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Integral formulas on a Riemannian foliation

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Abstract. In this paper, we study the infinitesimal automorphisms on a Riemannian foliation and establish the integral formulas for them.

1 Introduction

Let $(M, g_M, \mathcal{F})$ be a compact Riemannian manifold of dimension $p+q$ with a transversally oriented foliation $\mathcal{F}$ of codimension q and a bundle-like metric g_M with respect to $\mathcal{F}$. Let L be the tangent bundle of $\mathcal{F}$ and $Q = TM/L$ the normal bundle of $\mathcal{F}$. A vector field Y on M is called an infinitesimal automorphism of $\mathcal{F}$ if the flow generated by Y preserves the foliation, that is, maps leaves into leaves. In other words, for any $Z \in \Gamma L, [Y, Z] \in \Gamma L$. There has been extensive studies of geometric infinitesimal automorphisms of a minimal Riemannian foliation by many differential geometers. In this paper, we extend well-known integral formulas concerning infinitesimal automorphisms on a Riemannian manifold to a foliated manifold, which $\mathcal{F}$ is non-minimal.

2 Preliminaries

Let $(M, g_M, \mathcal{F})$ be a $(p+q)$-dimensional Riemannian manifold with a foliation $\mathcal{F}$ of codimension q and a bundle-like metric g_M with respect to $\mathcal{F}$. Let ∇^M be the Levi-Civita connection with respect to g_M. Let TM be the tangent bundle of M and L the integrable subbundle of TM given by $\mathcal{F}$. The normal bundle Q of $\mathcal{F}$ is given by $Q = TM/L$. Then there exists an exact sequence of vector bundles

$$0 \longrightarrow L \longrightarrow TM \underset{\sigma}{\overset{\pi}{\rightleftarrows}} Q \longrightarrow 0. \tag{2.1}$$

Let g_Q be the holonomy invariant metric on Q induced by g_M, that is,

$$g_Q(s,t) = g_M(\sigma(s), \sigma(t)) \quad \forall\ s,t \in \Gamma Q. \tag{2.2}$$

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This means that $\theta(X)g_Q = 0$ for $X \in \Gamma L$, where $\theta(X)$ is the transverse Lie derivative. A transversal Levi-Civita connection ∇ in Q is defined by ([5,11])

$$\nabla_X s = \begin{cases} \pi([X, Y_s]) & \forall X \in \Gamma L \\ \pi(\nabla^M_X Y_s) & \forall X \in \Gamma L^\perp, \end{cases} \tag{2.3}$$

where $s \in \Gamma Q$ and $Y_s \in \Gamma L^\perp$ corresponding to s under the canonical isomorphism $Q \cong L^\perp$. The curvature R^∇ of ∇ is defined by $R^\nabla(X,Y) = [\nabla_X, \nabla_Y] - \nabla_{[X,Y]}$ for $X, Y \in \Gamma TM$. Since $i(X)R^\nabla = 0$ for any $X \in \Gamma L$([11]), the operator $R^\nabla : Q \to Q$ is a well-defined endomorphism. Hence the transversal Ricci curvature ρ^∇ is defined by

$$\rho^\nabla(s_x) = \sum_{a=p+1}^{n} R^\nabla(s, e_a)e_a, \tag{2.4}$$

where $\{e_a\}_{a=p+1,\cdots,n}$ is an orthonormal basis of Q_x. And the transversal scalar curvature σ^∇ is given by $\sigma^\nabla = \mathrm{Tr}\rho^\nabla$. The foliation $\mathcal{F}$ is said to be (transversally) *Einsteinian* if the model space N is Einsteinian, that is,

$$\rho^\nabla = \frac{1}{q}\sigma^\nabla \cdot id \tag{2.5}$$

with constant transversal scalar curvature σ^∇. The *mean curvature vector* $\kappa^\sharp$ of $\mathcal{F}$ is defined by

$$\kappa^\sharp = \pi\Big(\sum_{i=1}^{p} \nabla^M_{E_i} E_i\Big), \tag{2.6}$$

where $\{E_i\}$ is a local orthonormal basis of L. The foliation $\mathcal{F}$ is said to be *minimal* if $\kappa^\sharp = 0$.

