碩士學位請求論文

THE VOLUME OF A PARAMETRIZED 3-SURFACE UNDER INVERSION

指導教授　玄　進　五

濟州大學校 教育大學院

數學教育專攻

安　成　意

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指導教授　玄　進　五

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提出者　安　成　意

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審 查 委 員　　　　印

審 查 委 員　　　　印

⟨*Abstract*⟩

THE VOLUME OF A PARAMETRIZED 3-SURFACE UNDER INVERSION

Ahn, Seong-Eui

Mathematics Education Major

Graduate School of Education, Cheju National University

Cheju, Korea

Supervised by professor Hyun, Jin-Oh

A mapping $f : E^3 - \{(0,0,0)\} \longrightarrow E^3$ which sends a point P into a point P' is called an inversion in an Euclidean space E^3 with respect to a given circle or sphere with center O and radius R, if $OP \cdot OP' = R^2$ and if the points P, P' are on the same side of O and O, P, P' are collinear.

This thesis shows that, for a parametrized 3-surface in E^3 is given by $X(u_1, u_2, u_3) = \Big(x(u_1, u_2, u_3), y(u_1, u_2, u_3), z(u_1, u_2, u_3) \Big)$, the volume of $f(X)$ is equal to $R^6 \int_U \frac{1}{|X|^6} \sqrt{g}\, du_1\, du_2\, du_3$,where $\sqrt{g}$ is the absolute value of Jacobian matrix of x, y, z with respect to u_1, u_2, u_3.

CONTENTS

Introduction

In this paper, we study the volume of the parametrized 3-surface in Euclidean space E^3.

In section 1, we present the basic concepts of a parametrized 3-surface in E^3 and a natural instrument to treat the volume of a parametrized 3-surface in E^3. And we also show how to find the volume of a parametrized 3-surface.

In section 2, we introduce the definition and some properties of inversion in E^3 and show that $f(X) : U \longrightarrow E^3$ is a parametrized 3-surface, and $\sqrt{\hat{g}} = \dfrac{R^6}{|X|^6}\sqrt{g}$.

Finally, in section 3, we show the volume of $f(X)$ under inversion is equal to $R^6 \displaystyle\int_U \frac{1}{|X|^6}\sqrt{g}\,du_1\,du_2\,du_3$, and give the example for the above theorem.

1. The volume of a parametrized 3-surface

In this section, we introduce the basic concepts of a parametrized 3-surface in E^3. And we define the volume of a parametrized 3-surface.

Definition 1.1 A parametrized 3-surface is a smooth map $X : U \longrightarrow E^3$ which is regular, where $U \subset E^3$ is open.

If we write $X(u_1, u_2, u_3) = (x(u_1, u_2, u_3), y(u_1, u_2, u_3), z(u_1, u_2, u_3))$ for any $(u_1, u_2, u_3) \in U \subset E^3$, then X is smooth if and only if $x(u_1, u_2, u_3)$, $y(u_1, u_2, u_3)$ and $z(u_1, u_2, u_3)$ have continuous partial derivatives of all orders in U. Regular condition means that dX_q is non-singular (has rank 3) for each $q \in U$. Let us compute the matrix of the linear map dX_q with respect to the canonical bases $e_1 = (1, 0, 0), e_2 = (0, 1, 0)$ and $e_3 = (0, 0, 1)$ of E^3 with coordinates (u_1, u_2, u_3) and $i_1 = (1, 0, 0), i_2 = (0, 1, 0)$ and $i_3 = (0, 0, 1)$ of E^3 with coordinates (x,y,z).

