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E_{lab} = 1503 MeV에서 McIntyre 파라미터 위상 이동량을 이용한 $^{16}O + {}^{40}Ca$ 탄성산란 분석

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Analysis of $^{16}O + {}^{40}Ca$ Elastic Scattering at E_{lab} = 1503 MeV Using McIntyre's Parametrized Phase Shift

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A semiclassical phase shift analysis of the $^{16}O + {}^{40}Ca$ elastic scattering at E_{lab} = 1503 MeV is presented using the McIntyre's parametrized phase shift. The calculated cross section is found to be good agreement with the experimental data. The presence of an nuclear rainbow is evidenced. The optical potential by inversion is calculated and compared with the one of optical model analysis.

I. INTRODUCTION

The study of elastic scattering is a basic ingredient to understand more complicated reaction process in heavy ion collisions. To understand and systematize heavy ion reactions, various models have been used. In recent year, a number of studies have been made to describe elastic scattering processes between heavy ions within the framework of the McIntyre strong absorption model (SAM)[1-4] . This model could provide a good description of nucleus-nucleus scattering data over a large energies range. Also, McIntyre SAM gives a realistic deflection function and allows, consequently, the nuclear rainbow effect observed in heavy ion scattering.

Elastic scatterings data for ^{12}C and ^{16}O ions at intermediate and high energies have been analyzed[2-4] sucessfully in the framework of the McIntyre SAM. Elastic scattering of $^{16}O + {}^{40}Ca$ system at E_{lab} = 1503 MeV was measured and analyzed with the optical model analysis[5] . Recently, there are several attempts[6-8] to evaluate optical potential from parametrized phase shift. A practical solution of the inversion problems using the

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quasi-classical limit of the high-energy approximation is reported[6]. Fayyad *et al.*[7] have shown by solving the inversion problem at high energies that the fundamental McIntyre parametrization of the S-matrix, for heavy ion collision, would correspond to a Woods-Saxon type optical model potential. The parameters of such a Woods-Saxon potential were directly related to the corresponding parameters of the McIntyre parametrization. And, Eldebawi *et al.* [8] use the simple Ericson parametrization of the phase shifts to analyze the experimental data and apply the Glauber approximation to evaluate the corresponding optical potential.

In the previous papers[9-11], the semiclassical phase shift analysis of the heavy ion elastic scattering by using the asymptotic Legendre function and McIntyre SAM was presented. It was applied satisfactorily to the elastic scattering data for $E_{lab}/A = 35$ MeV/nucleon ^{12}C beams on ^{90}Zr and ^{208}Pb target nuclei. Mermaz[3] have already fitted with 10 % error bars for $^{16}O + {}^{40}Ca$ at $E_{lab} = 1503$ MeV using McIntyre SAM.

This paper reproduces a semiclassical phase shift analysis of the elastic scattering data for $E_{lab} = 1503$ MeV ^{16}O beams on ^{40}Ca target nucleus. Furthermore, optical potential will be obtained by using the inversion procedure which McIntyre parametrization of the S-matrix is related to the Woods-Saxon type optical model potential ,and compared with one of optical model analysis[5]. In section II, we present the scattering amplitude. And the potential by inversion is presented in section III. Finally, we give results and conclusions in section IV.

II. SCATTERING AMPLITUDE

The elastic scattering amplitude for spin-zero particle via Coulomb and short-range central force is given by

$$f(\theta) = f_R(\theta) + \frac{1}{ik}\sum_{l=0}^{\infty}(l+\frac{1}{2})\exp(2i\sigma_l)(S_l^N - 1)P_l(\cos\theta). \qquad (1)$$

Here $f_R(\theta)$ is the usual Rutherford scattering amplitude, $\sigma_l = \arg\Gamma(l+1+i\eta)$ the Coulomb phase shifts. The nuclear S-matrix ,S_l^N, can be obtained from the nuclear phase shift $\chi(l)$ given by

$$S_l^N = \exp[i\chi(l)] = \exp[i(\chi_R(l) + i\chi_I(l))]. \qquad (2)$$

In this work, we use the McIntyre parametrization[1] of the S-matrix . The real and imaginary parts of nuclear phase shift for McIntyre parametrization [1] are expressed

