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Cyclotron transition linewidths due to electron–phonon interaction via piezoelectric scattering

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Abstract. A theory of cyclotron transitions in electron–phonon systems is presented with the help of the projection method and applied to obtaining the cyclotron resonance linewidth for the piezoelectric polaron system. The linewidth in the moderate-temperature quantum limit is proportional to the square root of temperature and independent of the magnetic field for 'adiabatic' processes, which is in qualitative agreement with the experiment of Baer *et al* and with theoretical values of Saitoh *et al* and Arora. The width for 'non-adiabatic' processes is proportional to the temperature and the inverse square root of the field. The results in the low-temperature regions in the quantum limit agree well with those of Larsen and Choi *et al*.

1. Introduction

The study of cyclotron resonance absorption lineshapes is useful in obtaining information about the scattering mechanisms for the charge carriers in solids. Since Meyer and Polder (1953) discussed the linewidth for piezoelectric scattering, many theoretical (Mahan and Hopfield 1964, Zook 1964, Larsen 1966, Kawabata 1967, Saitoh and Kawabata 1967, Lodder and Fujita 1968, Miyake 1968, Tam 1969, Argyres and Sigel 1974, Arora and Spector 1979, Choi and Fujita 1981, Choi and Chung 1983, Pal and Sharma 1974, Prasad 1982, Suzuki and Dunn 1982, Ryu and Choi 1984, Ryu *et al* 1985) and experimental (Baer and Dexter 1964, Sawamoto 1964, Ohyama *et al* 1986) studies have been made of the linewidths, dealing in particular with their dependence on the magnetic field (B) and temperature (T).

Especially, widths in the quantum-limit condition have drawn the attention of many theoreticians. Mahan and Hopfield (1964) noticed the importance of the effect of piezoelectric electron–phonon interaction upon the properties of low-energy electrons and obtained the width based on semiclassical arguments. Larsen (1966) undertook a quantum-mechanical calculation by extending the semiclassical theory and obtained a width proportional to $B^{1/4}$. Saitoh and Kawabata (1967) got a quantum-mechanical expression for the lineshape function using Mori's method (Mori 1965) and derived expressions applicable in both classical and quantum conditions. They predicted a $T^{1/2}$ dependence for the elastic and non-adiabatic scattering processes in the moderate quantum limit in which the magnetic field is strong but the temperature is comparatively high.

Miyake (1968) carried out a quantum-mechanical calculation of the width at moderate temperatures by using the thermal Green function method (Kadanoff and Baym 1963) and predicted $BT^{-3/2}$ dependence. Tam (1969) derived an expression for the energy and width using Kubo's (1957) formula and the method of double-time thermal Green functions (Zubarev 1960). Arora and Spector (1979) obtained a width that squares with that of Saitoh and Kawabata (1967) by using the density-matrix formalism method.

Choi and Fujita (1981), utilising Lodder and Fujita's (1968) quantum theory of width, obtained the same result as Larsen (1966) at zero temperature. Suzuki and Dunn (1982) developed a quantum theory using the Kubo formalism and the resolvent superoperator expansion method and predicted $T^{4.5}B^{-2}$ and T type dependences for low- and moderate-temperature cases, respectively. Ohyama *et al* (1986) discussed various scattering mechanisms, including acoustic piezoelectric scattering.

It is to be noted, however, that all these theoretical investigations have produced a bewildering variety of results. So the situation in the quantum-limit condition still remains unclear.

The purpose of the present paper is first to obtain the linewidth for piezoelectric potential scattering by utilising the projection method introduced by Mori (1965) in the case of adiabatic and non-adiabatic scattering processes, and secondly to compare the linewidth with other results in the quantum-limit condition in the moderate-temperature region. Comparison will also be made in the zero temperature region.

2. Theory of cyclotron resonance transitions

The average power P per unit volume delivered by a circularly polarised electromagnetic wave of frequency ω and amplitude E is given by (Kawabata 1967)

$$P = (E^2/2)\,\mathrm{Re}[\sigma_{+-}(\omega)] \tag{2.1}$$

where the symbol Re means 'the real part of'. The conductivity tensor $\sigma_{+-}(\omega)$ is the Fourier Laplace transform (FLT) of the response function $\Phi_{+-}(t)$ defined by

$$\sigma_{+-}(\omega) = \int_0^\infty \mathrm{d}t\,\Phi_{+-}(t)\exp(-\mathrm{i}\bar{\omega}t) \tag{2.2}$$

where $\bar{\omega} = \omega - \mathrm{i}/\tau_0$, τ_0 being the field-free relaxation time (Jones and March 1973), and the response function $\Phi_{+-}(t)$ is the current-correlation integral given by (Kubo 1957)

$$\Phi_{+-}(t) = \mathrm{Lim}(1/\Omega)\int_0^\beta \mathrm{d}\beta_1\,\mathrm{Tr}[\rho J^-(-\mathrm{i}\hbar\beta_1)J^+(t)]. \tag{2.3}$$