By the definition of the differential, we have

$$dX_q(e_1) = (\frac{\partial x}{\partial u_1}, \frac{\partial y}{\partial u_1}, \frac{\partial z}{\partial u_1}) = \frac{\partial X}{\partial u_1} = X_{u_1}, \tag{1.1}$$

$$dX_q(e_2) = (\frac{\partial x}{\partial u_2}, \frac{\partial y}{\partial u_2}, \frac{\partial z}{\partial u_2}) = \frac{\partial X}{\partial u_2} = X_{u_2}, \tag{1.2}$$

$$dX_q(e_3) = (\frac{\partial x}{\partial u_3}, \frac{\partial y}{\partial u_3}, \frac{\partial z}{\partial u_3}) = \frac{\partial X}{\partial u_3} = X_{u_3}. \tag{1.3}$$

Regular condition implies that for each $q \in U$,

$$\frac{\partial X}{\partial u_1} \cdot \frac{\partial X}{\partial u_2} \times \frac{\partial X}{\partial u_3} = \begin{vmatrix} \frac{\partial x}{\partial u_1} & \frac{\partial y}{\partial u_1} & \frac{\partial z}{\partial u_1} \\ \frac{\partial x}{\partial u_2} & \frac{\partial y}{\partial u_2} & \frac{\partial z}{\partial u_2} \\ \frac{\partial x}{\partial u_3} & \frac{\partial y}{\partial u_3} & \frac{\partial z}{\partial u_3} \end{vmatrix} \neq 0. \tag{1.4}$$

The mapping X is called a parametrization or a system of local coordinates in a neighborhood of $p \in U$.

Example 1.2 Let $X : U \longrightarrow E^3$ be defined by

$X(r, \theta, \phi) = (r\cos\theta\sin\phi, r\sin\theta\sin\phi, r\cos\phi)$,

where $U = \{(r, \theta, \phi) \,|\, 0 < r < a, 0 < \theta < 2\pi, 0 < \phi < \pi\}$.

Then $X : U \longrightarrow E^3$ is a parametrized 3-surface

and the image $X(U) = B \backslash \{(x_1, x_2, x_3) \in B \,|\, x_1 \geq 0, x_2 = 0\}$,

where $B = \{(x_1, x_2, x_3) \,|\, x_1^2 + x_2^2 + x_3^2 < a^2\}$.

Since $x(r,\theta,\phi) = r\cos\theta\sin\phi$, $y(r,\theta,\phi) = r\sin\theta\sin\phi$ and $z(r,\theta,\phi) = r\cos\phi$ have continuous partial derivatives of all orders in U, X is smooth.

Moreover, since

$$E_1(p) = \frac{\partial X}{\partial r} = (\cos\theta\sin\phi, \sin\theta\sin\phi, \cos\phi),$$
$$E_2(p) = \frac{\partial X}{\partial \theta} = (-r\sin\theta\sin\phi, r\cos\theta\sin\phi, 0) \text{ and}$$
$$E_3(p) = \frac{\partial X}{\partial \phi} = (r\cos\theta\cos\phi, r\sin\theta\cos\phi, -r\sin\phi) \text{ for any } p = (r,\theta,\phi) \in U,$$

we have $\left|\frac{\partial X}{\partial r}\cdot\frac{\partial X}{\partial \theta}\times\frac{\partial X}{\partial \phi}\right| \neq 0$. Hence the regular condition is satisfied.

Example 1.3 Let $X : U \to E^3$ be defined by $X(\rho,\phi,t) = (\rho\cos\phi, \rho\sin\phi, t)$, where $U = \{(\rho,\phi,t) \mid \rho \geq 0, 0 \leq \phi < 2\pi, -\infty < t < \infty\}$.

Then X is parametrized 3-surface.

Since $x(\rho,\phi,t) = \rho\cos\phi$, $y(\rho,\phi,t) = \rho\sin\phi$ and $z(\rho,\phi,t) = t$ have continuous partial derivatives of all orders in U, X is smooth.

