Estimation of the Steam Generator Water Level Variation

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1. Introduction

The steam generator is one of the major equipments of a nuclear plant and it comprises the boundary between the primary and secondary systems. For the safety, the content should always be maintained enough to cool the reactor and proper setting values and limitations are imposed to trip the reactor if the content deviates from the permissible operating conditions. But these considerations for the safety tends to induce spurious trips, particularly operators interfere with the system during the start up or during the severe grid load variations(Inaba, 1986). Therefore, to maintain the desirable availability and the safety as well, it can be thought to minimize the level fluctuations by introducing proper control elements to prevent spurious trips(Westinghouse, 1983).

For this, an exact estimation of the level is essential and from the view of real time application, the calculation procedure should be simple enough to permit the rapid yield of results. In the following, one method for the level estimation has been set up and its results are compared with those of other method.

2. Modeling and Governing Equations

Usually, the steam generator is modeled into several nodal volumes, and the basic equations of mass, energy and momentum are applied to each nodal volume to find major physical parameters. This multi nodes modeling gives a better result than a single node of course, but considering the

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calculation time required, the multi nodes modeling is not always preferable.

In the following, the steam generator has been treated as one nodal volume, including vapor region and liquid region, to establish simple equations to describe the transient of the key parameters.

Mass;

$$dM_1 = [w_f(t) + \dot{m}_r - \dot{m}_{fb}]dt \quad \cdots\cdots (1)$$

$$dM_2 = [\dot{m}_{fb} - \dot{m}_r - w_s(t)]dt \quad \cdots\cdots (2)$$

Enthalpy;

$$dH_1 = [w_f(t) \cdot h_i(t) + \dot{m}_r \cdot h_f - \dot{m}_{fb} \cdot h_g + \dot{Q}]\ dt \quad \cdots\cdots (3)$$

$$dH_2 = [\dot{m}_{fb} h - \dot{m}_r h_f - w_s(t) \cdot h_2(t)]\ dt \quad \cdots\cdots (4)$$

Internal energy;

$$dU_1 = dH_1 - C \cdot P \cdot \frac{dV_1}{dt} \cdot dt \quad \cdots\cdots (5)$$

$$dU_2 = dH_2 - C \cdot P \cdot \frac{dV_2}{dt} \cdot dt \quad \cdots\cdots (6)$$

where, M=mass, H=total enthalpy, U=total internal energy, V=volume, subscript 1= liquid region, 2=gas region, and

$w_f(t)$=inlet feed water flow rate,
$w_s(t)$=outlet steam flow rate,
$\dot{m}_r$=rain drop rate,
$\dot{m}_{fb}$=bubble generation rate,
$\dot{Q}$=heat generation rate,
$h_i(t)$=specific enthalpy of inlet water,
$h_2(t)$=specific enthalpy of outlet steam,
h_f, h_g=specific enthalpy of saturated liquid and steam,
c=conversion factor.

In the above equations, $w_f(t)$ and $w_s(t)$ as well as $h_i(t)$ are determined as operation conditions, and $\dot{m}_{fb}$ and $\dot{m}_r$ are caclulated by the use of bubble rise model as below (Nahabandi, 1976).

$$\dot{m}_{fb} = V_b \cdot \text{area} \cdot \rho_{gm}$$

$$\rho_{gm} = \frac{(1+C) \cdot m_b}{V_1} \quad , \text{ if } \alpha_{gm} = \frac{m_b \times v_g}{V_1} < \frac{1}{2}$$

$$= \frac{\alpha}{V_g} + \frac{(1-C) \cdot m_b}{V_1} \quad , \text{ if } \alpha_{gm} > \frac{1}{2}$$

$$\cdots\cdots (7)$$

$$\dot{m}_r = V_{rain} \times \text{area} \times (1. - \alpha) \quad \cdots\cdots (8)$$

where, V_b=bubble rise velocity,
area=cross sectional area,
m_b=mass of the bubble in the liquid region,
v_g=specific volume of the saturated steam,
α =void of the gas region.
V_{rain}=rain drop velocity.

