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The Transverse Conformal Field On The Non-Harmonic Foliations

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<초 록>

비 조화적 엽층구조상를 갖는 리만 다양체 상에서의 횡단적 공형장

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이 논문은 비 조화적 엽층구조를 갖는 리만 다양체 상에서의 횡단적 공형장에 대한 성질을 연구하여 다음의 정리를 증명한다.

(정리)

(M, g_M, F)은 $q\geq2$인 여차원을 갖는 compact인 다양체라 하자. s는 F의 횡단적 공형장이라 하자. 만약 ρ_∇이 M의 모든 곳에서 양이 아니라고 할 때, 평균 곡률을 따라 평행인 모든 s는 ∇-평행이다. 더구나 ρ_∇이 M 상의 모든 곳에서 양이 아니고 어떤 한 점에서 음이라고 할 때, 평균 곡률을 따라 평행인 모든 s는 ∇-평행이다.

<Abstract>

THE TRANSVERSE CONFORMAL FIELDS ON THE NON-HARMONIC FOLIATIONS

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In this thesis we syudy the transverse conformal fields on the non-harmonic foliation and prove the following theorem.

Theorem. Let (M, g_M, F) be the compact manifold with codimension q≥2. Let s be a transverse conformal field of F. If $\rho\nabla$ is non-positive everywhere on M, then every s paralled along the mean curvature vector is ∇-parallel. If $\rho\nabla$ is non-positive everywhere and negative at some point of M, then every s paralled along mean curvature vector is trivial.

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CONTENTS

1. Introduction

The foliation theory has its origin in the global analysis of solution of ordinary differential equations. The general notion of a folation was defined by Ehresmann and Reeb([1]). Over the last forty years the study of foliated manifolds has produced an extraordinarly rich collection of works.

In 1959, Rienhart([1]) introduced a particular type of foliation, namely, Riemannian foliation, which is quite intuitive. This imposes the existence of a " bundle -like" Riemannian metric g_M on M, that is , a metric for which the leaves of the foliation remain locally at constant distance from each other. Actually, the condition for a foliation to be Riemannian is a " transverse property", being given by the existence on the local quotient manifolds of a supplementary geometric structure that is invariant along the leaves.

In the case of a hamonic foliation, geometric transversal fields such as transversal killing, transversal affine, transversal projective, transversal confomal fields have been studied by F. W. Kamber and Ph. Tondeur, and many others. In particular, F. W. Kamber and Ph. Tondeur ([3]) proved the following Theorem A.

Theorem A. *Let $\mathcal{F}$ be a harmonic Riemannian foliation on a compact and oriented manifold M. Assume the transversal Ricci operator ρ_∇ of $\mathcal{F}$ to be ≤ 0 everywhere, and < 0 for at least one point $x \in M$. Then every transverse conformal field of $\mathcal{F}$ automorphism of $\mathcal{F}$ is tangential* *to $\mathcal{F}$.*

In this paper, we study the transverse conformal fields on the non-harmoic foliation and prove the following theorem.

Theorem. *Let $(M, g_M, \mathcal{F})$ be the compact manifold with codimension $q \geq 2$. Let s be a transverse conformal field of $\mathcal{F}$. If ρ_∇ is non-positive everywhere on M, then every s paralled along the mean curvature vector is ∇ - parallel. If ρ_∇ is non-positive everywhere and negative at some point of M, then every s paralled along mean curvature vector is trivial.*

We shall be in C^∞ - category and only with connected and oriented manifolds. We use the following convention on the range of indices :

$$1 \leq i, j \cdots \leq p,$$

$$p + 1 \leq \alpha, \beta \cdots \leq p + q.$$

2. Preliminaries

Let $(M, g_M, \mathcal{F})$ be a $(p+q)$-dimensional Riemannian manifold with a foliation $\mathcal{F}$ of codimension q and a bundle-like metric g_M with respect to $\mathcal{F}$. Let ∇^M be the Levi-Civita connection with respect to g_M. Let TM be the tangent bundle of M and L the integrable subbundle of TM given by $\mathcal{F}$. Let $\pi : TM \longrightarrow Q$ be the natural projection. The normal bundle Q of $\mathcal{F}$ is given by $Q = TM/L$. The metric g_M gives a splitting σ of the exact sequence