Moreover, since

$$E_1(p) = \frac{\partial X}{\partial \rho} = (\cos\phi, \sin\phi, 0),$$
$$E_2(p) = \frac{\partial X}{\partial \phi} = (-\rho\sin\phi, \rho\cos\phi, 0) \text{ and}$$
$$E_3(p) = \frac{\partial X}{\partial t} = (0,0,1) \qquad \text{for any} \quad p = (\rho,\phi,t) \in U,$$

we have $\left|\frac{\partial X}{\partial \rho} \cdot \frac{\partial X}{\partial \phi} \times \frac{\partial X}{\partial t}\right| \neq 0$. Hence the regular condition is satisfied.

Definition 1.4 Let $X : U \longrightarrow E^3$ be a parametrized 3-surface where $U \in E^3$ is open.

Then the volume of a parametrized 3-surface X denoted by $V(X)$ is defined by

$$V(X) = \int_U \left|\frac{\partial X}{\partial u_1} \cdot \frac{\partial X}{\partial u_2} \times \frac{\partial X}{\partial u_3}\right| du_1\, du_2\, du_3, \tag{1.5}$$

where (u_1, u_2, u_3) is a local coordinate system on U.

The function $|X_{u_1} \cdot X_{u_2} \times X_{u_3}|$ defined in U, measures the volume of a parallelepiped generated by the vectors $X_{u_1}, X_{u_2}, X_{u_3}$.

Proposition 1.5 Let $X : U \longrightarrow E^3$ be a parametrized 3-surface and let $g_{ij} = X_{u_i} \cdot X_{u_j} = \frac{\partial x}{\partial u_i} \cdot \frac{\partial x}{\partial u_j} + \frac{\partial y}{\partial u_i} \cdot \frac{\partial y}{\partial u_j} + \frac{\partial z}{\partial u_i} \cdot \frac{\partial z}{\partial u_j}$. Then

$$V(X) = \int_U \sqrt{g}\, du_1 du_2 du_3, \tag{1.6}$$

where $g = \left|\det \begin{pmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ g_{31} & g_{32} & g_{33} \end{pmatrix}\right|.$

proof.

$$
\begin{aligned}
\left|\frac{\partial X}{\partial u_1}\cdot\frac{\partial X}{\partial u_2}\times\frac{\partial x}{\partial u_3}\right|^2 &= \begin{vmatrix} \frac{\partial x}{\partial u_1} & \frac{\partial y}{\partial u_1} & \frac{\partial z}{\partial u_1} \\ \frac{\partial x}{\partial u_2} & \frac{\partial y}{\partial u_2} & \frac{\partial z}{\partial u_2} \\ \frac{\partial x}{\partial u_3} & \frac{\partial y}{\partial u_3} & \frac{\partial z}{\partial u_3} \end{vmatrix}^2 \\
&= \begin{vmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ g_{31} & g_{32} & g_{33} \end{vmatrix} \\
&= g.
\end{aligned}
$$

Thus $\left|\frac{\partial X}{\partial u_1}\cdot\frac{\partial X}{\partial u_2}\times\frac{\partial X}{\partial u_3}\right| = \sqrt{g}.$

Note that $\sqrt{g}$ is the absolute value of the Jacobian of x,y,z with respect to u_1, u_2, u_3.

Corollary 1.6 The parametrization X has the regularity condition iff $\sqrt{g}$ is never zero, that is $\sqrt{g} > 0$.

Example 1.7 Let $X : U \longrightarrow E^3$ be the parametrization in the example 1.2. Then

$$
g = \begin{vmatrix} 1 & 0 & 0 \\ 0 & r^2\sin^2\phi & 0 \\ 0 & 0 & r^2 \end{vmatrix} = r^4\sin^2\phi.
$$

Hence we get

$$\begin{aligned} V(X) &= \int_0^{\pi}\int_0^{2\pi}\int_0^{a} \sqrt{g}\, dr\, d\theta\, d\phi \\ &= \int_0^{\pi}\int_o^{2\pi}\int_0^{a} r^2 \sin\phi\, dr\, d\theta\, d\phi \\ &= \frac{4}{3}\pi a^3. \end{aligned}$$

Example 1.8 Let $X : U \to E^3$ be defined by $X(\rho, \phi, t) = (\rho\cos\phi, \rho\sin\phi, t)$,

where $U = \{(\rho, \phi, t)\,|\, 0 < \rho < a,\ 0 < \phi < 2\pi,\ 0 < t < b, \quad a, b > 0\}$.

