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# LPD(Linear Parameter Dependent) System Modeling and Control of Mobile Soccer Robot

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## ABSTRACT

In this paper, we studied linear parameter dependent (LPD) system modeling and control of mobile soccer robot. The mobile soccer robot is modeled by the LPD system and the controller which is based on the well known pole-placement algorithm is designed. In simulation, we use two random variables, zero mean white Gaussian noise, is added to the velocity of two wheels in order to realize for more real game environment. The maximum power of these random variables is 5% of maximum velocity of wheels. And we show that the LPD system modeling and control is more easy treatment of soccer robot.

**Key Words** : Linear parameter dependent system, pole placement, pole sensitivity, mobile soccer robot

## Ⅰ. Introduction

The robotic soccer systems and control schemes have been studied by many researchers with various degree of application and success [1-6]. Most of these studies are concentrated on the development, control and planning the strategy of mobile soccer robot.

In this paper, we studied linear parameter dependent (LPD) system modeling and control of mobile soccer robot. The mobile soccer robot is modeled by the LPD system and the controller which is based on the well known pole-placement algorithm is designed. In simulation, we use two random variables, zero mean white Gaussian noise, is added to the velocity of two wheels in order to realize for more real game environment. The maximum power of these random variables is 5% of maximum velocity of wheel. And we show that the LPD system modeling and control is more easy treatment of soccer robot.

## Ⅱ. Modeling of Soccer Robot

The structure of the mobile soccer robot, considered in this paper, is MIROSOT and which is shown in Fig. 1. The relation of forward velocity and wheel angular velocity is described by

$$\begin{bmatrix} v \\ \omega \end{bmatrix} = \begin{bmatrix} \frac{r}{2} & \frac{r}{2} \\ \frac{r}{2b} & -\frac{r}{2b} \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \end{bmatrix} \tag{1}$$

where, $v$ and $\omega$ are forward and rotation velocity of soccer robot respectively, $r$ is radius of wheel and $b$ is the displacement from center robot to center of

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wheel. The kinetic equation is

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ w \end{bmatrix} \qquad (2)$$

In order to derive the dynamic equations, we now define some variables

$I_c$ : robot inertia except wheels and rotor

$I_u$ : motor rotor inertia for wheels and wheel axis

$I_m$ : motor rotor inertia for wheels and wheel diameter

$m$ : mass of robot except wheels and motor rotor

$m_c$ : mass of wheels and motor rotor

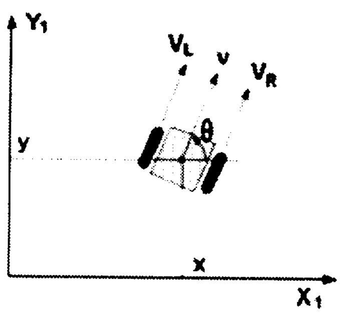


**Fig. 1.** The structure of soccer robot.

The dynamic equation of this type of robot is described by

$$M(q)\ddot{q} + V(q,\dot{q}) = E(q)\tau - A^T(q)\lambda \qquad (3)$$

where, $\lambda$ is Lagrangy multiplier and

$$M(q) = \begin{bmatrix} m & 0 & -m_c cd\sin\theta & m_c cd\sin\theta \\ 0 & m & m_c cd\cos\theta & -m_c cd\cos\theta \\ m_c cd\sin\theta & m_c cd\cos\theta & I_c^2 + I_u & -I_c^2 \\ m_c cd\sin\theta & -m_c cd\cos\theta & -I_c^2 & I_c^2 + I_u \end{bmatrix},$$

$$V(q,\dot{q}) = \begin{bmatrix} -m_c d\dot{\theta}^2\cos\theta \\ -m_c d\dot{\theta}^2\cos\theta \\ 0 \\ 0 \end{bmatrix}, E(q) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$\tau = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}, \lambda = \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix}$$