$$0 \longrightarrow L \longrightarrow TM \underset{\sigma}{\overset{\pi}{\rightleftarrows}} Q \longrightarrow 0 \tag{2.1}$$

with $\sigma(Q) = L^{\perp}$, where $L^{\perp}$ denotes the orthogonal complement bundle of L in TM with respect to g_M. Let g_M be the holonomy invariant metric on Q induced by g_M, that is,

$$g_Q(s,t) = g_M(\sigma(s), \sigma(t)) \quad \text{for all} \quad s,t \in \Gamma Q.$$

This means that

$$\theta(X)g_Q = 0 \quad \text{for} \quad X \in \Gamma L,$$

where $\theta(X)$ is Lie derivative. A connection ∇ in Q is defined by

$$\begin{aligned} &\nabla_X s = \pi([X, Z_s]), \text{ for } X \in \Gamma L, \ s \in \Gamma Q \text{ with } \pi(Z_s) = s, \\ &\nabla_X s = \pi(\nabla^M_X Z_s), \text{ for } X \in \Gamma L^{\perp}, \ s \in \Gamma Q \text{ with } \pi(Z_s) = s. \end{aligned} \tag{2.2}$$

Let ∇ be any connection in the normal bundle on Q of a foliation. its torsion is the Q-valud 2-form on M defined by

$$T_\nabla(X,Y) = \nabla_X \pi(Y) - \nabla_Y \pi(X) - \pi[X,Y] \tag{2.3}$$

for $X, Y \in \Gamma(TM)$. Thus we have

Proposition 2.1([3]). *The connection ∇ in Q is torsion-free and metrical with respect to g_M.*

The connection ∇ is called the *transversal Riemannian connection* of $\mathcal{F}$. The curvature R_∇ of ∇ is defined by

$$R_\nabla(X,Y)s = \nabla_X \nabla_Y s = \nabla_Y \nabla_X s - \nabla_{[X,Y]} s \tag{2.4}$$

for any $X, Y \in \Gamma TM$ and $s \in \Gamma Q$. Since $i(X)R_\nabla = 0$, where $i(X)$ denotes the interior product with respect to $X \in \Gamma L$([1]). Then we have the following fact.

Proposition 2.2([3]). *For any $\mu, v \in \Gamma Q$, the opreator $R_\nabla : Q \longrightarrow Q$ is a well-defined endomorphism.*

Let $x \in M$ and $\sigma \subset Q$, a 2-plane in the normal bundle spanned by two normal vector μ_x, v_x. Then the sectional curvature of $(\mathcal{F}, g_Q)$ at x in directions of σ is defined by

$$K_\nabla(\sigma) = g_Q(R_\nabla(\mu_x, v_x)v_x, \mu_x)/g_Q(\mu_x,\mu_x)g_Q(v_x,v_x) - g_Q(\mu_x, v_x)^2.$$

The Ricci curvature ρ_∇ is defined by

$$(\rho_\nabla \mu)_x = \sum_{\alpha=p+1}^{n} R_\nabla(\mu, e_\alpha)e_\alpha, \tag{2.5}$$

where $\{e_\alpha\}_{\alpha=p+1,\cdots,n}$ is an orthonormal basis of Q_x. And the scalar curvature σ_∇ finally is given by

$$\sigma_\nabla = Trace\rho_\nabla.$$

All these geometric quantities should be thought of as the corresponding curvature properties of a Riemannian manifold serving as model space for $\mathcal{F}$.

3. Infinitesimal automorphisms

Let $\mathcal{F}$ be an arbitary foliation on M. A vector field $Y \in \Gamma TM$ is an *infinitismal automorphism* of $\mathcal{F}$ if $[Y, Z] \in \Gamma L$ for all $Z \in \Gamma L$, where Y is preserves the foliation, i.e, maps leaves into leaves. Let $V(\mathcal{F})$ be the space of all vector field Y on M satisfying $[Y, Z] \in \Gamma L$ for all $Z \in \Gamma L$.