Here Ω is the volume of the system, $\beta = (k_B T)^{-1}$, $\rho = \rho(H)$ stands for the grand canonical density operator normalised to unity, $J^\pm = J_x \pm \mathrm{i}J_y$, $J(t)$ standing for the total current operator in the Heisenberg picture, the symbol Tr means the many-body trace with respect to the electrons and phonons, H is the Hamiltonian of the system, and the

symbol Lim means the bulk limit. By using the modified Hamiltonian formalism (Fujita 1966), equation (2.3) can be changed into

$$\Phi_{+-}(t) = \mathrm{Lim}(1/\Omega) \lim_{u_l \to 0} \partial/\partial u_l \langle \mathrm{Tr}^e[\rho' J^+(t)] \rangle. \tag{2.4}$$

Here Tr^e is the many-electron trace, $\boldsymbol{u}$ (u_x, u_y, u_z) is a constant vector, $\langle \ldots \rangle$ denotes the average over the phonon background, and

$$\rho' \equiv \exp(\alpha \bar{N} - \beta H') / \mathrm{Tr}^e[\exp(\alpha \bar{N} - \beta H')] \tag{2.5}$$

where $\bar{N}$ is the number of conduction electrons, $H' = H - \boldsymbol{J} \cdot \boldsymbol{u}$, and $\alpha \equiv \beta\zeta$, ζ being the chemical potential. It should be noted that the phonons are assumed not to be correlated with electrons in equation (2.4).

In the presence of a constant magnetic field along the z axis, characterised by the vector potential $A = (0, Bx, 0)$, the Hamiltonian H of a system of non-interacting electrons in a phonon field is given by

$$H = \sum_i^{\bar{N}} h^{(i)} + H_p \tag{2.6}$$

$$h \equiv h_0 + V \tag{2.7}$$

$$h_0 = (\boldsymbol{p} + e\boldsymbol{A})^2 / 2m \tag{2.8}$$

$$V = \sum_{\boldsymbol{q}} (\gamma_{\boldsymbol{q}} b_{\boldsymbol{q}} + \gamma_{\boldsymbol{q}}^+ b_{\boldsymbol{q}}^+) \tag{2.9}$$

$$\gamma_{\boldsymbol{q}} = C_{\boldsymbol{q}} \exp(\mathrm{i}\boldsymbol{q} \cdot \boldsymbol{r}) \tag{2.10}$$

$$H_p = \hbar \sum_{\boldsymbol{q}} \omega_{\boldsymbol{q}} b_{\boldsymbol{q}}^+ b_{\boldsymbol{q}} \tag{2.11}$$

where $b_{\boldsymbol{q}}^+$ and $b_{\boldsymbol{q}}$, respectively, are the creation and annihilation operators of a phonon with momentum $\hbar\boldsymbol{q}$ and energy $\hbar\omega_{\boldsymbol{q}}$, $C_{\boldsymbol{q}}$ is the interaction operator and $\boldsymbol{p}$ is the momentum of a conduction electron with effective mass m.

Then, neglecting the spins, we have

$$h_0 = [p_x^2 + (p_y + m\omega_0 x)^2 + p_z]/2m \tag{2.12}$$

where ω_0 is the resonance frequency given by $\omega_0 \equiv eB/m$. The eigenfunctions of h_0 are characterised by oscillator quantum numbers $N = 0, 1, 2, \ldots$ and two-dimensional k-vectors $\boldsymbol{k} = (k_y, k_z)$ as follows:

$$\Psi_{N,\boldsymbol{k}}(r) = \phi_N(x - X)(L_y L_z)^{-1/2} \exp(\mathrm{i}yk_y + \mathrm{i}zk_z) \tag{2.13}$$

$$\phi_N(x) = (2^N N! r_0 \sqrt{\pi})^{-1/2} H_N(x/r_0) \exp(-x^2/2r_0^2) \tag{2.14}$$

where $r_0 = (\hbar/m\omega_0)^{1/2}$, $X = -\hbar k_y/m\omega_0$, H_N is the Nth Hermite polynomial, and L_y and L_z are normalisation lengths. The energy eigenvalues are given by

$$E_{N,k} = (N + \tfrac{1}{2})\hbar\omega_0 + \varepsilon(k_z) \tag{2.15}$$

$$\varepsilon(k_z) = \hbar^2 k_z^2 / 2m. \tag{2.16}$$

Hereafter, $E_\alpha \equiv E_{N_\alpha, k_{\alpha z}}$ and $|\alpha\rangle \equiv \Psi_{N_\alpha, k_\alpha}$ will denote the energy and the eigenstate with $\alpha = (N_\alpha, \boldsymbol{k}_\alpha)$ respectively and $\alpha + 1$ will indicate $(N_\alpha + 1, \boldsymbol{k}_\alpha)$.