For the later use, we recall the divergence theorem on a foliated Riemannian manifold ([19]).

Theorem 2.1 *Let $(M, g_M, \mathcal{F})$ be a closed, oriented, connected Riemannian manifold with a transversally orientable foliation $\mathcal{F}$ and a bundle-like metric g_M with respect to $\mathcal{F}$. Then*

$$\int_M div_\nabla(X) = \int_M g_Q(X, \kappa^\sharp) \tag{2.7}$$

for all $X \in \Gamma Q$, where $div_\nabla(X)$ denotes the transversal divergence of X with respect to the connection ∇ defined by (2.3).

A differential form $\omega \in \Omega^r(M)$ is *basic* if

$$i(X)\omega = 0, \ \ \theta(X)\omega = 0, \quad \forall X \in \Gamma L. \tag{2.8}$$

Let $\Omega_B^r(\mathcal{F})$ be the set of all basic r-forms on M. The foliation $\mathcal{F}$ is said to be *isoparametric* if $\kappa \in \Omega_B^1(\mathcal{F})$, where κ is a g_Q-dual 1-form of $\kappa^\sharp$. Then it is well-known([11,12]) that on an isoparametric Riemannian foliation $\mathcal{F}$, the mean curvature form κ is closed, i.e., $d\kappa = 0$.

The *basic Laplacian* acting on $\Omega_B^*(\mathcal{F})$ is defined by

$$\Delta_B = d_B\delta_B + \delta_B d_B, \tag{2.9}$$

where δ_B is a formal adjoint of $d_B = d|_{\Omega_B^*(\mathcal{F})}$, which are locally given by

$$d_B = \sum_a E_a \wedge \nabla_{E_a}, \quad \delta_B = -\sum_a i(E_a)\nabla_{E_a} + i(\kappa^\sharp), \tag{2.10}$$

where $\{E_a\}$ is a local orthonormal basic frame on Q.

3 Integral formulas

Let $(M, g_M, \mathcal{F})$ be be a closed, oriented, connected Riemannian manifold of dimension $p+q$ with a transversally oriented foliation $\mathcal{F}$ of codimension q and a bundle-like metric g_M with respect to $\mathcal{F}$. Let $V(\mathcal{F})$ be the space of all vector fields Y on M satisfying $[Y, Z] \in \Gamma L$ for all $Z \in \Gamma L$. An element of $V(\mathcal{F})$ is called an *infinitesimal automorphism* of $\mathcal{F}$. Let

$$\bar{V}(\mathcal{F}) = \{\bar{Y} = \pi(Y) \in \Gamma Q | Y \in V(\mathcal{F})\}. \tag{3.1}$$

It is trivial that an element s of $\bar{V}(\mathcal{F})$ satisfies $\nabla_X s = 0$ for all $X \in \Gamma L$([6,11]).

Definition 3.1 For any vector field $Y \in V(\mathcal{F})$, we define an operator $A_Y : \Gamma Q \to \Gamma Q$ as

$$A_Y s = \theta(Y)s - \nabla_Y s. \tag{3.2}$$

Remark. Let $Y_s \in \Gamma TM$ with $\pi(Y_s) = s$. Then it is trivial that for any $Y \in V(\mathcal{F})$

$$A_Y s = -\nabla_{Y_s}\pi(Y). \tag{3.3}$$

So A_Y depends only on $s = \pi(Y)$ and is a linear operator. Moreover, A_Y extends in an obvious way to tensors of any type on Q (see [6] for details). In particular, for any basic 1-form $\phi \in \Omega_B^1(\mathcal{F})$, the operator A_Y is given by