A *transversal infinitesimal automorphim* s of $\mathcal{F}$ is an element of the set

$$(3.1) \qquad \bar{V}(\mathcal{F}) = \{s \in \Gamma Q | s = \pi Y,\ Y \in V(\mathcal{F})\}.$$

Lemma 3.1([4]). *An element s of $\bar{V}(\mathcal{F})$ satisfies $\nabla_X s = 0$ for all $X \in \Gamma L$.*

The *transversal Lie derivative* $\theta(Y)$ with respect to $Y \in V(\mathcal{F})$ is defined by

$$(3.2) \qquad \theta(Y)s = \pi([Y, Y_s]) \text{ for all } s \in \Gamma Q \text{ with } \pi(Y_s) = s.$$

Definition 3.2. *If $Y \in V(\mathcal{F})$ satisfies $\theta(Y)g_Q = 0$, then $s = \pi(Y)$ is called a transversal killing field $\mathcal{F}$.*

Definition 3.3. *If $Y \in V(\mathcal{F})$ satisfies $\theta(Y)g_Q = 2f_Y g_Q$, where f_Y is called a function on M, then $s = \pi(Y)$ is called a transversal conformal fields of $\mathcal{F}$ and f_Y is called the characteristic function of s.*

Definition 3.4. *If $Y \in V(\mathcal{F})$ satisfies $\theta(Y)\nabla = 0$, then $s = \pi(Y)$ is called a transverse affine field of $\mathcal{F}$.*

If g_M is a bundle-like metric on M and $Y \in \Gamma TM$ a conformal vector field (i.e. $\theta(Y)g_M = f_Y g_M$ for some function f_Y on (M, g_M)), then $\pi(Y)$ is transversal conformal field for g_M. In fact, $\theta(Y)g_M(s,t) = \theta(Y)g_Q(s,t)$ for all $s, t \in \Gamma(Q)$. But the converse is not necessarily true: $Y \in V(\mathcal{F})$ may satisfy $\theta(Y)g_Q = f_Y g_Q$ without satisfying $\theta(Y)g_M = \tilde{f}_Y g_M$, where f_Y and $\tilde{f}_Y$ are some funtions on M. For the relation of Killing fields, the following is well known([2]).

Proposition 3.5([2]). *Let $\pi(Y)$ be the transversal Killing field on M. Then Y is a Killing field on M if and only if*

$$g_M(\nabla^M_Z Y, W) + g_M(\nabla^M_W Y, Z) = 0 \tag{3.3}$$

for any $Z, W \in \Gamma L$.

Now, we study the relations of conformal vector fields. First we calculate $(\theta(Y)g_M)(Z,W)$ for any $Z, W \in TM$. By properties of $\theta(Y)$, we have

(3.4)

$$\begin{aligned}
&(\theta(Y)g_M)(Z,W)\\
&\qquad = Yg_M(Z,W) - g_M(\theta(Y)Z, W) - g_M(Z, \theta(Y)W)\\
&\qquad = (\theta(Y)g_L)(\pi^\perp Z, \pi^\perp W) + (\theta(Y)g_Q)(\pi Z, \pi W)\\
&\qquad - g_M(\theta(Y)\pi Z, \pi^\perp W) - g_M(\theta(Y)\pi W, \pi^\perp Z),
\end{aligned}$$

where $g_M = g_L + g_Q$ and $\pi^\perp : TM \longrightarrow L$ is the projection. Since $\theta(Y)s = \pi[Y, Y_s]$, $\pi(Y_s) = s$ for $s \in Q$, the last two terms on the above equation are zero. Also, by long calculation, we get

(3.5) $(\theta(Y)g_L)(\pi^\perp Z, \pi^\perp W) = g_L(\nabla^M_{\pi^\perp Z}Y, \pi^\perp W) + g_L(\pi^\perp Z, \nabla^M_{\pi^\perp W}Y).$