Then $\sigma_{+-}(\omega)$ is reduced to the single-electron formalism (Kawabata 1967, Lodder and Fujita 1968, Suzuki and Dunn 1982):

$$\sigma_{+-}(\omega) = (\hbar\omega_0)^{-1} \sum_{\alpha} [f(E_\alpha + \hbar\omega_0) - f(E_\alpha)]\tilde{Y}_\alpha(\bar{\omega}). \tag{2.17}$$

Here $f(E)$ stands for the electron distribution function and $\tilde{Y}_\alpha(\bar{\omega})$ is the FLT of $(j^-, j^+(t))_\alpha$, where $j^+(t) = \exp(iLt/\hbar)j^+$, $j^\pm = j_x \pm ij_y$, j being the single-electron current operator,

$$(A, B)_\alpha \equiv (AB)_{\alpha\alpha} \equiv \sum_{\mu} \langle\alpha|A|\mu\rangle\langle\mu|B|\alpha\rangle \tag{2.18}$$

and

$$L = L_0 + L_1 + L_p \tag{2.19}$$

where L_0, L_1 and L_p respectively denote the Liouville operators corresponding to the unperturbed part of the single-electron Hamiltonian h_0, the electron–phonon interaction Hamiltonian V and the phonon Hamiltonian H_p.

For the calculation of $\tilde{Y}_\alpha(\bar{\omega})$, we now define the projection operators P_α and $\bar{P}_\alpha$ as

$$P_\alpha X = (j^-, X)_\alpha/(j^-, j^+)_\alpha \tag{2.20}$$

$$\bar{P}_\alpha = 1 - P_\alpha \tag{2.21}$$

where X is any linear operator. Following Mori (1965), we separate $j^+(t)$ into the projective and vertical components with respect to the j^+ as

$$j^+(t) = P_\alpha j^+(t) + \bar{P}_\alpha j^+(t) = Z_\alpha(t)j^+ + \int_0^t Z_\alpha(s)f_1(t-s)\,ds \tag{2.22}$$

where

$$Z_\alpha(t) = (j^-, j^+(t))_\alpha/(j^-, j^+)_\alpha \tag{2.23}$$

$$f_1(t) = \exp(i\bar{P}_\alpha Lt/\hbar)f_1 \tag{2.24}$$

$$f_1 = i\bar{P}_\alpha Lj^+/\hbar. \tag{2.25}$$

In order to obtain $(j^-, j^+(t))_\alpha$ or $Z_\alpha(t)$, we differentiate equation (2.23) as

$$\frac{dZ_\alpha(t)}{dt} = i\omega_\alpha Z_\alpha(t) - \int_0^t \Gamma_\alpha(t-s)Z_\alpha(s)\,ds \tag{2.26}$$

where

$$\Gamma_\alpha(t) = (j^-, iLf_1(t)/\hbar)_\alpha/(j^-, j^+)_\alpha \tag{2.27}$$

$$\omega_\alpha = (j^-, Lj^+/\hbar)_\alpha/(j^-, j^+)_\alpha = \omega_0 + (V_{\alpha+1,\alpha+1} - V_{\alpha\alpha})/\hbar. \tag{2.28}$$

We see that $\omega_\alpha = \omega_0$ if $\langle V_{\alpha\alpha}\rangle = 0$ is assumed (Kawabata 1967, Lodder and Fujita 1968, Suzuki and Dunn 1982). Then comparison of equation (2.23) and the FLT of equation (2.26) leads to

$$\tilde{Y}_\alpha(\bar{\omega}) = \frac{(j_\alpha^+)^2}{i\hbar(\bar{\omega} - \omega_0) + \hbar\tilde{\Gamma}_\alpha(\bar{\omega})} \tag{2.29}$$

where $\langle\mu|j^+|\alpha\rangle = (\langle\alpha|j^-|\mu\rangle)^* = ie[2\hbar\omega_0(N_\alpha + 1)/m]^{1/2}\delta_{\mu,\alpha+1}$, $\tilde{\Gamma}_\alpha(\bar{\omega})$ is the FLT of

$\Gamma_\alpha(t)$ defined by equation (2.27), ω_α has been replaced by ω_0 in approximation, and $X_\alpha \equiv \langle \alpha+1|X|\alpha\rangle$. By taking into account the relations $\langle \alpha+1|\bar{P}_\alpha X|\alpha\rangle = 0$ and $P_\alpha L_0 G_{\rm op}\bar{P}_\alpha = 0$, from equation (2.27) we have

$$j_\alpha^+(i\hbar\bar{\Gamma}_\alpha(\bar{\omega})) = \left\langle \sum_{n=1}^{\infty} [(L_1 G_{\rm op}\bar{P}_\alpha)^n L j^+]_\alpha \right\rangle \tag{2.30}$$

where $G_{\rm op} = (\hbar\bar{\omega} - L_0 - L_{\rm p})^{-1}$.

We define the phonon state as

$$|q\rangle = |n_{q_1}, n_{q_2}, \ldots, n_{q_i}, \ldots\rangle \tag{2.31}$$

where n_{q_i} is the occupation number of the ith oscillator, and take into account the following formula:

$$\langle q|\langle\alpha|G_{\rm op}X|\mu\rangle|q'\rangle = \frac{\langle q|\langle\alpha|X|\mu\rangle|q'\rangle}{\hbar\bar{\omega} - E_\alpha + E_\mu - \langle q|H_{\rm p}|q\rangle + \langle q'|H_{\rm p}|q'\rangle} \tag{2.32}$$

$$\sum_\mu (\bar{P}_\alpha X)_{\alpha+1,\mu} = \sum_{\mu(\neq\alpha)} X_{\alpha+1,\mu}. \tag{2.33}$$