By using the results of example 1.3, we have

$$g = \begin{vmatrix} 1 & 0 & 0 \\ 0 & \rho^2 & 0 \\ 0 & 0 & 1 \end{vmatrix} = \rho^2.$$

Hence we get

$$\begin{aligned} V(X) &= \int_0^{b}\int_0^{2\pi}\int_0^{a} \sqrt{g}\, d\rho\, d\phi\, dt \\ &= \int_0^{b}\int_0^{2\pi}\int_0^{a} \rho\, d\rho\, d\phi\, dt \\ &= \pi a^2 b. \end{aligned}$$

2. Definition and some properties of an inversion

In this section, we define an inversion in E^3 and study some properties of an inversion.

Let the symbol $(O)_R$ denote the sphere with center O and radius R

Definition 2.1 Two points P and P' of E^3 are said to be inverse with respect to a given sphere $(O)_R$ if

$$OP \cdot OP' = R^2. \tag{2.1}$$

and if P, P' are on the same side of O and the points O, P, P' are collinear.

A $(O)_R$ is called the sphere of inversion, and the transformation which sends a point P into P' is called an inversion.

Note that the center O of the sphere of inversion has no inverse point.

From now on, we take the center O as an origin in E^3, and denote the distance from O to a point X by $|X|$.

Then we have the following properties.

Proposition 2.2 An inversion in a space E^3 is a mapping $f : E^3 - \{(0,0,0)\} \longrightarrow E^3$ such that

$$f(X) = \frac{R^2 X}{< X, X >} = \frac{R^2 X}{|X|^2}, \tag{2.2}$$

where $< X, X >= X \cdot X$ is the dot product.

proof. Since f is an inversion and O, X and $f(X)$ are collinear.

Hence $f(X) = kX$ for some positive real number k.

Since $f(X)$ is inverse point of X, by means of (2.1),

$$|X||f(X)| = R^2.$$

$$k|X|^2 = R^2.$$

Since $|X| \neq 0$, we have

$$k = \frac{R^2}{|X|^2}.$$

The inverse point $f(X) = \dfrac{R^2 X}{|X|^2}$ is the vector of length $R^2|X|^{-1}$ on the ray of X.

Theorem 2.3

(1) A plane through O inverts into a plane through O.

(2) A plane not through O inverts into a sphere through O.

(3) A sphere through O inverts into a plane not through O.

(4) A sphere not through O inverts into a sphere not through O.

proof. Let B be any nonzero constant vector in E^3, and consider the equation

$$a|X|^2 + < B, X > + c = 0, \tag{2.3}$$

where $a, c \in R$.

Then the equation (2.3) represents a sphere for $a \neq 0$ and a plane for $a = 0$.

For $|X| \neq 0$, multiplying both sides of (2.3) by $\dfrac{R^2}{|X|^2}$, we have

$$R^2 a + \frac{R^2 < B, X >}{|X|^2} + \frac{R^2 c}{|X|^2} = 0. \tag{2.4}$$

Let $Y = \dfrac{R^2 X}{|X|^2}$. Then we have

$$\frac{c}{R^2}|Y|^2 + < B, Y > + R^2 a = 0. \tag{2.5}$$

Thus (2.3) is transformed into (2.5) under inversion. Hence we get:

(1) When $a = 0, c = 0$, (2.3) and (2.5) represent a plane through O.

(2) When $a = 0, c \neq 0$, (2.3) represents a plane not through O and (2.5)

represents a sphere through O.

(3) When $a \neq 0, c = 0$, (2.3) represents a sphere through O and (2.5) represents a plane not through O.