We note that the displacement from the center of mass to the center of movement is zero i.e., $d=0$, then the dynamic equation becomes

$$\begin{bmatrix} 1&0&0&0&0&0 \\ 0&1&0&0&0&0 \\ 0&0&1&0&0&0 \\ 0&0&0&1&0&0 \\ 0&0&0&0&e_{11}&e_{12} \\ 0&0&0&0&e_{21}&e_{22} \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\varpi}_1 \\ \dot{\varpi}_2 \\ \ddot{\varpi}_1 \\ \ddot{\varpi}_2 \end{bmatrix} =$$

$$\begin{bmatrix} 0&0&0&0&cb\cos\theta & cb\cos\theta \\ 0&0&0&0&cb\sin\theta & cb\sin\theta \\ 0&0&0&0&1&0 \\ 0&0&0&0&0&1 \\ 0&0&0&0&I_c^2+I_u & -I_c^2 \\ 0&0&0&0&-I_c^2 & I_c^2+I_u \end{bmatrix} \begin{bmatrix} x \\ y \\ \varpi_1 \\ \varpi_2 \\ \dot{\varpi}_1 \\ \dot{\varpi}_2 \end{bmatrix} + \begin{bmatrix} 0&0 \\ 0&0 \\ 0&0 \\ 0&0 \\ 1&0 \\ 0&1 \end{bmatrix} \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} \qquad (4)$$

where

$$\begin{aligned} e_{11} &= m(cb)^2 + I_c^2 + I_u \\ e_{12} &= m(cb)^2 - I_c^2 \\ e_{21} &= m(cb)^2 - I_c^2 \\ e_{22} &= m(cb)^2 + I_c^2 + I_w \end{aligned}$$

The equation (4) is descriptor type equation but we need to slight modification because the system equation is uncontrollable. We note here that the linear velocity $(\dot{x}, \dot{y})$ is proportional to the angular velocity $(\varpi_1, \varpi_2)$, thus, the linear velocity $(\dot{x}, \dot{y})$ can be deleted from the dynamic equation. So, the equation (4) becomes

$$E\dot{x} = Ax + Bu$$

where,

$$x = [\varpi_1 \ \varpi_2 \ \dot{\varpi}_1 \ \dot{\varpi}_2], \ u = [\tau_1 \ \tau_2]$$

$$E = \begin{bmatrix} 1&0&0&0 \\ 0&1&0&0 \\ 0&0&e_{11}&e_{12} \\ 0&0&e_{21}&e_{22} \end{bmatrix},$$

$$A = \begin{bmatrix} 0&0&1&0 \\ 0&0&0&1 \\ 0&0&I_c^2+I_u & -I_c^2 \\ 0&0&-I_c^2 & I_c^2+I_u \end{bmatrix}, B = \begin{bmatrix} O \\ I \end{bmatrix}$$

and the output equation becomes

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} cb\cos\theta & cb\cos\theta & 0 & 0 \\ cb\sin\theta & cb\sin\theta & 0 & 0 \end{bmatrix}$$

## III. Pole-placement Control of LPD system

We now describe LPD systems and some theoretical results and present a new algorithm.

## 3.1. LPD System

Many of the physical system can be modeled by linear parameter dependent system. Before introducing the LPD system, we need to define the set of all admissible parameter trajectories.

Definition 1[7]. Given a compact set $P \subset R^S$, the parameter set $F_P$ denote the set of all piecewise continuous functions mapping $R^+$ into $P$ with finite number of discontinuities in any interval.

By the definition 1, the parameter value $\rho_i \in F_P$ are differentiable with respect to time

A state space realization of LPD system is

$$\dot{x}(t) = A(\rho)x(t) + Bu(t) \tag{5}$$

where, $\rho \in F_P, x(t) \in R^n, u(t) \in R^{n_u}$ and $y(t) \in R^{n_y}$.

## 3.2. Pole Sensitivity[7]

Now, we define the pole-sensitivity which can be used robust pole placement.

Definition 2. The pole sensitivity, defined as the ratio of pole displacement with respect to the parameter variation, is described by

$$S_{ij} := \frac{\partial \lambda_i}{\partial \rho_j} = \frac{u_i \frac{\partial A(\rho)}{\partial \rho_j} v_i}{u_i v_i} \tag{6}$$

where, $u_i, v_i$ is left and right eigen-vector of i-th system pole respectively.

By the definition 2, $S_{ij}$ means i-th pole displacement with j-th parameter variation.