Then, with the help of equations (2.31), (2.32) and (2.33) the lineshape function $\bar{\Gamma}_\alpha(\bar{\omega})$ for the second-order scattering strength is obtained from equation (2.30) as

$$\begin{aligned} i\hbar\bar{\Gamma}_\alpha(\bar{\omega}) &= \hbar\Delta_{N_\alpha,k_\alpha}(\omega) + i\hbar\gamma_{N_\alpha,k_\alpha}(\omega) \\ &= \sum_q (1+n_q)\Bigg(\sum_{\mu(\neq\alpha+1)} \frac{(\gamma_q)_{\alpha+1,\mu}[(\gamma_q^+)_{\mu,\alpha+1} - (\gamma_q^+)_{\mu-1,\alpha} j_{\mu-1}^+/j_\alpha^+]}{\hbar\bar{\omega} - E_\mu + E_\alpha - \hbar\omega_q} \\ &\quad + \sum_{\mu(\neq\alpha)} \frac{[(\gamma_q)_{\alpha\mu} - (\gamma_q)_{\alpha+1,\mu+1} j_\mu^+/j_\alpha^+](\gamma_q^+)_{\mu\alpha}}{\hbar\bar{\omega} - E_{\alpha+1} + E_\mu + \hbar\omega_q}\Bigg) \\ &\quad + \sum_q n_q\Bigg(\sum_{\mu(\neq\alpha+1)} \frac{(\gamma_q^+)_{\alpha+1,\mu}[(\gamma_q)_{\mu,\alpha+1} - (\gamma_q)_{\mu-1,\alpha} j_{\mu-1}^+/j_\alpha^+]}{\hbar\bar{\omega} - E_\mu + E_\alpha + \hbar\omega_q} \\ &\quad + \sum_{\mu(\neq\alpha)} \frac{[(\gamma_q^+)_{\alpha\mu} - (\gamma_q^+)_{\alpha+1,\mu+1} j_\mu^+/j_\alpha^+](\gamma_q)_{\mu\alpha}}{\hbar\bar{\omega} - E_{\alpha+1} + E_\mu - \hbar\omega_q}\Bigg) \end{aligned} \tag{2.34}$$

where n_q is the phonon distribution function and the $\gamma_{N_\alpha,k_\alpha}(\omega)$ and $\Delta_{N_\alpha,k_\alpha}(\omega)$, respectively, describe the relaxation rate and shift. The $\mu \neq \alpha+1$ in the summation means that the terms $(N_\mu, k_\mu) = (N_\alpha+1, k_\alpha)$ should be excluded. Therefore we need to include only terms such as $(N_\mu \neq N_\alpha+1, k_\mu \neq k_\alpha)$, $(N_\mu = N_\alpha+1, k_\mu \neq k_\alpha)$ and $(N_\mu \neq N_\alpha+1, k_\mu = k_\alpha)$. However, the $(N_\mu \neq N_\alpha+1, k_\mu = k_\mu = k_\alpha)$ is excluded since the matrix elements of the interaction operators, γ_q and γ_q^+, in the numerators do not contain $k_\mu = k_\alpha$. The $(N_\mu \neq N_\alpha+1, k_\mu \neq k_\alpha)$ describes the contribution of the non-adiabatic processes, while the $(N_\mu = N_\alpha+1, k_\mu \neq k_\alpha)$ describes that of the adiabatic processes (Shin *et al* 1973). The difference of these processes depends on whether the matrix element of the interaction operator is diagonal or not. Similarly, the $\mu \neq \alpha$ in the summation includes only terms such as $(N_\mu \neq N_\alpha, k_\mu \neq k_\alpha)$ and $(N_\mu = N_\alpha, k_\mu \neq k_\alpha)$. Equation (2.34) is reduced to Choi and Chung's (1983) formula if $\hbar/\tau_0$ approaches zero in the energy denominators. That we allow $\hbar/\tau_0$ to go to zero is acceptable in the quantum limit (Jones and March 1973). We will follow the recipe in what follows, on the understanding that $\hbar/\tau_0$ can be reintroduced if necessary.

In order to rewrite equation (2.34) in more convenient form, we introduce the K matrices defined by (appendix 1)

$$K(N, N'; t) = \begin{cases} -\dfrac{N!}{N'} t^{N'-N} \exp(-t) L_N^{(N'-N)}(t) L_{N+1}^{(N'-N-1)}(t) & N' > N \\ \dfrac{N'!}{(N+1)} t^{N-N'+1} \exp(-t) L_{N'}^{(N-N')}(t) L_{N'}^{(N-N'+1)}(t) & N \geqslant N' \end{cases} \tag{2.35}$$

where $L_n^{(m)}(t)$ is the associated Laguerre polynomial

$$L_n^{(m)}(t) = (n!)^{-1} \exp(t) t^{-n} \frac{\mathrm{d}^n}{\mathrm{d}t^n} [t^{n+m} \exp(-t)] \tag{2.36}$$