$$(A_Y\phi)(X) = -\phi(A_Y X) \quad \forall X \in \Gamma Q. \tag{3.4}$$

Now, we introduce the operator $\nabla_{tr}^*\nabla_{tr} : \Omega_B^*(\mathcal{F}) \to \Omega_B^*(\mathcal{F})$ as

$$\nabla_{tr}^*\nabla_{tr}\phi = -\sum_a \nabla^2_{E_a,E_a}\phi + \nabla_{\kappa^\sharp}\phi, \tag{3.5}$$

where $\nabla^2_{X,Y} = \nabla_X\nabla_Y - \nabla_{\nabla^M_X Y}$ for any $X, Y \in TM$. Then we have the following.

Theorem 3.2 *On the Riemannian foliation $\mathcal{F}$ on M, we have*

$$\Delta_B\phi = \nabla_{tr}^*\nabla_{tr}\phi + A_{\kappa^\sharp}\phi + F(\phi) \tag{3.6}$$

for $\phi \in \Omega_B^r(\mathcal{F})$, where $F(\phi) = \sum_{a,b} E^a \wedge i(E_b)R^\nabla(E_b, E_a)\phi$. In particular, if ϕ is a basic 1-form, then $F(\phi)^\sharp = \rho^\nabla(\phi^\sharp)$.

Proof. Fix $x \in M$ and let $\{E_a\}$ be an orthonormal basis for Q with $(\nabla E_a)_x = 0$. Then from (2.10) we have

$$d_B\delta_B\phi = -\sum_{a,b} E^a \wedge i(E_b)\nabla_{E_a}\nabla_{E_b}\phi + d_B i(\kappa^\sharp)\phi$$

and

$$\delta_B d_B\phi = \sum_a -\nabla_{E_a}\nabla_{E_a}\phi + \sum_{a,b} E^a \wedge i(E_b)\nabla_{E_b}\nabla_{E_a}\phi + i(\kappa^\sharp)d_B\phi.$$

Summing up the above two equations, we have

$$\begin{aligned}
\Delta_B\phi &= d_B i(\kappa^\sharp)\phi + i(\kappa^\sharp)d_B\phi - \sum_a \nabla_{E_a}\nabla_{E_a}\phi + \sum_{a,b} E^a \wedge i(E_b)R^\nabla(E_b, E_a)\phi \\
&= \theta(\kappa^\sharp)\phi - \sum_a \nabla_{E_a}\nabla_{E_a}\phi + \sum_{a,b} E^a \wedge i(E_b)R^\nabla(E_b, E_a)\phi \\
&= \nabla_{tr}^*\nabla_{tr}\phi + F(\phi) + A_{\kappa^\sharp}\phi.
\end{aligned}$$

On the other hand, let ϕ be a basic 1-form and $\phi^\sharp$ its g_Q-dual vector field. Then

$$\begin{aligned}
g_Q(F(\phi), E^c) &= \sum_{a,b} g_Q(E^a \wedge i(E_b)R^\nabla(E_b, E_a)\phi, E^c) \\
&= \sum_b i(E_b)R^\nabla(E_b, E_c)\phi = \sum_b g_Q(R^\nabla(E_b, E_c)\phi^\sharp, E_b) \\
&= \sum_b g_Q(R^\nabla(\phi^\sharp, E_b)E_b, E_c) = g_Q(\rho^\nabla(\phi^\sharp), E_c).
\end{aligned}$$

This yields that for any basic 1-form ϕ, $F(\phi)^\sharp = \rho^\nabla(\phi^\sharp)$. □

From (3.4) and (3.6), we have the following corollary.