$$\chi_R(l) = 2\mu\{1 + \exp[(l - \Lambda_1)/\Delta_1)]\}^{-1} \qquad (3)$$

and

$$\chi_I(l) = \ln[1 + \exp\{(\Lambda_2 - l)/\Delta_2\}]. \qquad (4)$$

¿From the above two equations, there are five adjustible parameters available for fitting the cross section data : $\Lambda_1, \Lambda_2, \Delta_1, \Delta_2$ and μ. The two grazing angular momenta Λ_1 and Λ_2 are related semiclassically to the interaction radius of the colliding nuclei while the corresponding widths $\Delta_1$and $\Delta_2$are related to the thickness of the region in which the nuclear interaction between the colliding nuclei takes place without destruction of the identity of either of the nuclei. The reduced radius $r_{1/2}$ and diffusivity d are related to the grazing angular momentum Λ_2 and angular momentum width Δ_2 through the following semi-classical relationship[3]:

$$\Lambda_2 = kR_{1/2}(1 - \frac{2\eta}{kR_{1/2}})^{1/2}, \tag{5}$$

$$\Delta_2 = kd(1 - \frac{\eta}{kR_{1/2}})(1 - \frac{2\eta}{kR_{1/2}})^{-1/2} \tag{6}$$

where $R_{1/2} = r_{1/2}(A_1^{1/3} + A_2^{1/3})$, k is the wave number and $\eta = mZ_1Z_2e^2/(\hbar^2 k)$ the Sommerfeld parameter. The remaining parameter, μ, is required to introduce the strength of the nuclear phase shift.

The semiclassical approximation assumes that contributions to the cross section come mainly from the large angular momenta. The asymptotic form of the Legendre function can be written as[12]

$$P_l^m(\cos\theta) \simeq (l + \frac{1}{2})^m (\frac{\theta}{\sin\theta})^{1/2} J_{-m}[(l + \frac{1}{2})\theta], \tag{7}$$

which is valid for all m and all angles except when $(\pi - \theta) \leq l^{-1}$. The scattering amplitude can now be written as an integral over the continuous variable $\lambda = l + \frac{1}{2}$, using the asymptotic form Eq.(7) with $m = 0$ and replacing S_l^N by a continuous differential function $S_N(\lambda)$,

$$f_N(\theta) \simeq \frac{1}{ik}(\frac{\theta}{\sin\theta})^{1/2} \int_{1/2}^{\infty} \lambda \exp[2i\sigma(\lambda)][S_N(\lambda) - 1] J_0(\lambda\theta) d\lambda. \tag{8}$$

III. POTENTIAL BY INVERSION

The real and imaginary parts of nuclear phase shift can be rewritten in terms of impact parameter as

$$\chi_R(b) = \frac{2\mu}{1 + \exp[(b - b_0')/d']} \tag{9}$$

and

$$\chi_I(b) = \ln[1 + \exp(\frac{b_0 - b}{d}) \tag{10}$$

where b and b_0 are the impact parameters normally given by $kb = l + \frac{1}{2}$, $kb_0 = \Lambda_2 + \frac{1}{2}$ and d is diffusivity with $d = \Delta_2/k$.

Using the relation between nuclear phase shift and optical potential

$$\chi(b) = -\frac{k}{E}\int_b^\infty \frac{r(V(r)+iW(r))}{\sqrt{r^2-b^2}}dr, \tag{11}$$

we can write real and imaginary nuclear phase shifts as

$$\chi_R(b) = -\frac{k}{E}\int_b^\infty \frac{rV(r)}{\sqrt{r^2-b^2}}dr \tag{12}$$

and

$$\chi_I(b) = -\frac{k}{E}\int_b^\infty \frac{rW(r)}{\sqrt{r^2-b^2}}dr. \tag{13}$$

Equation (12) and (13) are of Abel's [7] type and the inversion solutions of these equations are given by

$$V(r) = \frac{2E}{k\pi}\frac{1}{r}\frac{d}{dr}\int_r^\infty \frac{\chi_R(b)}{\sqrt{b^2-r^2}}bdb \tag{14}$$

and

$$W(r) = \frac{2E}{k\pi}\frac{1}{r}\frac{d}{dr}\int_r^\infty \frac{\chi_I(b)}{\sqrt{b^2-r^2}}bdb. \tag{15}$$

