is one of the most widely used Operations Research/Management Science/ Industrial Engineering techniques. Recently, multiple criteria decision making or multiple objective linear programming has been well established as a practical approach to seeking satisfactory solutions to real-world decision problems. Also, much attention has been focused on a personal computer as an economical management tool.

A bicriteria linear programming problem arises as a special case of multiple objective linear programming problems when only two objective functions are of interest. In this paper we develop a software package for solving bicriteria linear programming problems by using bicriteria revised simplex method combined the compromise techique on a personal computer. As an application of the software package Micro-BLP implemented here. We also demonstrate multiple criteria objective linear programming problems and design a conversational and user-friendly system.

I. Introduction

In 1972. Turban [22] reported on an Operations Research activities that statistical analysis,

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Cheju National University. Dept of Business Education, Assistant Professor.
Ashikaga Institute of Tech, Japan, Assistant Professor *
Ashikaga Institute of Tech, Japan. Assistant Instructor. **

simulation, and linear programming were currently the most widely used techniques. In 1977, Ledbetter and Cox [18] reported on a survey utilization of Operations research techniques that regression anlaysis, linear programming. and simulation were the most popular, thereby reinforcing the results of the Turban's study. In 1981, Whitehouse, et al [23] reported on a industrial Engineering in microcomputer application that 40% of the respondents expressed interest in linear programming and 35% desired multiple linear regression. Recently, Lee, Gen, and Shim reported n a state–of–the–art survey of goal programming for decision making that about 89% of all applications used linear goal programming which is an extension of linear programming.

Recently, multiple criteria decision making or multiple objective decision making has been fairly well established as a practical decision making approach with limited resources, information. and cognitive ability of the decision maker [2, 9, 25, 24, 25], Hannan [13] stated that three of the most promising methodologies for dealing with multiple objective linear programming problems are vector maximum methods, goal programming, and interactive techniques. Bicriteria linear programming problem arises as a special case of multiple objective linear programming problems when only two objective functions are of interest. The method for solving a bicriteria mathematical programming problem have also been studied by many authors [1, 3, 4, 6, 8, 12, 14, 16, 17]. Most solution methods are based on the parametric techniques of taking weighted sum of objective function and imposing varying minimum levels to of the objective. Choo [7] modified the simplex method so that the pivoting process generates those nondominated extreme points whose images describe the efficient frontier in the two dimensional criterion space. However, it is a difficult in the evaluation phase that the decidion maker finds a compromise solution which is as close as possible to the ideal solution from nondominated extreme points.

Much attention has been focused on a microcompulter or personal computer as an economical management tool to apply systematic approaches to decision making in Industrial Engineering problems [10. 23]. Microcomputer are making important contributions to the operation of industrial, business, educational and nonprofit organizations [20]. Recently, Schrage [21] developed the software package LINDO (Linear INteractive Discrete Optimizer) which is one of the mostly used linear programming codes on minicomputer. However there is no a routine for solving a bicriteria or multiple criteria linear programming problem in LINDO.

In this paper we develop a software package of the bicriteria revised simplex method combined the compromise technique for interactively solving bicriteria linear programming problems on a personal computer. In the software package Microl–BLP implemented here, we can also solve multiple criteria linear programming problems and design a conversational and user–friedly system. In Section Ⅱ. we discuss a bicriteria linear programming problem and describe a bicriteria revised simplex method combined the compromise technique in Section Ⅲ. In Section Ⅳ. an implementation on a personal computer is described and numerical example of a multiple criteria linear programming problem running on personal computers is demonstrated by the Micro–BLP.

Ⅱ. Bicriteria Linear Programming Model

Bicriteria linear programming is a linear programming having two conflicting objective functions and arises as a special case of multiple objective linear programming when only two objective functions are of interest. Methods for solving multiple objective linear programming problems are obviously applicable to bicriteria linear programming problems. However, bicriteria linear programming problems have a simple structure in comparison to multiple objective linear programming problems and specialized algorithm exploiting this simplicity of structure.