Once the mass and internal energy of each region are obtained from above equations, the system pressure can be searched by applying the Newton-Raphson method. First, as an inner loop, the enthalpy of each region is searched with the assumed values of the pressure(usually, the value of prior step).

$$h_2 = \frac{U_2}{M_2} + c \cdot p \cdot v_2 \quad \cdots\cdots (9)$$

where h_2 and v_2 are specific enthalpy and volume of gas region

Then,

$$F_2(p, h_2) = h_2 - \frac{U_2}{M_2} - c \cdot p \cdot v_2(p, h_2) \quad \cdots (10)$$

$$\frac{\partial F_2}{\partial h_2} = 1 - c \cdot p \cdot \frac{\partial v_2}{\partial h_2} - c \cdot v_2 \quad \cdots\cdots\cdots (11)$$

$$h_2(i+1) = h_2(i) - \frac{F_2}{\left(\frac{\partial F_2}{\partial h_2}\right)} \quad \cdots\cdots\cdots\cdots\cdots (12)$$

Therefore,

$$h_1 = (U_1 + U_2 + c \cdot p \cdot V_{geo} - h_2 \cdot M_2)/M_1 \cdots (13)$$

where V_{geo} is the total volume of steam generator. With these values of enthalpies and with the volume constraint,

$$F_1 = V_{geo} - V_1 - V_2 = V_{geo} - M_1 \cdot v_1 - M_2 \cdot v_2 \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (14)$$

$$\frac{dF_1}{dp} = -M_1 \frac{dv_1}{dp} - v_1 \frac{dM_1}{dp} - M_2 \frac{dv_2}{dp} - v_2 \frac{dM_2}{dp} \quad \cdots (15)$$

where, v_1=specific volume of liquid region,
v_2=specific volume of gas region.

Equations (14) and (15) are used for the outer loop of the Newton-Raphson to search the pressure, whereas equations (10) and (11) comprise the inner loop to determine the enthalpies. This procedure iterates until the pressure converges up to desirable tolerances. In addition, the supplementary state equations and their derivatives as below are used.

For saturated states;

$$\frac{dv_f}{dh} = \frac{-1}{\rho_f^2} \frac{\partial \rho_f}{\partial h} - \frac{\partial \rho_f}{\partial \rho} \frac{dp}{dh_f} \cdot \frac{1}{\rho_f^2} \quad \cdots\cdots (16)$$

$$\frac{dv_g}{dh} = \frac{-1}{\rho_g^2} \cdot \frac{\partial \rho_g}{\partial h} - \frac{1}{\rho_g^2} \cdot \frac{\partial \rho_g}{\partial p} \cdot \frac{dp}{dh_g} \quad \cdots\cdots (17)$$

$$\frac{dv_f}{dp} = \frac{-1}{\rho_f^2} \cdot \frac{\partial \rho_f}{\partial \rho} - \frac{1}{\rho_f^2} \cdot \frac{\partial \rho_f}{\partial h} \cdot \frac{dh_f}{dp} \quad \cdots\cdots (18)$$

$$\frac{dv_g}{dp} = \frac{-1}{\rho_g^2} \cdot \frac{\partial \rho_g}{\partial p} - \frac{1}{\rho_g^2} \cdot \frac{\partial \rho_g}{\partial h} \cdot \frac{dh_g}{dp} \quad \cdots\cdots (19)$$

For the single phase;

$$\frac{dv_l}{dh} = -\frac{1}{\rho_l^2} \cdot \frac{d\rho_l}{dh} \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (20)$$

$$\frac{dv_v}{dh} = -\frac{1}{\rho_v^2} \cdot \frac{d\rho_v}{dh} \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (21)$$

$$\frac{dv_l}{dp} = -\frac{1}{\rho_l^2} \cdot \frac{d\rho_l}{dp} \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (22)$$

$$\frac{dv_v}{dp} = -\frac{1}{\rho_v^2} \cdot \frac{d\rho_v}{dh} \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (23)$$

For the two phase;

$$\left.\frac{\partial v_1}{\partial h_1}\right)_p = \frac{\partial v_f}{\partial h} + x_1 \cdot \left(\frac{\partial v_g}{\partial h} - \frac{\partial v_f}{\partial h}\right) + v_{fg} \cdot \left.\frac{\partial x_1}{\partial h_1}\right)_p \cdots (24)$$

$$\left.\frac{\partial v_1}{\partial p}\right)_{h1} = \frac{\partial v_f}{\partial p} + x_1 \cdot \left(\frac{\partial v_g}{\partial p} - \frac{\partial v_f}{\partial p}\right) + v_{fg} \cdot \left.\frac{\partial x_1}{\partial p}\right)_{h1} \cdots (25)$$

$$\left.\frac{\partial v_2}{\partial h_2}\right)_p = \frac{\partial v_f}{\partial h} + x_2 \cdot \left(\frac{\partial v_g}{\partial h} - \frac{\partial v_f}{\partial h}\right) + v_{fg} \cdot \left.\frac{\partial x_2}{\partial h_2}\right)_p \cdots (26)$$

$$\left.\frac{\partial v_1}{\partial p}\right)_{h2} = \frac{\partial v_f}{\partial p} + x_2 \cdot \left(\frac{\partial v_g}{\partial p} - \frac{\partial v_f}{\partial p}\right) + v_{fg} \cdot \left.\frac{\partial x_2}{\partial p}\right)_{h2} \cdots (27)$$

Once the enthalpy and pressure ar known, quality and specific volume of each region can be found, and accordingly the water level can be found.