Approximating the phase shift $\chi_R(b)$ and $\chi_I(b)$ in terms of sums of Gaussian shape and inserting this approximated phase shift forms into Eqs.(14) and (15), we get

$$V(r) = -\frac{4\mu E}{\pi k\alpha}\sum_{n=1}^{N} c_n\sqrt{\pi n}\exp(-\frac{nr^2}{\alpha^2}) \tag{16}$$

and

$$W(r) = -\frac{2E}{\pi k\beta}\sum_{n=1}^{N} b_n\sqrt{\pi n}\exp(-\frac{nr^2}{\beta^2}). \tag{17}$$

To relate above $V(r)$ and $W(r)$ with the familiar Woods-Saxon forms, let us rewrite $V(r)$ and $W(r)$ in the forms

$$V(r) = -\frac{4\mu E}{\pi k\alpha}\frac{1}{1+\exp[(r-R')/\Delta']} \tag{18}$$

and

$$W(r) = -\frac{2E}{\pi k\beta}\frac{1}{1+\exp[(r-R)/\Delta]}. \tag{19}$$

With the above parametrization and as is shown in the Appendix of Ref.[7], the following relations, between the parameters of the corresponding phase shifts, will hold. For the real part,

$$\frac{I_4(R',\Delta')}{I_2(R',\Delta')} = \frac{3}{2}\frac{I_3(b_0',d')}{I_1(b_0',d')} \tag{20}$$

and

$$\frac{I_6(R',\Delta')}{I_4(R',\Delta')} = \frac{5}{4}\frac{I_5(b_0',d')}{I_3(b_0',d')}. \tag{21}$$

For the imaginary part,

$$\frac{I_4(R,\Delta)}{I_2(R,\Delta)} = \frac{3}{4}\frac{I_4(b_0,d)}{I_2(b_0,d)} \tag{22}$$

and

$$\frac{I_6(R,\Delta)}{I_4(R,\Delta)} = \frac{5}{6}\frac{I_6(b_0,d)}{I_4(b_0,d)} \tag{23}$$

where $I_\nu(x_0,a_0)$ is an integral given by

$$I_\nu(x_0,a_0) = \int_0^\infty \frac{x^\nu}{1+\exp[(x-x_0)/a_0]}dx. \tag{24}$$

We can obtain parameters R' and Δ' from solving two nonlinear simultaneous Eqs.(20) - (21) and R and Δ from Eqs.(22)-(23), respectively, by using a certain iteration procedure such as the Newton method[13]. The parameters α and β in Eqs.(18) and (19) are given by

$$\alpha = \frac{2}{\pi}[1+\exp(-b_0'/d')]I_0(R',\Delta') \tag{25}$$

and

$$\beta = \frac{2}{\pi}\frac{I_0(R,\Delta)}{\ln[1+\exp(b_0/d)]} \tag{26}$$

where $I_0(R',\Delta')$ is given by

$$I_0(R',\Delta') = R' + \Delta'\ln[1+\exp(-R'/\Delta')]. \tag{27}$$

Now, Eqs.(18) and (19) can now be rewritten in the form

$$V(r) = \frac{-V_0}{1+\exp[(r-R')/\Delta']} \tag{28}$$

and

$$W(r) = \frac{-W_0}{1+\exp[(r-R)/\Delta]} \tag{29}$$

where $V_0(=4\mu E/\pi k\alpha)$ and $W_0(=2E/\pi k\beta)$ represent the depth of the optical model potential.

IV. RESULTS AND CONCLUSIONS

The elastic differential scattering cross sections are obtained from the scattering amplitude using Eq.(1) and Eq. (8). The elastic scattering angular distributions for ^{16}O + ^{40}Ca system fitted with five independent parameters are presented in Fig. 1. The fits are satisfactory and the corresponding parameters are given in table 1. The full and broken curves represent the calculated cross sections obtained by using Eqs.(1) and (8), respectively. The two numerical results agree well with the observed data[5]. In table 1, r_{ph} and d_{ph} are the reduced radius and diffusivity for the nuclear phase shift $\chi_R(l)$. These parameters are related to Λ_1 and Δ_1 by means of the semiclassical relationship, Eq. (5) and (6). As shown in table 2, our calculated $\chi^2/N = 2.29$ was improved compared with $\chi^2/N = 4.3$ by Mermaz[3]. The terms θ_g in table 2 is the grazing angle equal to $2\text{ArcTan}(\eta/\Lambda_2)$.