$$t = r_0^2 (q_x^2 + q_y^2)/2. \tag{2.37}$$

Then we have

$$\begin{aligned} [\mathrm{i}\hbar\bar{\Gamma}_\alpha(\bar{\omega})]_{\text{non-ad}} = \sum_q |C_q|^2 \Big[& (1+n_q) \Big(\sum_{N_\mu (\neq N_\alpha + 1)} \sum_{k_{\mu z} (\neq k_{\alpha z})} \sum_{k_{\mu y} (\neq k_{\alpha y})} (\hbar\bar{\omega} - E_\mu + E_\alpha - \hbar\omega_q)^{-1} \\ & + \sum_{N_\mu (\neq N_{\alpha+1})} \sum_{k_{\mu z} (\neq k_{\alpha z})} \sum_{k_{\mu y} (\neq k_{\alpha y})} (\hbar\bar{\omega} - E_{\alpha+1} + E_\mu + \hbar\omega_q)^{-1} \Big) \\ & + n_q \Big(\sum_{N_\mu (\neq N_\alpha + 1)} \sum_{k_{\mu z} (\neq k_{\alpha z})} \sum_{k_{\mu y} (\neq k_{\alpha y})} (\hbar\bar{\omega} - E_\mu + E_\alpha + \hbar\omega_q)^{-1} \\ & + \sum_{N_\mu (\neq N_\alpha)} \sum_{k_{\mu z} (\neq k_{\alpha z})} \sum_{k_{\mu y} (\neq k_{\alpha y})} (\hbar\bar{\omega} - E_{\alpha+1} + E_\mu - \hbar\omega_q)^{-1} \Big) \Big] \\ & \times K(N_\alpha, N_\mu; t) \delta_{k_{\mu z}, k_{\alpha z} - q_z} \delta_{k_{\mu y}, k_{\alpha y} - q_y} \end{aligned}$$

(for non-adiabatic processes) (2.38)

and

$$\begin{aligned} [\mathrm{i}\hbar\bar{\Gamma}_\alpha(\bar{\omega})]_{\text{ad}} = \sum_q |C_q|^2 \Big[& (1+n_q) \Big(\sum_{k_{\mu z} (\neq k_{\alpha z})} \sum_{k_{\mu y} (\neq k_{\alpha y})} [\hbar\bar{\omega} - \hbar\omega_0 + \varepsilon(k_{\mu z}) \\ & - \varepsilon(k_{\alpha z}) - \hbar\omega_q]^{-1} K(N_\alpha, N_\alpha + 1; t) \\ & + \sum_{k_{\mu z} (\neq k_{\alpha z})} \sum_{k_{\mu y} (\neq k_{\alpha y})} [\hbar\bar{\omega} - \hbar\omega_0 - \varepsilon(k_{\alpha z}) \\ & + \varepsilon(k_{\mu z}) + \hbar\omega_q]^{-1} K(N_\alpha, N_\alpha; t) \Big) \\ & + n_q \Big(\sum_{k_{\mu z} (\neq k_{\alpha z})} \sum_{k_{\mu y} (\neq k_{\alpha y})} [\hbar\bar{\omega} - \hbar\omega_0 - \varepsilon(k_{\mu z}) + \varepsilon(k_{\alpha z}) + \hbar\omega_q]^{-1} \\ & \times K(N_\alpha, N_{\alpha+1}; t) + \sum_{k_{\mu z} (\neq k_{\alpha z})} \sum_{k_{\mu y} (\neq k_{\alpha y})} [\hbar\bar{\omega} - \hbar\omega_0 - \varepsilon(k_{\alpha z}) \\ & + \varepsilon(k_{\mu z}) - \hbar\omega_q]^{-1} K(N_\alpha, N_\alpha; t) \Big) \Big] \delta_{k_{\mu z}, k_{\alpha z} - q_z} \delta_{k_{\mu y}, k_{\alpha y} - q_y} \end{aligned}$$

(for adiabatic processes) (2.39)

where $+\hbar\omega_q$ and $-\hbar\omega_q$ are energies involved in emission and absorption of an acoustic phonon, respectively (appendix 2).

The presence of $\hbar\omega_q \simeq \eta\hbar s(q_\perp^2 + q_z^2)^{1/2}$ in the denominator of equations (2.38) and (2.39) makes the integration quite difficult. Some simplification is therefore inevitable. We follow Kubo *et al* (1965) to make an approximation $q_\perp^2 \gg q_z^2$, which will make $\hbar\omega_q = \hbar s q_\perp$. Now, we make a simple assumption that the phonons have wavenumbers of the order of $1/r_0$. Therefore, the phonon energy $\hbar\omega_q$ is replaced by $\eta\hbar s/r_0$ for the acoustic phonon energy (Pal and Sharma 1974, Arora 1976), where η is a parameter that describes the extent of inelasticity of the collision (Arora 1976).