Hence we have

(3.6)
$$(\theta(Y)g_M)(Z,W) = (\theta(Y)g_Q)(\pi Z, \pi W) + g_M(\nabla^M_{\pi^\perp Z}Y, \pi^\perp W) + g_M(\pi^\perp Z, \nabla^M_{\pi^\perp W}Y).$$

From this equality, we have

Proposition 3.6. *Let $\pi(Y)$ be the transversal conformal vector field on M. If the infinitesimal automorphism Y satisfies*

(3.7) $$g_M(\nabla^M_Z Y, W) + g_M(\nabla^M_W Y, Z) = 0 \text{ for any } Z, W \in \Gamma L,$$

then Y is a conformal vetor field on M.

Corollary 3.7. *Let $\pi(Y)$ be the transversal conformal vetor field. If Y is parallel along the leaves, then Y is a conformal vetor field.*

Since the Riemannian foliation can be considered as Riemannian submersion locally, we can introduce the following tensors ([4]):

(3.8)
$$A_X Y = \pi\nabla^M_{\pi X}\pi^\perp Y + \pi^\perp\nabla^M_{\pi X}\pi Y$$
$$T_X Y = \pi\nabla^M_{\pi^\perp X}\pi^\perp Y + \pi^\perp\nabla^M_{\pi^\perp X}\pi Y$$

for any vector field $X, Y \in TM$. And the following properties hold:

$$A_X = 0 \quad \text{and} \quad A_U V = -A_V U$$
(3.9)
$$T_U = 0 \quad \text{and} \quad T_X Y = T_Y X$$

for any $X, Y \in \Gamma L$ and $U, V \in \Gamma Q$. The Riemannian foliation is said to be totally geodesic if all the leaved are *totally geodesic* submanifolds, that is, $T = 0$. Moreover the normal bundle $L^{\perp} \equiv Q$ is ***integrable*** if and only if $A = 0$ (in this case the integral submanifolds of $L^{\perp}$ are totally geodesic). By these properties of A and T, we have

$$(3.10)\quad g_M(\nabla^M_{\pi^{\perp} Z} Y, \pi^{\perp} W) = g_M(T_Z Y, \pi^{\perp} W) + g_M(\nabla^M_{\pi^{\perp} Z} \pi^{\perp} Y, \pi^{\perp} W).$$

From this equation, we have

Corollary 3.8. *Let $\mathcal{F}$ be the totally geodesic foliation. Let $\pi(Y)$ be the transversal conformal vector field of M. If the tangential part of the infinitesimal automorphism Y is parallel along the leaves, then Y is a conformal vector fiels of M.*

4. Transversal conformal fields

Let $\Omega^r(M,Q) \cong \Gamma(Q) \otimes \Omega^r(M)$ be the space of Q- valud r-forms over M, where $\Omega^r(M)$ is a space of differential r-forms on M. For any $s \in \Gamma(Q)$ and $\eta \in \Omega^r(M)$, the element $s \otimes \eta \in \Omega^r(M,Q)$ is usually abbreviated to $s\eta$. We can consider the connection ∇ given in (2.2) as an R-Liner map $\nabla : \Omega^0(M,Q) \to \Omega^1(M,Q)$ such that

$$\nabla(fs) = f\nabla s + sdf \tag{4.1}$$

for any $f \in \Omega^0(M), s \in \Gamma(Q)$ and such that

$$d < s_1, s_2 >= \nabla s_1 \wedge s_2 + s_1 \wedge \nabla s_2 \tag{4.2}$$

for any $s_1, s_2 \in \Omega^0(M,Q)$, where we define

$$s_1\eta_1 \wedge s_2\eta_2 =< s_1, s_2 > \eta_1 \wedge \eta_2$$

for any $s_1\eta_1 \in \Omega^r(M,Q)$ and $s_2\eta_2 \in \Omega^s(M,Q)$. By the usual algebraic formalism, $\nabla : \Omega^0(M,Q) \to \Omega^1(M,Q)$ can be extended to an anti-derivation