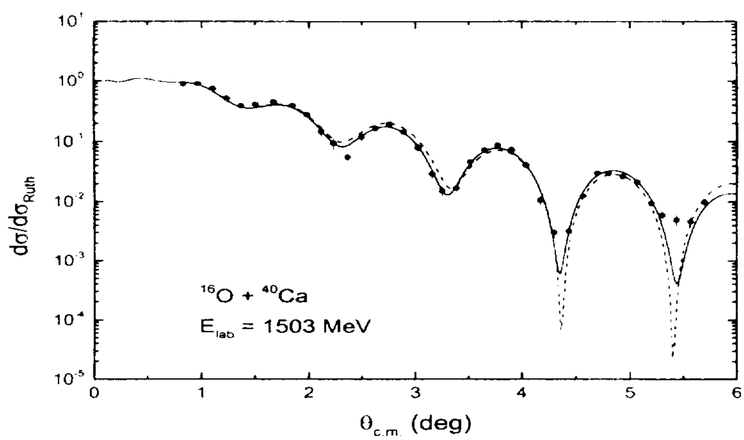

FIG. 1. Elastic scattering angular distributions for $^{16}O + {}^{40}Ca$ system at $E_{lab} = 1503$ MeV. The solid circles denote the observed data taken from Roussel-Chomaz *et al.* [5]. The full and broken curves are the calculated results from Eq.(1) and Eq.(8).

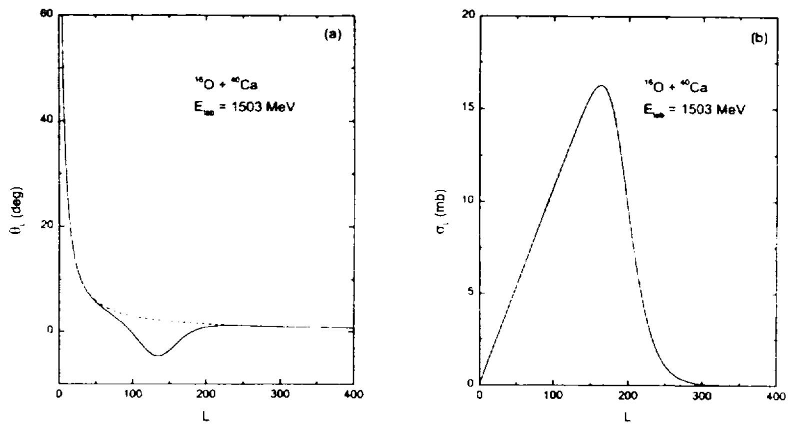

FIG. 2. (a) Deflection function and (b) partial wave reaction cross section for the ^{16}O + ^{40}Ca system at E_{lab} = 1503 MeV plotted against the angular momentum L.

The deflection function given by the formula, $\frac{d}{dl}(2\sigma_l + \chi_R(l))$, is plotted in figure 2 along with the partial wave reaction cross section. The quantities θ_{CR} and θ_{NR} ,in table 2, are, respectively, the Coulomb and nuclear rainbow angles obtained from the deflection function. We can see in Fig.2(a) that an nuclear rainbow exists in the system of ^{16}O + ^{40}Ca at E_{lab} = 1503 MeV. The partial wave contributions, $\sigma_l = \pi/k^2(2l+1)(1-|S_l^N|^2)$, to the total reaction cross section as a function of l is also presented in figure 2(b). Figure 2(b) show that regions of higer partial waves do not nearly contribute to the total reaction cross section. We can find from in table 2 that the strong absorption radius (R_{SAR}), corresponding to $|S_l^N|^2 = 1/2$, gives a good measure of the reaction cross section in terms of $\sigma_{R,1/2} = \pi R_{SAR}^2$.

TABLE I. Input values of the McIntyre strong absorption model for ^{16}O + ^{40}Ca at E_{lab}=1503 MeV.