A general bicriteria linear programming model with m constraints and n decision variables can be representet as follows:

$$\begin{aligned} \max \quad & z_1=(x)=\sum_{j=1}^{n} c_{1j}x_j \\ \max \quad & z_2=(x)=\sum_{j=1}^{n} c_{2j}x_j \end{aligned} \qquad (1)$$

$$\text{subj. to} \quad \sum_{j=1}^{n} a_{ij}x_j = b_i, \quad i=1,2,\cdots\cdots, m \qquad (2)$$

$$x_j \geq 0, \quad j=1,2,\cdots\cdots, n, \qquad (3)$$

where

c_{1j}, c_{2j}: the jth coefficient of the 1st and 2nd objective function, respectively.

a_{ij}: the jth technological coefficient of the ith resousce.

b_i: the ith resource.

x_j: the jth decision variable.

We note that it is always possible to express the objective function in their maximizing forms since a minimization problem can be transformed to maximization one by proper sign manipulation. As for the constraints. greater and less than form (>=or <=) are transformable to their equivalent equal forms by introducing slack or surplus variables.

It is impossible to simultaneously maximize two conflicting objective functions because of the individual objective functions are merely listed in (1) and they are not added, multiplied, or combined in any way. In other words, the increase in any one of the objective functions will decrease the other. In bicriteria linear programming, such a concept is thus replaced by the concept of a best compromise solution which, in turn, depends on the decision maker's preference [2].

Ⅲ. Bicriteria Revised Simplex Method

In order to solve bicriteria linear programming problems interactively on personal computers, we develop a bicriteria revised simplex method combined the compromise technique to decide a best compromise solution which is as close as possible to the ideal solution from nondominated extreme points. We summarize the bicriteria revised simplex method using the compromise technique as follows:

Step 1: Solve the linear programming problem including the 1st objective function by the revised simplex algorithm to find x_1 and z_1^* with the basis inverse B^{-1} Construct PIS(positive ideal solution) and NIS(negative ideal solution) pay-off table

	PIS	NIS	x^*	x^-
LP-1	z_1^*	z_1^-	x_1^*	x_1^-
LP-2	z_2^*	z_2^-	x_2^*	x_2^-

after solving each linear programming problem and set t=0.

Step 2: Calculate the reduced cost of the 2nd objective function:

$$\bar{c}_{2j} = c_{2B}^T B^{-1} a_j - c_{2j}, \quad j \in I_N \qquad (4)$$

where C_{2B}: the basic cost vector of the 2nd objective function.

a_j: the nonbasic coefficient vector.

I_N: the index set of nonbasic vectors.

Step 3: if $\bar{c}_{2j} \leq 0$, $j \in I_N$, then set $x_2^* = x_t^\circ$, $z_2^* = f_{2t}^\circ$, $z_1^\circ = f_{1t}^\circ$, and $q = t-1$, and go to Step 7. Other wise, set $t = t+1$ and go to Step 4.

Step 4: Determine an incoming vector such that

$$-\frac{\bar{c}_{1s}}{\bar{c}_{2s}} = \min_j \{-\frac{\bar{c}_{1j}}{\bar{c}_{2j}} \mid \bar{c}_{2j} > 0,\ j \in I_N\} \quad (5)$$

Step 5: Calculate a basis representation $\alpha_s = B^{-1}$ ad2s. If all the elements of α_s re zero or negative, stop with the unbounded bicriterial linear programming problem. Otherwise, determine an outgoing vector a_{ir} by the minimum ratio test.

$$\frac{\beta_r}{\alpha_{rs}} = \min_i \{\frac{\beta_i}{\alpha_{is}} \mid \alpha_{is} > 0,\ i = 1, 2, \cdots, m\} \quad (6)$$

Step 6: update the basis inverse, the basic feasible solution, the 1st and 2nd reduced cost vectors by the elementary matrix:

$$B^{-1},\ \beta,\ \bar{c}_{1N},\ \bar{c}_{2N}.$$

Set the nondominated extreme solution and objective functions:

$$x_t^\circ = \beta,\ f_{1t}^\circ = f_1(x_t^\circ),\ f_{2t}^\circ = f_2(x_t^\circ)$$

Return to Step 3.