(4) When $a \neq 0, c \neq 0$, (2.3) and (2.5) represents a sphere not through O.

Define $f \circ X : U \longrightarrow E^3$ by $(f \circ X)(u_1, u_2, u_3) = \frac{R^2 X}{|X|^2}$, where $(u_1, u_2, u_3) \in U$ and $X = (x(u_1, u_2, u_3), y(u_1, u_2, u_3), z(u_1, u_2, u_3))$.

Theorem 2.4 Let $X : U \longrightarrow E^3 - \{(0,0,0)\}$ be a parametrized 3-surface for $U \subset E^3$ and $f : E^3 - \{(0,0,0)\} \longrightarrow E^3$ be an inversion.

Then $f(X) = f \circ X$ is a parametrized 3-surface.

proof. Since X is a parametrized 3-surface and $f \circ X = f(X) = \frac{R^2 X}{|X|^2}$, $f(X)$ is smooth and regular.

Hence $f(X)$ is a parametrized 3-surface.

Theorem 2.5 Let $f(X) : U \longrightarrow E^3$ be an inversion of a parametrized 3-surface X. Then

$$\widehat{g_{ij}} = < \frac{\partial f(X)}{\partial u_i}, \frac{\partial f(X)}{\partial u_j} > = \frac{R^4}{|X|^4} g_{ij}, \tag{2.6}$$

where $g_{ij} = < \frac{\partial X}{\partial u_i}, \frac{\partial X}{\partial u_j} >$.

proof. Since f is an inversion, from (2.2), we have $f(X) = \frac{R^2}{|X|^2} X$.

By the equation $\frac{\partial f(X)}{\partial u_i} = \frac{R^2}{|X|^2}\frac{\partial X}{\partial u_i} - \frac{2R^2}{|X|^3}\frac{\partial |X|}{\partial u_i} X$,

$$\begin{aligned}
\widehat{g_{ij}} &= < \frac{\partial f(X)}{\partial u_i}, \frac{\partial f(X)}{\partial u_j} > \\
&= < \frac{R^2}{|X|^2}\frac{\partial X}{\partial u_i} - \frac{2R^2}{|X|^3}\frac{\partial |X|}{\partial u_i} X, \frac{R^2}{|X|^2}\frac{\partial X}{\partial u_j} - \frac{2R^2}{|X|^3}\frac{\partial |X|}{\partial u_j} X > \\
&= < \frac{R^2}{|X|^2}\frac{\partial X}{\partial u_i}, \frac{R^2}{|X|^2}\frac{\partial X}{\partial u_j} > \\
&= \frac{R^4}{|X|^4} < \frac{\partial X}{\partial u_i}, \frac{\partial X}{\partial u_j} > \\
&= \frac{R^4}{|X|^4} g_{ij}.
\end{aligned}$$

Corollary 2.6 Let $X : U \longrightarrow E^3 - \{(0,0,0)\}$ be a parametrized 3-surface and $f : E^3 - \{(0,0,0)\} \longrightarrow E^3$ be an inversion.

Then the $\sqrt{\widehat{g}}$ of $f(X)$ is equal to $\frac{R^6}{|X|^6}\sqrt{g}$.

proof.

$$\begin{aligned}
\hat{g} &= \begin{vmatrix} \widehat{g}_{11} & \widehat{g}_{12} & \widehat{g}_{13} \\ \widehat{g}_{21} & \widehat{g}_{22} & \widehat{g}_{23} \\ \widehat{g}_{31} & \widehat{g}_{32} & \widehat{g}_{33} \end{vmatrix} \\
&= \begin{vmatrix} \frac{R^4}{|X|^4} g_{11} & \frac{R^4}{|X|^4} g_{12} & \frac{R^4}{|X|^4} g_{13} \\ \frac{R^4}{|X|^4} g_{21} & \frac{R^4}{|X|^4} g_{22} & \frac{R^4}{|X|^4} g_{23} \\ \frac{R^4}{|X|^4} g_{31} & \frac{R^4}{|X|^4} g_{32} & \frac{R^4}{|X|^4} g_{33} \end{vmatrix} \\
&= \left(\frac{R^4}{|X|^4}\right)^3 \begin{vmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ g_{31} & g_{32} & g_{33} \end{vmatrix} \\
&= \frac{R^{12}}{|X|^{12}} g.
\end{aligned}$$

Hence $\sqrt{\hat{g}} = \frac{R^6}{|X|^6}\sqrt{g}$.