## 3.3. Pole-Placement

The matrix $F_0$ is used for the pole-placement which makes the closed loop poles locate on the desired location or lie in the desired region. We firstly, state a method of finding state feedback gain matrix $F_0$ which locates the closed loop poles on the desired location. The input matrix $B$ the rank of which is $m$, is partitioned as

$$B = [U_1 \ U_2]\begin{bmatrix} Z \\ 0 \end{bmatrix} \tag{7}$$

where, $U_1, U_2$ are unitary matrix and $Z$ is nonsingular matrix with rank $m$. Let $\Lambda_D, V_D$ be desired closed loop pole matrix and right eigenvector matrix, respectively. Then, the following equation is necessary and sufficient condition for the existence of the state feedback gain matrix which places the closed loop poles on the desired location.

$$U_2^T(A_0 V_D - V_D \Lambda_D) = 0 \tag{8}$$

If the equation (8) is hold then the state feedback gain matrix $F_0$ is

$$F_0 = Z^{-1}U_1^T(A_0 - V_D \Lambda_D V_D^{-1}) \tag{9}$$

The proof of the equation (8) and (9) can be found many books and papers which treat the linear system control.

## 3.4. Pole-Placement with Pole-sensitivity

To make the pole-sensitivity defined by equation (6), one of the following equation must be hold

$$I_n - (U_1 Z)(Z^{-1}U_1^T) = 0 \tag{10}$$

$$A_i - BF_i = V_D^{\perp} \tag{11}$$

$$V_D V_D^{\perp} = 0 \ \text{or} \ V_D^{\perp} V_D = 0$$

Generally, equations (10) and (11) are not hold because the second term of the equation (10) has rank $m\langle n$ and the rank of the matrix is equal to $n$. We use two tricks which guarantee the small pole sensitivity. Rewrite the pole sensitivity equation as

$$S_{ij} = \frac{ui\left(\begin{vmatrix} A_j^{11} & A_j^{12} \\ A_j^{21} & A_j^{22} \end{vmatrix} - [\ U_1 \ \ U_2]\begin{bmatrix} Z \\ 0 \end{bmatrix}[\ F_j^1 \ \ F_j^2]\right)v_i}{u_i v_i}$$

It becomes

$$S_{ij} = u_i^1(A_j^{11} - U_1ZF_j^1)v_i^1 + u_i^1(A_j^{12} - U_1ZF_j^2)v_i^2 + u_i^2A_j^{21}v_i^1 + u_i^2A_j^{21}v_i^2 \tag{12}$$

And by proper selection of auxiliary feedback gain matrix, we can obtain the pole sensitivity as

$$S_{ij} = u_i^2A_j^{21}v_i^1 + u_i^2A_j^{21}v_i^2 \tag{13}$$

If the parameters are not first order, then the pole sensitivity as

$$S_{ij} = u_i^1\frac{\partial}{\partial\rho_j}(A^{11} - I_1ZF^1)v_i^1 + u_i^2\frac{\partial}{\partial\rho_j}A^{21}v_i^1 + u_i^1\frac{\partial}{\partial\rho_j}(A^{12} - I_1ZF^2)v_i21 + u_i^2\frac{\partial}{\partial\rho_j}A^{21}v_i^2 \tag{14}$$

The equation (14) becomes

$$S_{ij} = u_i^2\frac{\partial}{\partial\rho_j}A^{21}v_i^1 + u_i^2\frac{\partial}{\partial\rho_j}A^{21}v_i^2 \tag{15}$$

By proper selection of left and right eigenvectors, the pole sensitivity can be minimized.

## Ⅳ. Simulation

In simulation, the robot considered is MIROSOT and detailed specifications are summarized in the Table 1.

**Table 1.** The specifications of MIROSOT robot

| | |
|---|---|
| size | 70x70x70 mm |
| Wheel diameter | 45 mm |
| rpm | 8000 |
| Gear ratio | 8:1 |

The mass of the robot is 0.6Kg and used parameters are

$$b = 35mm, \quad c = r/2b$$

The sampling time of this simulation is 15 ms and the maximum velocity of the wheel is 2/3 of the maximum velocity of the motor specification. Also, we use two random variables, zero mean white Gaussian noise, is added to the velocity of two wheels in order to realize for more real game environment. The maximum power of these random variables is 5% of maximum velocity of wheel.

Fig. 2 is the tracking result of 5 point and Fig. 3 is the desired velocity and actual velocity.