Corollary 3.3 *On the Riemannian foliation $\mathcal{F}$ on M, we have that for any $X \in \bar{V}(\mathcal{F})$*

$$\Delta_B X = \nabla_{tr}^* \nabla_{tr} X + \rho^{\nabla}(X) - A_{\kappa^\sharp}^t X. \tag{3.7}$$

Proposition 3.4 *For any basic function f on M, it holds that*

$$\int_M \Delta_B f = 0. \tag{3.8}$$

Proof. From (2.9), we have

$$\Delta_B f = \delta_B d_B f = -\sum_a i(E_a)\nabla_{E_a} d_B f + i(\kappa^\sharp) d_B f = -div_\nabla(d_B f) + i(\kappa^\sharp) d_B f.$$

Then the divergence theorem (2.7) implies

$$\int_M \Delta_B f = -\int_M div_\nabla(d_B f) + \int_M g_Q(\kappa^\sharp, d_B f) = 0. \quad \square$$

Note that on M, the direct calculation gives

$$\frac{1}{2}\Delta_B f^2 = (\Delta_B f)f - |\nabla_{tr} f|^2, \tag{3.9}$$

which yields

$$\int_M \{(\Delta_B f)f - |\nabla_{tr} f|^2\} = 0. \tag{3.10}$$

Hence we have the following proposition.

Proposition 3.5 *On the Riemannian foliation $\mathcal{F}$ on M, if a basic function f satisfies $\Delta_B f \geq 0$ (or$\Delta_B f \leq 0$), then f is constant on M.*

Proof. By Proposition 3.4, if $\Delta_B f \geq 0$, then $\Delta_B f = 0$. So f is constant from (3.10). □

Proposition 3.6 *For any basic function f and a constant λ on M, if $\Delta_B f = \lambda f$, then λ is positive.*

Proof. From (3.10), if $\Delta_B f = \lambda f$, then we have

$$\int_M \{(\lambda f)f - |\nabla_{tr} f|^2\} = 0,$$

which implies $\lambda > 0$. □

Proposition 3.8 *On the Riemannian foliation $\mathcal{F}$ on M, any vector field $X \in \bar{V}(\mathcal{F})$ satisfies*

$$\begin{aligned}
&-div_\nabla(A_X X) - div_\nabla(div_\nabla(X)X) \\
&\quad = g_Q(\rho^\nabla(X), X) + \frac{1}{2}|\theta(X)g_Q|^2 - |\nabla_{tr}X|^2 - (\delta_T X)^2 \\
&\quad = g_Q(\rho^\nabla(X), X) - \frac{1}{2}|d_B\xi|^2 + |\nabla_{tr}X|^2 - (\delta_T X)^2.
\end{aligned}$$

Proof. By a direct calculation with (3.3), it holds that for any $X \in \bar{V}(\mathcal{F})$

$$\begin{aligned}
div_\nabla(A_X X) &= -\sum_a g_Q(\nabla_{E_a}\nabla_X X, E_a), \\
div_\nabla(div_\nabla(X)X) &= X div_\nabla(X) + (div_\nabla(X))^2.
\end{aligned}$$

Since $X div_\nabla(X) = X g_Q(\nabla_{E_a}X, E_a) = g_Q(\nabla_X\nabla_{E_a}, E_a)$, we have

$$\begin{aligned}
&div_\nabla(div_\nabla(X)X) + div_\nabla(A_X X) \\
&\quad = \sum_a g_Q(\nabla_X\nabla_{E_a}X - \nabla_{E_a}\nabla_X X, E_a) + (div_\nabla(X))^2 \\
&\quad = \sum_a g_Q(R^\nabla(X, E_a)X + \nabla_{[X,E_a]}X, E_a) + (div_\nabla(X))^2 \\
&\quad = -g_Q(\rho^\nabla(X), X) - \sum_a g_Q(A_X A_X E_a, E_a) + (div_\nabla(X))^2.
\end{aligned}$$