3. The volume of a parametrized 3-surface under inversion

In this section, we show that the volume of $V(f(X))$ under inversion is equal to $R^6 \int_U \frac{1}{|X|^6} \sqrt{g} du_1 du_2 du_3$ and give example for the following theorem.

Theorem 3.1 Let $X : U \longrightarrow E^3 - \{(0,0,0)\}$ be a parametrized 3-surface defined by

$X(u_1, u_2, u_3) = \big(x(u_1, u_2, u_3), y(u_1, u_2, u_3), z(u_1, u_2, u_3)\big)$ for (u_1, u_2, u_3) $\in U \subset E^3$, and let $f : E^3 - \{(0,0,0)\} \longrightarrow E^3$ be an inversion of X, then the volume of $f(X)$ under inversion is equal to

$$V(f(X)) = R^6 \int_U \frac{1}{|X|^6} \sqrt{g}\, du_1\, du_2\, du_3. \tag{3.1}$$

proof. By corollary 2.6 and proposition 1.5, we have

$$\begin{aligned} V(f(X)) &= \int_U \sqrt{\hat{g}}\, du_1\, du_2\, du_3 \\ &= \int_U \frac{R^6}{|X|^6} \sqrt{g}\, du_1\, du_2\, du_3 \\ &= R^6 \int_U \frac{1}{|X|^6} \sqrt{g}\, du_1\, du_2\, du_3. \end{aligned}$$

Example 3.2 Let $X : U \longrightarrow E^3 - \{(0,0,0)\}$ be a mapping defined by

$X(r,\theta,\phi) = (r\cos\theta\sin\phi, r\sin\theta\sin\phi, r\cos\phi)$,

where $U = \{(r,\theta,\phi) \mid 1 < r < 2, 0 < \theta < 2\pi, 0 < \phi < \pi\}$.

Then, by example (1.2), X is a parametrized 3-surface, and

$$\begin{aligned} |X|^2 &= r^2\cos^2\theta\sin^2\phi + r^2\sin^2\theta\sin^2\phi + r^2\cos^2\phi \\ &= r^2\sin^2\phi(\cos^2\theta + \sin^2\theta) + r^2\cos^2\phi \\ &= r^2(\sin^2\phi + \cos^2\phi) \\ &= r^2. \end{aligned}$$

Thus $|X|^6 = r^6$ and $\sqrt{g} = r^2\sin\phi$. Hence

$$\begin{aligned} V(f(X)) &= R^6 \int_0^{\pi}\int_0^{2\pi}\int_1^2 \frac{1}{r^4}\sin\phi\, dr\, d\theta\, d\phi \\ &= \frac{7}{24}\cdot 2\pi \cdot 2R^6 \\ &= \frac{7}{6}\pi R^6. \end{aligned}$$

On the other hand,

$$\begin{aligned} f(X) &= \frac{R^2}{r^2}X \\ &= \frac{R^2}{r^2}(r\cos\theta\sin\phi, r\sin\theta\sin\phi, r\cos\phi) \\ &= \frac{R^2}{r}(\cos\theta\sin\phi, \sin\theta\sin\phi, \cos\phi). \end{aligned}$$