$r_{1/2}(fm)$	$d(fm)$	μ	$r_{ph}(fm)$	$d_{ph}(fm)$	Λ_1	Δ_1	Λ_2	Δ_2
1.276	0.732	4.329	0.940	0.746	132.653	18.078	181.015	17.737

TABLE II. Analysis results from the McIntyre strong absorption model (SAM) for ^{16}O + ^{40}Ca at E_{lab}=150 3MeV.

θ_g(deg)	$\theta_{C.R.}$(deg)	$\theta_{N.R.}$(deg)	$R_{SAR}(fm)$	$\sigma_{R,1/2}(mb)$	$\sigma_R^{SAM}(mb)$	$\sigma_R^{OM}(mb)^a$	χ^2/N	χ^2/N [b]
1.646	2.246	-4.615	8.137	2080	2163	1996	2.29	4.3

[a]Values of the optical model analysis are taken from Roussel-Chomaz *et al.* [5].
[b]Value of χ^2/N is taken from Mermaz [3] using 10 % error bars analysis.

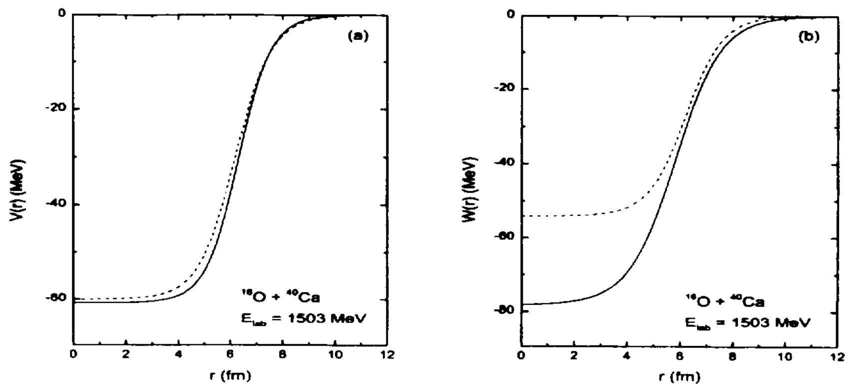

FIG. 3. (a) Real $V(r)$ and (b) imaginary $W(r)$ parts of the optical potential. The full curves are the calculated results by inversion procedure. The dashed curves are the results from the optical model analysis taken from Ref.[5].

We have calculated the optical potential through the inversion procedure[7] . Using the McIntyre parameters,$\Lambda_1, \Lambda_2, \Delta_1, \Delta_2$ and μ, obtained from fitting the data and calculating the other related parameters (such as η, k,etc), we got the phase shift parameters as $b_0 = 7.49171\ fm, b_0' = 5.49565\ fm$. Note that Coulomb effects have been substracted off in evaluating the parameters cited above. Using these parameters and solving two sets of nonlinear simultaneous Eqs (20)-(21) and Eqs(22)-(23) with the iteration procedure such as the Newton method[13], we can solve for R, Δ, R', and Δ'. Their values are $R = 5.79223\ fm, \Delta = 0.87921\ fm, R' = 6.31477\ fm$, and $\Delta' = 0.63274\ fm$. We get from Eqs. (25) and (26) and above parameter that $\alpha = 4.02267\ fm$ and $\beta = 0.36040\ fm$. Accordingly, we obtain $V_0 = -60.713$ MeV and $W_0 = -78.270$ MeV. The comparison of the real and imaginary parts of optical potential with those of Woods-Saxon potentials obtained in optical model analysis[5] and the potentials previously deduced using McIntyre parametrization of S-matrix is shown in Fig. 3. The potential obtained in the present paper agree with one obtained in the optical model analysis near the strong absorption radius. The depth of imaginary potential by inversion give more deeper than one of imaginary potential by optical model analysis[5]. We can find that the real potential provide better agreements with the optical model result compared to imginary one.

In this paper, we have found that using the asymptotic formula for the Legendre functions we can convert the partial wave sum for the scattering amplitude to an integral over the continuous variable λ. We have also shown that the integral form gives cross sections for the elastic scatterings of $^{16}O + ^{40}Ca$ system at $E_{lab} = 1503$ MeV in satisfactory agreement with the direct sum of partial waves. The presence of an nuclear rainbow is also evidenced by the deflection function. We can see that the agreement of real potential obtained from inversion procedure with the Woods-Saxon potential by optical model analysis is more good compared to one of the imaginary potential.

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