3. Resonance linewidth for piezoelectric potential scattering

We assume that all electrons occupy Landau states with $N_\alpha = 0$. That is, the distribution function for electrons with non-zero quantum number vanishes. Then, equations (2.17) and (2.29) reduce to (Lax 1958, Ciobanu and Banyai 1963)

$$\mathrm{Re}\,\sigma_{+-}(\omega) = (2e^2/m)\,\mathrm{Lim}(1/\Omega)\sum_{k_z}\frac{f_{0,k_z}\gamma_{0,k_z}(\omega)}{[(\omega-\omega_0)^2+\gamma_{0,k_z}^2(\omega)]} \equiv \frac{n_e e^2\tau}{m} \tag{3.1}$$

where $k_{\alpha z} \equiv k_z$, n_e is the density of electrons, τ is the mean relaxation time, the inverse of which gives the linewidth $(2/\tau)$ at $\omega = \omega_0$, and we have dropped the energy shift since we are interested in finding the width (Kawabata 1967, Lodder and Fujita 1968, Suzuki and Dunn 1982, Prasad 1982).

The interaction between the electrons and piezoelectric acoustic phonons is defined as (Meyer and Polder 1953, Saitoh and Kawabata 1967)

$$|C_q|^2 = 2\pi K^2 s\hbar^2/ma\Omega q \tag{3.2}$$

where K is the electromechanical coupling constant, $a = \varepsilon\hbar^2/me^2$ the effective Bohr radius, ε the static dielectric constant of the medium, and s the speed of sound defined by

$$\omega_q = sq. \tag{3.3}$$

3.1. Moderate-temperature region

In the temperature region specified by $\hbar\omega_0 \gg k_B T \gg \hbar s/r_0$, we may approximate $n_q + 1$ and n_q by $(\beta\hbar sq)^{-1}$ and consider only the transition between states with $N_\alpha = 0$ and $N_\mu = 0$ in equations (2.38) and (2.39) as Saitoh and Kawabata (1967) did.

Then, recalling that the relaxation rate $\gamma_{0,k_z}(\omega)$ is the imaginary part of $i\hbar\bar{\Gamma}_0(\omega)$ for $N_\alpha = 0$, considering equation (3.2), changing the summation in equations (2.38) and (2.39) into the integral as (Arora 1976, Prasad 1982)

$$\sum_q \to \Omega(2\pi)^{-3}\int_{-\infty}^{\infty} dq_z \int_0^{\infty} (2\pi/r_0^2)\,dt \tag{3.4}$$

and considering the relations

$$\lim_{b\to 0^+}(x - ib)^{-1} = p(1/x) + i\pi\delta(x) \tag{3.5}$$

$$\delta[f(q_z)] = \sum\frac{\delta(q_z - q_{z0})}{|f'(q_z)|_{q_z=q_{z0}}} \tag{3.6}$$

we easily obtain from equations (2.38) and (2.39)

$$\gamma_{0,k_z}(\omega_0) = (K^2 k_B T/2\hbar a)\{A_-[2 - a^-\exp(a^-)E_1(a^-) - b^-\exp(b^-)E_1(b^-)] + A_+[2 - a^+\exp(a^+)E_1(a^+) - b^+\exp(b^+)E_1(b^+)]\}$$
(for non-adiabatic processes) (3.7)

and

$$\gamma_{o,k_z}(\omega_0) = \frac{K^2 k_B T}{2\hbar a}\{B_-[2 - c^- \exp(c^-)E_1(c^-) - d^- \exp(d^-)E_1(d^-)] + B_+[2 - c^+ \exp(c^+)E_1(c^+) - d^+ \exp(d^+)E_1(d^+)]\}$$

(for adiabatic processes) (3.8)

where

$$A_\pm = \left| k_z^2 - \frac{2m}{\hbar}\left(\omega \pm \frac{\eta s}{r_0}\right)\right|^{-1/2} \tag{3.9}$$

$$B_\pm = \left| k_z^2 \pm \frac{2m\eta s}{\hbar r_0}\right|^{-1/2} \tag{3.10}$$

$$a^\pm = \frac{r_0^2}{2}\left\{k_z + \left[k_z^2 - \frac{2m}{\hbar}\left(\omega \pm \frac{\eta s}{r_0}\right)\right]^{1/2}\right\}^2 \tag{3.11}$$

$$b^\pm = \frac{r_0^2}{2}\left\{k_z - \left[k_z^2 - \frac{2m}{\hbar}\left(\omega \pm \frac{\eta s}{r_0}\right)\right]^{1/2}\right\}^2 \tag{3.12}$$

$$c^\pm = \frac{r_0^2}{2}\left[k_z + \left(k_z^2 \pm \frac{2m\eta s}{\hbar r_0}\right)^{1/2}\right]^2 \tag{3.13}$$

$$d^\pm = \frac{r_0^2}{2}\left[k_z - \left(k_z^2 \pm \frac{2m\eta s}{\hbar r_0}\right)^{1/2}\right]^2 \tag{3.14}$$

$$E_1(b) = \int_b^\infty \frac{\exp(-y)}{y}\,dy. \tag{3.15}$$

In the case of the elastic scattering for $k_z \to 0$, which is $\eta = 0$ in equations (3.7) and (3.8), the width for the non-adiabatic processes is finite. That for adiabatic processes is infinite, which is identical with the result of Suzuki and Dunn (1982).