$$d_\nabla : \Omega^r(M,Q) \to \Omega^{r+1}(M,Q)$$

by the following rule : if $s\eta \in \Omega^r(M,Q)$, then

$$d_\nabla(s\eta) = \nabla s \wedge \eta + s(d\eta) \tag{4.3}$$

for $s \in \Gamma(Q), \eta \in \Omega^r(M,Q)$. For a Riemannian metric g_M on M, we extend the star operator $* : \Omega^r(M) \to \Omega^{n-r}(M)(n = dim\mathrm{M})$ to

$$* : \Omega^r(M,Q) \to \Omega^{n-r}(M,Q)$$

as follows : if $s \in \Gamma(Q)$ and $\eta \in \Omega^r(M)$, then

(4.4) $$*(s\eta) = s(*\eta).$$

Moreover the operator $d_\nabla^* : \Omega^r(M,Q) \to \Omega^{r-1}(M,Q)$ given by

(4.5) $$d_\nabla^*\phi = (-1)^{n(r+1)+1} * d_\nabla * \phi, \quad \phi \in \Omega^r(M,Q)$$

is adjoint of d_∇ with respect to an inner product $< \cdot,\cdot >$ defined by

(4.6) $$< s_1\eta_1, s_2\eta_2 >= g_Q(s_1,s_2)(\eta_1,\eta_2).$$

The Laplacian Δ for $\Omega^*(M,Q)$ is given by

(4.7) $$\Delta = d_\nabla d_\nabla^* + d_\nabla^* d_\nabla.$$

Let $e_1,\cdots,e_n$ be orthonomal basis of T_xM and $E_1,\cdots,E_n$ a local framing of TM in a neighborhood of x, coinciding with $e_1,\cdots,e_n$ at x and satisfying $\nabla^M_{e_A}E_B = (\nabla^M_{E_A}E_B)_x = 0 \quad (A,B=1,\cdots,n)$, where ∇^M denotes the Riemannian connection of (M,g_M). Let w^A be a coframe field of e_A. Then on $\Omega^*(M,Q)$, we have

(4.8) $$d_\nabla = \sum w^A \wedge \tilde{\nabla}_{e_A}, \quad d_\nabla^* = -\sum i(e_a)\tilde{\nabla}_{e_A},$$

where $\tilde{\nabla}$ is a connection on $\Omega^*(M,Q)$ defined as

$$\tilde{\nabla}_X(s\eta) = (\nabla_X s)\eta + s(\nabla^M_X\eta)$$

and

$$i(X)(s\eta) = s[i(X)\eta]$$

for $s \in \Gamma(L), \eta \in A^*(M)$. The Q-valued bilinear form α on M is defined by

$$\alpha(X,Y) = -(\tilde{\nabla}_X \pi)(Y) \tag{4.9}$$

for all $X, Y \in \Gamma TM$([9]). Since $\alpha(X,Y) = \pi(\nabla^M_X Y)$ for all $X, Y \in \Gamma L$, we call α the *second fundamental form* of $\mathcal{F}$([9]). The tension field τ of $\mathcal{F}$ is defined by

$$\tau = \sum_{i=1}^{p} \alpha(E_i, E_i), \tag{4.10}$$

where $\{E_i\}_{i=1,\cdots,p}$ is an orthonormal basis of L. We remark that $\tau = d^*_\nabla \pi \in \Gamma Q$([9]). The foliation $\mathcal{F}$ is *minimal* (or *harmonic*) if $\tau = 0$ ([9]). For $Y \in V(\mathcal{F})$, we define an operator $A_\nabla(Y) : \Gamma(Q) \to \Gamma(Q)$ by

$$A_\nabla(Y)s = \theta(Y)s - \nabla_Y s. \tag{4.11}$$

Then we have

$$A_\nabla(Y)s = -\nabla_{Y_s}\pi(Y), \tag{4.12}$$

for $s = \pi(Y_s)$. This shows that (i) $A_\nabla(Y)$ depends only on $s = \pi(Y)$, (ii) $A_\nabla(Y)$ is a linear operator of $\Gamma(Q)$. Thus we can use $A_\nabla(s)$ instead of $A_\nabla(Y)$([4]).