Step 7: Calculate L_pk–metrics for the nondominated extreme solution.

$$L_{rt} = \{\sum_{k=1}^{2} \frac{z_k^\circ - f_{kt}^\circ}{z_k^* - z_k^-}\}^{1/r},$$
$$r = 1, 2, \quad t = 1, 2, \cdots\cdots, q, \quad (7)$$

$$L_{rt} = \max_k \{\frac{z_k^* - f_{kt}^\circ}{z_k^* - z_k^-} \mid k = 1, 2\}$$
$$r = \infty, \quad t = 1, 2, \cdots\cdots, q. \quad (8)$$

Determine a best compromise solution s_{t*} and $\tilde{z}_k = F_{kt*}$ such that

$$L_{rt}^* = \min_t \{\sum_r L_{rt} \mid t = 1, 2, \cdots\cdots, c\}. \quad (9)$$

Ⅳ. Numerical Example

In developing a Micro–BLP package, we designed a conversational and user-frendly system. Also, in order to prevent the problems of the degeneracy and cycling we adopted Bland's method [5].

In the paper [9], we illustrated the bicriteria revised simplex methed combined the compromise technique by solving the bicriteria transportation problem treated by Aneja and Nair [3]. Now, to demonstrate the micro–BLP package on a microcomputer, the following multiple criteria linear programming problem with four objective functions, three constraints, and four decision variables are considered.

$$\begin{aligned}
z\text{-max} \quad & z_1 = 2x_1 - x_2 + 2x_3 + 3x_4 \\
& z_2 = 5x_3 - 2x_4 \\
& z_3 = -6x_4 + 2x_2 + x_4 \\
& z_4 = x_4 + 2x_2 + x_3 + 4x_4 \\
\text{subj. to} \quad & 2x_1 + 2x_2 + 6x_3 + 4x_4 \leq 24 \\
& 5x_1 + 2x_2 + x_3 + 2x_4 = 11 \\
& 3x_1 + x_2 + 3x_3 - 3x_4 \geq 3 \\
& x_1,\ x2,\ x_3,\ x_4 \geq 0
\end{aligned}$$

After entering data of the multiple criteria linear programming problem, it will be shown on the CRT screen as in Table 1. Tabel 2 shows a computing process by Micro–BLP in which there are nondominated solution table, PIS and NIS, Lp–metric for the nondominated solutions, the best compromise solution in a bicriteria linear programming problem, objective function, and row information of constraints. In Table 3, it shows a best compromise solution for the multiple criteria linear programming problem.

Table 1. Multiple Criteria Linear Programming Problem

* Title of Promblem: MLP ex.2 H − M

	X 1 RHS	X 2	X 3	X 4
MAX. Z 1	2.0000	− 1.0000	2.0000	3.0000
MAX. Z 2	0.0000	0.0000	5.0000	2.0000
MAX. Z 3	− 6.0000	2.0000	0.0000	1.0000
MAX. Z 4	2.0000	2.0000	1.0000	4.0000
S.T. G 1	24.0000	2.0000	6.0000	4.0000
	5.0000			
G 2	11.0000	2.0000	1.0000	2.0000
	3.0000			
G 3	3.0000	1.0000	3.0000	− 3.0000

Table 2. A Computing Process by Micro—BLP

* BLP Problem 1 with Objective Funcs. Z 1 and Z 2
* nondominated solutions table

	X 1 Z 2	X 2	X 3	X 4	X 1
E 1	0.8846	0.0000	2.2692	2.1538	12.7692
	7.0385				
E 2	1.5000	0.0000	3.5000	0.0000	10.0000
	17.5000				

* positive ideal and negative ideal values.