For a more quantitative discussion in the elastic scattering approximation ($\eta = 0$), we must include the effect of the electron distribution. Assuming the Boltzmann distribution for low electron densities (of the order of $10^{14}\,\mathrm{cm}^{-3}$), we obtain from equations (3.1), (3.7) and (3.8) at $\omega = \omega_0$

$$(\tau)_{\text{non-ad}} = \frac{5\hbar^2 a}{2^{3/2}(\pi m)^{1/2}K^2(k_B T)^{3/2}}\int_{-\infty}^{\infty} dk_z \left|\frac{2m\omega_0}{\hbar} - k_z^2\right|^{1/2} \exp\left(-\frac{\beta\hbar^2 k_z^2}{2m}\right) \tag{3.16}$$

$$(\tau)_{\text{ad}} = \frac{2^{1/2}\hbar^2 a}{1.08(\pi m)^{1/2}K^2(k_B T)^{3/2}}\int_{-\infty}^{\infty} dk_z\, k_z \exp\left(-\frac{\beta\hbar^2 k_z^2}{2m}\right) \tag{3.17}$$

where we have made approximations $a^\pm \exp(a^\pm)E_1(a^\pm) = b^\pm \exp(b^\pm)E_1(b^\pm) \simeq 0.6$ since $a^\pm = b^\pm = 1$, $d^\pm = 0$, in the quantum-limit condition. We have made another approximation $c^\pm = 2r_0^2 k_z^2 \simeq 2k_B T/\hbar\omega_0 \simeq 11.26$ for the experimental temperatures 2–10 K and resonant frequency 70 GHz (Baer and Dexter 1964). This rough approximation has

been adopted for the sake of mathematical convenience. Performing a Taylor expansion and integration over k_z, we obtain

$$(\tau)_{\text{non-ad}} = \frac{3.53m^{1/2}a}{K^2k_BT}\left|\hbar\omega_0 - \frac{k_BT}{2}\right|^{1/2} \tag{3.18}$$

$$(\tau)_{\text{ad}} = \frac{3.7m^{1/2}a}{(2\pi k_BT)^{1/2}K^2}. \tag{3.19}$$

The calculated width that corresponds to the inverse mean relaxation time is proportional to the inverse square root of the magnetic field and temperature for non-adiabatic processes in the quantum-limit condition. The T dependence for elastic scattering is identical with that of Suzuki and Dunn (1982). The width for adiabatic processes is proportional to the square root of the temperature, which agrees well with those of Baer and Dexter (1964), Saitoh and Kawabata (1967) and Arora and Spector (1979) obtained under the quantum-limit condition and those of Meyer and Polder (1953), Mahan and Hopfield (1964) and Zook (1964) calculated in the classical limit. However, the conditions are different from one another. For CdS, the material constants are $K^2 = 0.03$, $m/m_0 = 0.165$ and $a = 2.4 \times 10^{-7}$ cm, where m_0 is the bare electron mass (Saitoh and Kawabata 1967). From equation (3.19) the mean relaxation time is given by

$$(\tau)_{\text{ad}} = 1.23 \times 10^{-11}T^{-1/2}\ \text{s} \tag{3.20}$$

for elastic scattering, while the experimental value is (Baer and Dexter 1964)

$$(\tau)_{\text{exp}} = 1.36 \times 10^{-11}T^{-1/2}\ \text{s}. \tag{3.21}$$

3.2. *Low-temperature region*

At low temperatures specified by $\hbar\omega_0 \gg \hbar s/r_0 \gg k_BT$ and $T \to 0$ K in the extreme quantum limit, we may make approximations $\omega_q \simeq s/r_0$ and $n_q \simeq 0$. Then, for slow enough electrons for which $k_z \to 0$ (Choi and Fujita 1981) at $\omega = \omega_0$, we obtain from equations (2.38) and (2.39)

$$(2/\tau)_{\text{non-ad}} = \frac{K^2s}{2^{1/2}a}\left(1 + \sum_{N=2}^{N_D}(N-1)^{-1/2} + \sum_{N=1}^{N_D}N^{-1/2}\right)$$

(for elastic and inelastic scattering) (3.22)

$$(2/\tau)_{\text{ad}} = \begin{cases}\infty & \text{(for elastic scattering)} \quad (3.23)\\ 2^{3/2}K^2s^{1/2}(\hbar\omega_0)^{1/4}/m^{1/4}a & \text{(for inelastic scattering).} \quad (3.24)\end{cases}$$

Here $N \equiv N_\mu - N_\alpha$ for $N_\alpha = 0$, and N_D is the cut-off Landau level index characterised by the Debye temperature θ_D as

$$N_D \leq k_B\theta_D/\hbar\omega_0. \tag{3.25}$$

The summation in equation (3.22) would tend to infinity if the cut-off in equation (3.25) were not taken into consideration (Choi and Chung 1984). In other words, we have assumed that there is phonon scattering only up to θ_D. The width is independent of temperature and magnetic field for non-adiabatic scattering processes, while the width for adiabatic scattering processes is proportional to $B^{1/4}$ for inelastic scattering, which is in qualitative agreement with the results of Larsen (1966) and Choi and Fujita (1981).

But the present result does not agree with the result of Suzuki and Dunn (1982), which is proportional to $T^{4.5}B^{-2}$.