Proposition 4.1([7]). *For $Y \in V(\mathcal{F})$, it holds that*

$$(\theta(Y)\nabla)_{Y_s} t = R_\nabla(\pi(Y), s)t - (\nabla_{Y_s} A_\nabla(\pi(Y))t$$

for any $s, t \in \Gamma(Q)$ with $Y_s = \sigma(s)$.

Proposition 4.2 ([7]). *If $s \in \bar{V}(\mathcal{F})$, then it holds that*

$$\Delta s = d^*_\nabla d_\nabla s = \nabla_\tau s + \sum_{\alpha=p+1}^{n} (\nabla_{E_\alpha} A_\nabla(s))E_\alpha$$

Theorem 4.3([7]). *If $s \in \bar{V}(\mathcal{F})$ is a transversal conformal field of $\mathcal{F}$, then we have*

$$\Delta s = \nabla_\tau s + \rho_\nabla(s) + (1 - \frac{2}{q})grad(div_\nabla s),$$

where $div_\nabla s = g_Q(\nabla_{E_\alpha} s, E_\alpha)$.

Let $B_\nabla(s) : \Gamma(Q) \to \Gamma(Q)$ $(s \in \bar{V}(\mathcal{F}))$ be an operator defined by

$$B_\nabla(s) = A_\nabla(s) +^t A_\nabla(s) + \frac{2}{q}(div_\nabla s)I, \tag{4.13}$$

where I denotes the identity map of ΓQ. Note that the operator $B_\nabla(s)$ is symmetric.

Proposition 4.4 ([7]). *A transversal infintisimal antomorphism s of $\mathcal{F}$ is a transversal conformal field of $\mathcal{F}$ if and only if $B_\nabla(s) = 0$.*

Theorem 4.5 ([12]). *Suppose that M is compact. It holds that*

$$\int_M div_\nabla s dM = \ll \tau, s \gg$$

for any $s \in \Gamma Q$.

Theorem 4.6 ([12]). *Suppose that M is compact. It holds that*

$$\ll \Delta s, t \gg = \ll \nabla s, \nabla t \gg$$

for any $s, t \in \bar{V}(\mathcal{F})$, where $\ll \nabla s, \nabla t \gg = \int_M g_Q(\nabla_{E_\alpha} s, \nabla_{E_\alpha} t)dM$.

Proposition 4.7 ([7]). *For all $s \in \tilde{V}(\mathcal{F})$, it holds that*

$$(i) Ric_{\nabla}(s) + TrA_{\nabla}(s)A_{\nabla}(s) - (div_{\nabla})^2 + div_{\nabla}(A_{\nabla}(s)s) + div_{\nabla}(div_{\nabla}s)s = 0.$$

$$(ii) TrA_{\nabla}(s)A_{\nabla}(s) = -Tr^tA_{\nabla}(s)A_{\nabla}(s) + \frac{1}{2}Tr(A_{\nabla}(s) + ^t A_{\nabla}(s))^2,$$

where TrC denotes the trace of an operator $C : \Gamma(Q) \to \Gamma(Q)$ with respect to g_Q, and $^tA_{\nabla}(s)$ denotes the transposed operator of $A_{\nabla}(s)$ with respect to g_Q.

Theorem 4.8 ([8]). *Let $(M, g_M, \mathcal{F})$ be a closd, oriented, connected Riemannian manifold of dimension $p+q$ with a transversally oriented foliation $\mathcal{F}$ of codimension $q \geq 2$ and a bundle-like metric g_M with respect to $\mathcal{F}$. Let s be a transversal infinitesimal automorphism of $\mathcal{F}$. Then s is a transversal conformal field of $\mathcal{F}$ if and only if s satisfies*

$$\Delta s = \nabla_T s + \rho_{\nabla}(s) + (1 - \frac{1}{q}) grad(div_{\nabla} s).$$

By the direct calculation, we have

$$g_Q(grad(div_{\nabla} div_{\nabla} s, s) = \sigma(s)(div_{\nabla} s) \tag{4.14}$$