3. Numerical Handling

The satisfation of the real time requirement depends on the speed of the convergence. But the Newton method is not so efficient when the derivative is small. Hence to boost the convergence, the bisection method is mixed with the Newton in the manner of below.

1) Enthalpy searching

The Newton is used through the iterations until the sign changes and then switches into bisection.

2) Pressure searching

Equation (14) shows that when the pressure is high, the quality decreases and therefore V_1 and V_2 decrease also, which makes $F_1(p)$ positive and vice versa. Taking this into account, a certain value of Δp is added (negative F_1) or substracted (positive F_1) during the iteration of the bisection and then the procedure is switched to the Newton when the signs of the i th and i+1 the step are different, say, $F_1(i) * F_1(i+1) < 0$.

The results of this switching procedure show a more efficient convergence than the Newton alone, since the bisection does not call the subroutines which calculate the derivatives of F_1 or F_2.

4. Application and Tests

To test the program developed in the above, a fictitious model is taken as an example of which overall height is 10 ft, cross sectional area 1 ft^2, and the pressure is 1000 psi. As an initial condition, both regions are assumed to be saturated.

Three cases are considered. The first is the simple compression with inlet flow rate of 1 lb/sec, and no outlet steam. The second is the extraction with outlet flow rate of 1 lb/sec and no inlet feed. Finally, the third one is of simultaneous inlet and outlet of 1 lb/sec.

On the other hand, for the purpose of comparison, a different approach (Motamed, 1983) has been studied to make its corresponding program, hereinafter called UMI, and has been applied to the same

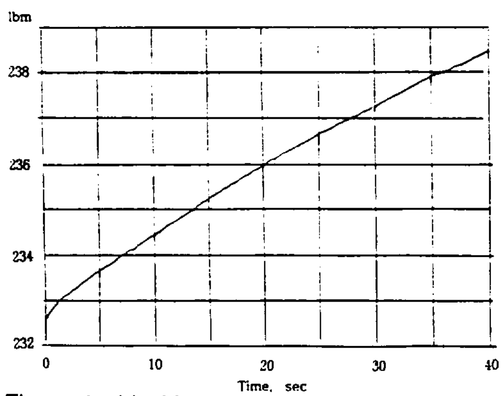

Figure 1. Liquid mass.

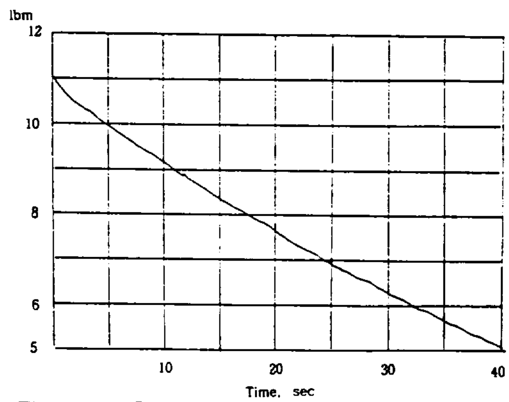

Figure 2. Gas mass.

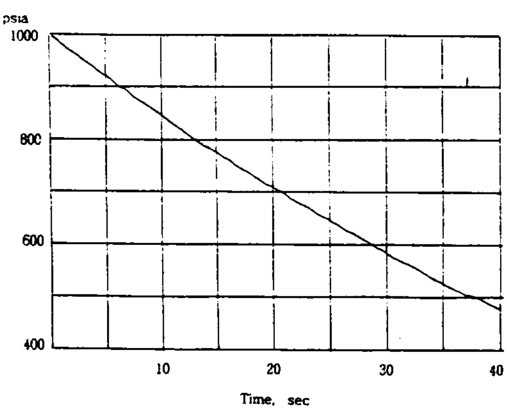

Figure 3. Pressure.