From Lemma 3.7, the proof is completed. □

Corollary 3.9 *On the Riemannian foliation $\mathcal{F}$ on M, any vector field $X \in \bar{V}(\mathcal{F})$ satisfies*

$$\begin{aligned}
&\int_M [g_Q(\rho^\nabla(X), X) + \frac{1}{2}|\theta(X)g_Q|^2 - |\nabla_{tr}X|^2 - (\delta_T X)^2] \\
&\quad + \int_M [div_\nabla(A_X X) + div_\nabla(div_\nabla(X)X)] = 0
\end{aligned} \tag{3.15}$$

or

$$\begin{aligned}
&\int_M [g_Q(\rho^\nabla(X), X) - \frac{1}{2}|d_B\xi|^2 + |\nabla_{tr}X|^2 - (\delta_T X)^2] \\
&\quad + \int_M [div_\nabla(A_X X) + div_\nabla(div_\nabla(X)X)] = 0.
\end{aligned} \tag{3.16}$$

From (3.7), (3.15) and (3.16), we have the following corollary.

Lemma 3.7 *For any vector field $X \in \bar{V}(\mathcal{F})$ on M, it holds that*

$$\begin{aligned} TrA_XA_X &= -\frac{1}{2}|d_B\xi|^2 + |\nabla_{tr}X|^2 \\ &= \frac{1}{2}|\theta(X)g_Q|^2 - |\nabla_{tr}X|^2, \end{aligned}$$

where ξ is g_Q-dual 1-form of X.

Proof. For any basic 1-form ϕ, it is well-known that

$$(d_B\phi)(Y,Z) = Y\phi(Z) - Z\phi(Y) - \phi([Y,Z]), \quad \forall X,Y \in \Gamma Q.$$

Since $[E_a, E_b] = 0$, we have that at $x \in M$

$$\begin{aligned} |d_B\xi|^2 =& \sum_{a,b}\{(d_B\xi)(E_a,E_b)\}^2 \\ =& \sum_{a,b}\{E_a\xi(E_b) - E_b\xi(E_a)\}^2 = \sum_{a,b}\{g_Q(\nabla_{E_a}X,E_b) - g_Q(\nabla_{E_b}X,E_a)\}^2 \\ =& 2|\nabla X|^2 - 2\sum_{a,b} g_Q(\nabla_{E_a}X,E_b)g_Q(\nabla_{E_b}X,E_a). \end{aligned} \tag{3.11}$$

On the other hand, from (3.2) it is trivial that

$$TrA_XA_X = \sum_{a,b} g_Q(\nabla_{E_a}X,E_b)g_Q(\nabla_{E_b}X,E_a). \tag{3.12}$$

Hence the first equation in Lemma 3.7 is proved from (3.11) and (3.12). Next, it is well-known that

$$\begin{aligned} TrA_XA_X &= -TrA_X^tA_X + \frac{1}{2}Tr(A_X + A_X^t)^2 \\ &= -|\nabla_{tr}X|^2 + \frac{1}{2}Tr(A_X + A_X^t)^2. \end{aligned} \tag{3.13}$$

Moreover, since $(\theta(X)g_Q)(Y,Z) = g_Q(\nabla_Y X, Z) + g_Q(\nabla_Z X, Y)$ for any $X,Y,Z \in \Gamma Q$, we have

$$\begin{aligned} |\theta(X)g_Q|^2 =& \sum_{a,b}\{g_Q(\nabla_{E_a}X,E_b) + g_Q(\nabla_{E_b}X,E_a)\}^2 \\ =& \sum_{a,b} g_Q((A_X + A_X^t)E_a,E_b)^2 = Tr(A_X + A_X^t)^2. \end{aligned} \tag{3.14}$$

From (3.13) and (3.14), the second equation is proved. □

Corollary 3.10 *On the Riemannian foliation $\mathcal{F}$ on M, any vector field $X \in \bar{V}(\mathcal{F})$ satisfies*

$$\int_M \{g_Q(\Delta_B X, X) - 2g_Q(\rho^\nabla(X), X) - \frac{1}{2}|\theta(X)g_Q|^2 + (\delta_T X)^2\} \tag{3.17}$$
$$+ \int_M \{g_Q(A_{\kappa^\sharp} X, X) - div_\nabla(A_X X) - div_\nabla(div_\nabla(X)X)\} = 0,$$

or

$$\int_M \{g_Q(\Delta_B X, X) - \frac{1}{2}|d_B\xi|^2 - (\delta_T X)^2\} \tag{3.18}$$
$$+ \int_M \{g_Q(A_{\kappa^\sharp} X, X) + div_\nabla(A_X X) + div_\nabla(div_\nabla(X)X)\} = 0.$$