	Z −	Z *
Z 1	− 5.5000	12.7692
Z 2	− 1.7143	17.5000

* LP−metric for the nondominated solutions.

	L 1	L 2	L oo
E 1	0.5445	0.5445	0.5445
E 2	0.1516	0.1516	0.1516

* best compromise solution in BLP 1.

X	1	1.5000
X	2	0.0000
X	3	3.5000
X	4	0.0000

* objective functions.

Z	1	10.0000
Z	2	17.5000

* row information of constraints.

		G X	SK(or SP)	RHS
G	1	24.0000	0.0000	24.0000
G	2	11.0000	0.0000	11.0000
G	3	15.0000	12.0000	3.0000

Table 3. A Best Compromise Solution of the Problem

* nondominated solutions table.

		X 1 / X 2	X 2 / Z 3	X 3 / Z 4	X 4	Z 1
F	1	1.5000	0.0000	3.5000	0.0000	10.0000
		17.5000	9.0000	5.0000		
F	2	0.0000	2.7600	1.8000 8	1.8000	6.4000
		5.8000	7.3200	14.6000		
F	3	7.0385	0.0000	2.2692	2.1538	12.7692
		0.0000	− 3.1538	11.7692		
F	4	13.0000	4.2000	2.6000	0.0000	1.0000
		0.0000	8.4000	11.0000		
F	5	13.0000	4.2000	2.6000	0.0000	1.0000
		0.0000	2.7600	11.0000		
F	6	5.8000	7.3200	1.8800	1.8000	6.4000
		0.8816	8.4000	14.6000		

* positive ideal and negative ideal values

		Z −	Z *
Z	1	− 5.5000	12.7692
Z	2	− 1.7143	17.5000
Z	3	− 13.2000	11.0000
Z	4	2.0000	14.6000

* LP-metric for the nondominated solutions.

		L 1	L 2	L oo
E	1	1.7522	1.1425	0.0000
E	2	1.1096	0.7180	0.0000
E	3	1.3576	0.8310	0.0000
E	4	1.2762	0.7521	0.1074
E	5	1.2762	0.7521	0.1074
E	6	1.1096	0.7180	0.0000

* best compromise solution for MOLP.

X	1	0.0000
X	2	2.7600
X	3	1.8800
X	4	1.8000

* object functions.

Z	1	6.1000
Z	2	5.8000
Z	3	7.3200
Z	4	14.6000

* row information of constraints.

		G(X)	SK(or SP)	RHS
G	1	24.0000	− 0.0000	24.0000
G	2	11.0000	0.0000	11.0000
G	3	3.0000	− 0 0000	3.0000

computing time : 00:02:38

V. Conclusion

Recently, multiple criteria decisin making has been fairly well established as a practical decision making approach with limited resources, information, and cognitive of the decision maker. Multiple objective linear programming is one of the widely formulated multiple criteria decision making tools that reflect the actual decision making process in real-world situations (11).

Bicriteria linear programming is a linear programming having two conflicting objective functions and arises as a special case of multiple objective linear programming when only two objective functions are of interest.

In this paper, we developed a software package of a bicriteria revised simplex method combined the compromise technique for interactively solving a bicriteria linear programming on the personal computer. A numerical example of the multiple criteria linear programming problem funning on the personal computer was demonstrated by the Micro–BLP.

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國 文 抄 錄

線型計劃(linear programming)은 가장 널리 사용되는 OR, MS, IE技法 중의 하나이다. 最近에 多特性意思決定(multible criteria decision making) 혹은 多目標線型計劃(multiple objective linear programming)은 現實의 問題에 대한 答을 얻기 위해 연구되고 있다.

二目的線型計劃은(bicriteria lineal programning) 目的函數가 2個인 경우의 多目標線型計劃의 特殊한 형태이다. 이 論文에서는 二目的修正심플렉스法을 사용하여 퍼스널컴퓨터로 二目的線型計劃을 둘 수 있는 컴퓨터 패키지를 開發했다.