Thus

$$\widehat{E_1(p)} = \frac{\partial f(X)}{\partial r} = -\frac{R^2}{r^2}(\cos\theta\sin\phi, \sin\theta\sin\phi, \cos\phi),$$
$$\widehat{E_2(p)} = \frac{\partial f(X)}{\partial \theta} = \frac{R^2}{r}(-\sin\theta\sin\phi, \cos\theta\sin\phi, 0) \text{ and}$$
$$\widehat{E_3(p)} = \frac{\partial f(X)}{\partial \phi} = \frac{R^2}{r}(\cos\theta\cos\phi, \sin\theta\cos\phi, -\sin\phi).$$

Hence

$$\begin{aligned}
\hat{g} &= \begin{vmatrix} \widehat{g}_{11} & \widehat{g}_{12} & \widehat{g}_{13} \\ \widehat{g}_{21} & \widehat{g}_{22} & \widehat{g}_{23} \\ \widehat{g}_{31} & \widehat{g}_{32} & \widehat{g}_{33} \end{vmatrix} \\
&= \begin{vmatrix} \frac{R^4}{r^4} & 0 & 0 \\ 0 & \frac{R^4}{r^2}\sin^2\phi & 0 \\ 0 & 0 & \frac{R^4}{r^2} \end{vmatrix} \\
&= \frac{R^4}{r^4} \cdot \frac{R^8}{r^4}\sin^2\phi.
\end{aligned}$$

Thus

$$\begin{aligned}
V(f(X)) &= R^6 \int_0^{\pi}\int_0^{2\pi}\int_1^2 \frac{1}{r^4}\sin\phi\, dr\, d\theta\, d\phi \\
&= \frac{7}{6}\pi R^6.
\end{aligned}$$

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〈초 록〉

전위에 의한 매개 삼차곡면의 부피

안 성 의

제주대학교 교육대학원 수학교육전공

지도교수 현 진 오

중심이 O 이고 반지름의 길이가 R 인 주워진 원 또는 구에서 Euclid 공간 E^3 의 두 점 P 와 P' 이 중심 O 의 같은 쪽에 있고 점 P 에서 P' 으로 $OP \cdot OP' = R^2$ 가 되도록 보내는 변환 $f : E^3 - \{(0,0,0)\} \longrightarrow E^3$ 를 전위라 한다. 이 논문은 Euclid 공간 E^3 에서 주어진 매개 3 차 곡면, X는

$$X(u_1, u_2, u_3) = \left(x(u_1, u_2, u_3), y(u_1, u_2, u_3), z(u_1, u_2, u_3) \right)$$

에 대하여 전위에 의한 매개 3 차 곡면의 부피 $V(f(X))$ 는 x, y, z 가 u_1, u_2, u_3에 대한 Jacobian 행렬의 고유치하에서 $R^6 \int_U \frac{1}{|X|^6} \sqrt{g}\, du_1\, du_2\, du_3$ 와 같다는 것을 보인다.

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감 사 의 글

본 논문이 완성되기까지 연구에 바쁘신 가운데도 항상 세심한 지도와 격려를 해 주신 현진오 교수님과, 자세한 검토와 조언을 해 주신 김도현 교수님 그리고 대학원을 다니는 5학기 동안 많은 가르침과 격려를 해 주신 수학교육과, 수학과 모든 교수님께 깊은 감사를 드리며, 함께 강의를 받으며 서로 의지하고 어려운 일에 협조를 아끼지 않는 현태영 선생님, 강순구 선생님께도 고마운 마음을 전하고 싶습니다.

그리고 학교 일과 진행의 어려움 속에서도 대학원의 과정을 마칠 수 있도록 배려해 주신 아라중학교 교장선생님 및 모든 선생님에게 감사 드리며, 특히 많은 격려와 용기를 주신 모든 분들께도 감사를 드립니다.

끝으로, 자식을 위해 헌신적으로 도와준 부모님, 많은 어려움 속에서도 인내와 사랑으로 따뜻한 커피로 위로해준 소중한 아내, "아빠 힘내세요!" 하며 밝게 웃어 주는 지희, 동준이와 함께 이 기쁨을 같이 하고자 합니다.

1995년 7월

안 성 의 드림