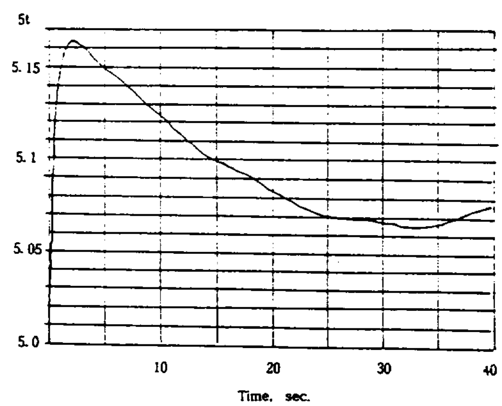

Figure 4. Level.

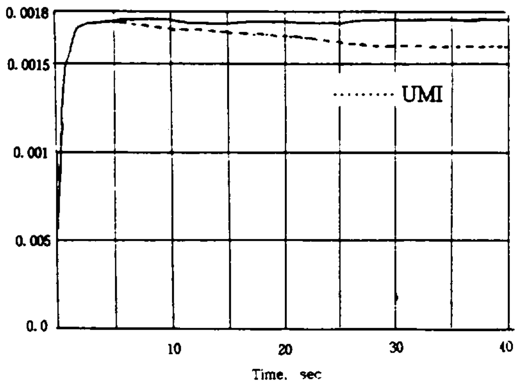

Figure 5. Liquid quality.

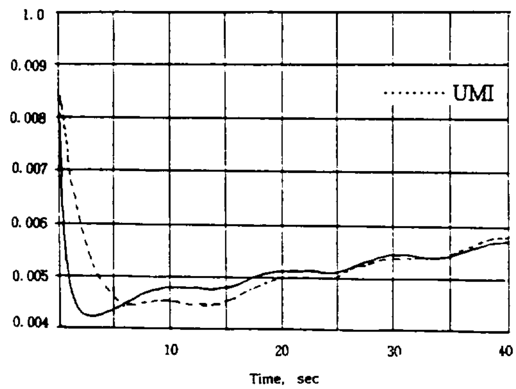

Figure 6. Steam quality.

model. The briefs of UMI is that four cases are considered in accordance with the states of each region (superheated or saturated in vapor region, and saturated or subcooled in liquid region), and mass and energy equations are applied for each case.

As shown in the figures, the results are almost same each other. But the computing time of the switching method is less than that of UMI, particularly when the time step interval is large. However both approaches yield a bad convergence efficiency when the time step is too large.

The figures above are the results of case 3 explained already. Figure 1 through 4 describes liquid mass, gas mass, pressure and level respectively. Those four figures are exaclty same for two approaches. Of interest is the level. As the pressure drops, more bubble generates (flashes) in the liquid region. This leads to the increase of liquid volume, accordingly the increase of the level. As the pressure drops on, the liquid quality reaches a certain saturated value and the level decreases by the specific volume differences of liquid and vapor.

Figures 5 and 6 show the quality of liquid and vapor for two methods. The values are somewhat different each other. Particularly, the vapor quality of the mixing method shows the minimum while UMI not. In a physical sense, the resut of UMI is preferable since it shows a smooth variation. However the absolute values are very small, and the difference does not effect the level variation as shown in the figures.

5. Discussions and Limitation

The model above has its own intrinsic drawbacks. First of all, the pressure is common to liquid and vapor region. But when the physical size is large and the flow resistance is not negligible, the pressure varies along the height. This results in the variation of major parameters along the height, and the heat transfer rate becomes different by the presence of the gas in the liquid region. In addition, the bubble rise model requires an exact value of bubble velocities which are to be determined experimentally. To eliminate these drawbacks, the moving boundary model can be considered. In this model, the boundary between the gas and liquid region is a time dependent junction of which the position is the water level. Further, by introducing the moving boundary in the liquid region which bounds the single and two phase zone, more exact value of heat transfer can be obtained.

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적 요

원자력 발전소의 증기발생장치는 발전소의 안전성을 위하여 항상 일정수위를 유지하도록 되어 있으나, 운전중 뜻하지 않은 가동정지를 유발시켜 효율성을 저하시키는 사례가 많이 발생되고 있다. 따라서 안전성 및 효율성을 동시에 만족시키기 위해서 설정치의 조절, 가동 정지논리의 개선, 운전조건 및 제어계통의 개선등이 모색되고 있으며, 이러한 것들을 위해서는 일단 정확한 수위예측이 필요하게 된다. 본고에서는 운전중 제어계통의 입력으로 사용될 수 있게끔 계산속도가 증가된 모델을 만들어 그의 결과들을 다른 방법과 비교 검토하였다.