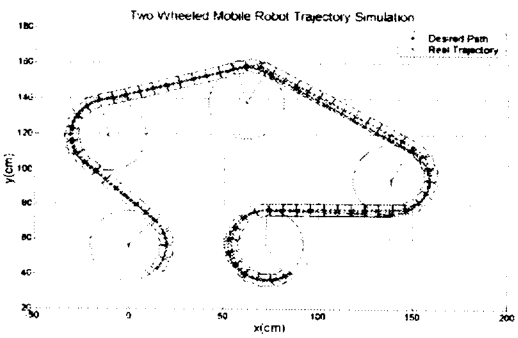


**Fig. 2.** The tracking result of 5 points.

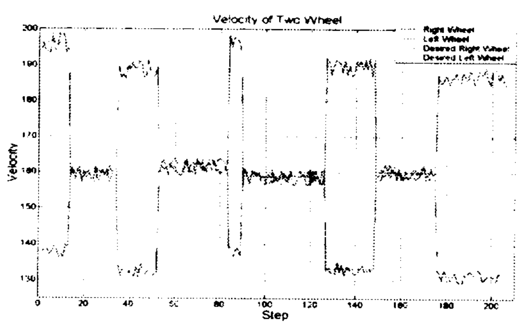


**Fig. 3.** is the desired velocity and actual velocity.

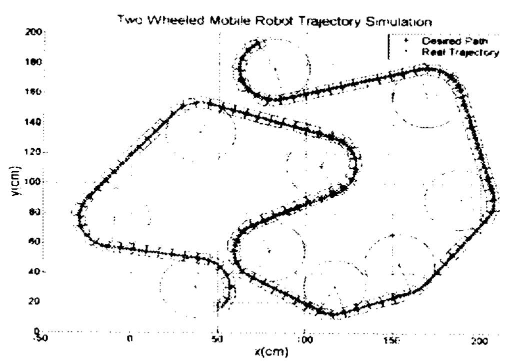


**Fig. 4.** The tracking result of 10 points.

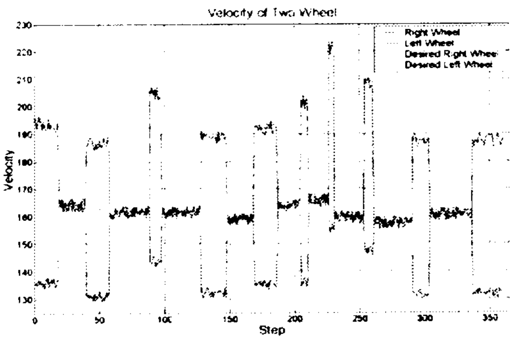


**Fig. 5.** is the desired velocity and actual velocity.

And Fig. 4 is the tracking result of 10 point and Fig. 5 is the desired velocity and actual velocity. It is shown in Fig. 2 and Fig. 4. that the robot tracks the desired path very well.

## V. Conclusion

In this paper. we studied the modeling and control of soccer robot via LPD system. The pole-sensitivity is defined and a control algorithm is presented by well-known pole-placement. It is shown in this paper that the soccer robot can be treated more easily via LPD framework which is a type of linear system

## References

1) Sun-Gi Hong. Tae-Dok Eom. Il-Kwon Jeong. Changkyu Chio. Jin-Ho Shin and Ju Jang Lee. 1997. Development of Soccer-Playing Robots Symposium of the Control Measurement. Automation and Robotics.. pp.303-308

2) Seung-Min Baek. Woong-Gie Han and Tae-Yong Kuc. 1997. Cell-Based Motion control of Mobile Robots for Soccer Game Proc. of the 12th KACC.. 819-824

3) Sung-Wook Park. Eun-Hee Kim. Do-Hyun Kim and Jun-Ho Oh. 1997. Development of Soccer-Playing Robots Using Visual Tracking Proc. of the 12th KACC.. 617-620

4) Sung Ho Kim. Jong Suk Choi. Jae Kwon Kim and Byung Kook Kim. 1997. A Cooperative Strategy for Soccer Robots Symposium of Control Measurement. Automation and Robotics.. pp. 283-287

5) Xiaoshu Wang and Masanori Sugisaka. 1999. Learning Soccer Robot Using Genetic Programming Proc. of the 14th KACC.. E-292-297

6) Jung-Min Yang and Jong-Hwan Kim. 1997. Tracking Control of Wheeled Mobile Robots Using Variable Structure Systems Proc. of the 2nd Asian Control Conference. I-381-384

7) Jin-Shig Kang and Seong-Bo Oh. 2001. Pole Placement of Linear Parameter Dependent System Proc. Of the ICMIT2001. pp.37-40