,

$$div_{\nabla}((div_{\nabla} s)s) = \sigma(s)(div_{\nabla} s) + (div_{\nabla} s)^2 \tag{4.15}$$

for any $s \in \Gamma Q$. From Theorem 4.8, (4.14) and (4.15), we have

$$g_Q(\Delta s, s) = g_Q(\nabla_T s, s) + g_Q(\rho_{\nabla}(s), s) + (1 - \frac{2}{q})\{div_{\nabla}(div_{\nabla} s)s - (div_{\nabla} s)^2\}.$$

From Proposition 4.7, the above equation becomes

$$(4.16)\quad g_Q(\nabla s, s) = (2-\frac{2}{q})g_Q(\nabla_\tau s, s) + \frac{2}{q}g_Q(\rho_\nabla(s), s) + (1-\frac{2}{q})g_Q(\nabla s, \nabla s) - \frac{1}{2}(1-\frac{2}{q})TrB_\nabla(s)^2 - \frac{1}{2}(1-\frac{2}{q})\frac{4}{q}(2div_\nabla s)^2$$

By integrating (4.16) and using Theorem 4.6, we get

(4.17)

$$\ll \nabla s, \nabla s \gg = (q-1) \ll \nabla_\tau s, s \gg + \ll \rho(s), s \gg - \frac{q-2}{4}\int_M TrB_\nabla(s)^2 dM - \frac{q-2}{4q}\int_M (div_\nabla s)^2 dM.$$

The Ricci operator ρ_∇ of $\mathcal{F}$ is non-positive (rest. negative) at $x \in M$ if $g_Q(\rho_\nabla(s), s)_x \le 0$ (resp.< 0) for any $s \in \Gamma(Q)$ (resp. $s(x) \neq 0$). If ρ_∇ is non-positive everywhere on M, then we have $\ll \rho_\nabla(s), s \gg \le 0$ for any $s \in \Gamma(Q)$. If $s \in \Gamma(Q)$ satisfies $\nabla s = 0$, that is , $\nabla_x s = 0$ for any $X \in \Gamma(TM)$, then s is called ∇ - *parallel*. From Proposition 4.4 and (4.17), we have the following theorem.

Therorem 4.9. *Let $(M, g_M, \mathcal{F})$ be the compact manifold with codimension $q \ge 2$. Let s be a transverse conformal field of $\mathcal{F}$. If ρ_∇ is non-positive everywhere on M ,then every s paralled along the mean curvature vector is ∇-parallel. If ρ_∇ is non-positive everywhere and negative at some point of M, then every s paralled along mean curvature vector is trivial.*

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<초 록>

비 조화적 엽층구조상를 갖는 리만 다양체 상에서의 횡단적 공형장

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이 논문은 비 조화적 엽층구조를 갖는 리만 다양체 상에서의 횡단적 공형장에 대한 성질을 연구하여 다음의 정리를 증명한다.

(정리)

(M, g_M, F)은 $q\geq 2$인 여차원을 갖는 compact인 다양체라 하자. s는 F의 횡단적 공형장이라 하자. 만약 ρ_∇이 M의 모든 곳에서 양이 아니라고 할 때, 평균 곡률을 따라 평행인 모든 s는 ∇-평행이다. 더구나 ρ_∇이 M 상의 모든 곳에서 양이 아니고 어떤 한 점에서 음이라고 할 때, 평균 곡률을 따라 평행인 모든 s는 ∇-평행이다.

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감사의 글

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세월의 흐름 속에 벌써 성장하여 출가를 한 딸의 장래를 늘 걱정하고 계시는 부모님, 한 집안의 며느리로서의 역할도 제대로 하지 못하는 저에게 언제나 너그러운 말씀과 도움을 주셨던 시부모님, 그리고 집안 어르신들께 이 지면을 빌어 깊은 감사를 드립니다. 그리고 결혼을 해서도 객지에 떨어져 살아야 했던 사랑하는 저의 남편에게도 감사와 기쁨의 소식을 전하고 싶습니다. 이 논문을 완성하기까지 도움을 주신 모든 분들에게 행복이 가득하시기를 빕니다.

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