Lemma 3.11 *On the Riemannian foliation $\mathcal{F}$ on M, any vector field $X \in \bar{V}(\mathcal{F})$ satisfies*

$$|\theta(X)g_Q + \frac{2}{q}(\delta_T X)|^2 = |\theta(X)g_Q|^2 - \frac{4}{q}(\delta_T X)^2.$$

Proof. A direct calculation gives

$$\begin{aligned}
|\theta(X)g_Q + \frac{2}{q}(\delta_T X)|^2 &= |\theta(X)g_Q|^2 + \frac{4}{q}(\delta_T X)^2 + \frac{4}{q}(\delta_T X)\sum_a(\theta(X)g_Q)(E_a, E_a) \\
&= |\theta(X)g_Q|^2 + \frac{4}{q}(\delta_T X)^2 - \frac{8}{q}(\delta_T X)^2 \\
&= |\theta(X)g_Q|^2 - \frac{4}{q}(\delta_T X)^2. \quad \square
\end{aligned}$$

From Corollary 3.10 and Lemma 3.11, we have the following.

Corollary 3.12 *On the Riemannian foliation $\mathcal{F}$ on M, any vector field $X \in \bar{V}(\mathcal{F})$ satisfies*

$$\int_M \{g_Q(\Delta_B X, X) - 2g_Q(\rho^\nabla(X), X) - \frac{1}{2}|\theta(X)g_Q + \frac{2}{q}(\delta_T X)|^2 + \frac{q-2}{q}(\delta_T X)^2\}$$
$$+ \int_M \{g_Q(A_{\kappa^\sharp} X, X) - div_\nabla(A_X X) - div_\nabla(div_\nabla(X)X)\} = 0. \tag{3.19}$$

Lemma 3.13 *On the Riemannian foliation $\mathcal{F}$ on M, any vector field $X \in \bar{V}(\mathcal{F})$ satisfies*

$$\int_M \{g_Q(A_{\kappa^\sharp} X, X) + div_\nabla(A_X X)\} = -\int_M X g_Q(\kappa^\sharp, X), \tag{3.20}$$
$$\int_M div_\nabla(div_\nabla(X)X) = -\int_M (\delta_T X) g_Q(X, \kappa^\sharp). \tag{3.21}$$

Proof. Equation (3.21) is followed from the divergence theorem. From (3.3) and the divergence theorem, (3.20) is proved. □

Now we denote $VK^{\perp}(\mathcal{F})$ by

$$VK^{\perp}(\mathcal{F}) = \{X \in \bar{V}(\mathcal{F}) | g_Q(X, \kappa^{\sharp}) = 0\}. \tag{3.22}$$

Then we have the following theorem.

Theorem 3.14 *Let $(M, g_M, \mathcal{F})$ be a compact Riemannian manifold with a foliation $\mathcal{F}$ and a bundle-like metric g_M. For any vector field $X \in VK^{\perp}(\mathcal{F})$ we have*

$$\int_M \{g_Q(\Delta_B X, X) - 2g_Q(\rho^{\nabla}(X), X) - \frac{1}{2}|\theta(X)g_Q + \frac{2}{q}(\delta_T X)|^2\} \tag{3.23}$$
$$+ \int_M \{\frac{q-2}{q}(\delta_T X)^2\} + 2g_Q(A_{\kappa^{\sharp}} X, X)\} = 0.$$

Proof. From Corollary 3.12 and Lemma 3.13, it is trivial. □

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