In the case of $\hbar s/r_0 \gg \hbar\omega_0 \gg k_B T$ at $\omega = \omega_0$, we have from equations (2.38) and (2.39)

$$(2/\tau)_{\text{non-ad}} = \begin{cases} \dfrac{K^2 s^{1/2}(\hbar\omega_0)^{1/4}}{2^{1/2}m^{1/4}a}\left(1 + \sum_{N=2}^{N_D} + \sum_{N=1}^{N_D}\right) & \text{(for inelastic scattering)} \quad (3.26) \\ \text{the same as equation (3.22)} \qquad \text{(for elastic scattering)} \quad (3.27) \end{cases}$$

$$(2/\tau)_{\text{ad}} = \text{the same as equations (3.23) and (3.24), respectively} \quad (3.28)$$

where we have assumed that electrons are also slowly moving. The width in the non-adiabatic processes is independent of the temperature for both elastic and inelastic scattering. However, it is independent of magnetic field for elastic scattering, but proportional to $B^{1/4}$ for inelastic scattering. The width in the adiabatic processes gives the same values as equations (3.23) and (3.24). The difference between elastic scattering and inelastic scattering is explained by Ito *et al* (1966).

4. Conclusions

So far we have obtained cyclotron resonance linewidths for piezoelectric potential scattering in the quantum limit. Our result for the adiabatic and elastic scattering processes in the moderate-temperature region agrees fairly well with the result of Baer and Dexter (1964), while that for the non-adiabatic and elastic scattering processes is identical with that of Suzuki and Dunn (1982) on the temperature dependence. The width at low temperatures is in qualitative agreement with those of Larsen (1966) and Choi and Fujita (1981).

The result will be improved if higher-order transitions, non-parabolic band models, degeneracy of valence bands and impurity scattering are considered.

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Appendix 1. Derivation of K functions

We define $J_{NN'}(X, q_x, X')$ by

$$J_{NN'}(X, q_x, X') \equiv \int_{-\infty}^{\infty} dx\, \phi_N^*(x - X)\exp(iq_x x)\phi_{N'}(x - X')$$

$$= (2^{N+N'}N!N'!\pi r_0^2)^{-1/2}\int_{-\infty}^{\infty} dx \exp\left(-\frac{(x - X)^2 + (x - X')^2}{2r_0^2} + iq_x x\right)$$

$$\times H_N\left(\frac{x-X}{r_0}\right) H_{N'}\left(\frac{x-X'}{r_0}\right). \tag{A1.1}$$

It is seen from equation (A1.1) that $J_{NN'}(X, q_x, X')$ satisfies the following equation:

$$J_{NN'}(X, q_x, X') = J_{N'N}(X', q_x, X) = J^*_{NN'}(X, -q_x, X') = J^*_{N'N}(X', -q_x, X). \tag{A1.2}$$

Changing the variable as $z = [x - \frac{1}{2}(X + X' + ir_0^2 q_x)]/r_0$ and using the relation (Gradshteyn and Ryzhik 1963)

$$\int_{-\infty}^{\infty} dz \exp(-z^2) H_N(z + a) H_{N'}(z + b) = 2^{N'}\sqrt{\pi} N! b^{N'-N} L_N^{(N'-N)}(-2ab) \tag{A1.3}$$

for $N \leq N'$ we obtain

$$J_{NN'}(X, q_x, X') = \left(\frac{N_<!}{N_>!}\right)^{1/2} \left(\frac{X_< - X_> + iq_x r_0^2}{\sqrt{2} r_0}\right)^{N_> - N_<} \exp\left[-\left(\frac{X - X'}{2r_0}\right)^2 - \left(\frac{r_0 q_x}{2}\right)^2 + \frac{iq_x(X + X')}{2}\right] L_{N_<}^{(N_> - N_<)}\left(\frac{(X - X')^2 + q_x^2 r_0^4}{2r_0^2}\right) \tag{A1.4}$$

where $N_>$ and $N_<$ are the larger and the smaller of the two numbers N and N', and $X_>$ and $X_<$ correspond to $N_>$ and $N_<$ respectively.

We now define K_1 and K_2 functions as

$$K_1(N, N'; t) \equiv J_{NN'}(X, q_x, X \pm r_0^2 q_y) J_{N'N}(X \pm r_0^2 q_y, -q_x, X) \tag{A1.5}$$

and

$$K_2(N, N'; t) \equiv \begin{cases} J_{NN'}(X, q_x, X \pm r_0^2 q_y) J_{N'+1, N+1}(X \pm r_0^2 q_y, -q_x, X) \\ J_{N+1, N'+1}(X, q_x, X \pm r_0^2 q_y) J_{N'N}(X \pm r_0^2 q_y, -q_x, X) \end{cases} \tag{A1.6}$$

respectively. These functions can be re-expressed as

$$K_1(N, N'; t) = \frac{N_<!}{N_>!} t^{(N_> - N_<)} \exp(-t)[L_{N_<}^{(N_> - N_<)}(t)]^2 \tag{A1.7}$$

and

$$K_2(N, N'; t) = \left(\frac{N_< + 1}{N_> + 1}\right)^{1/2} \frac{N_<!}{N_>!} t^{(N_> - N_<)} \exp(-t) L_{N_<}^{(N_> - N_<)}(t) L_{N_< + 1}^{(N_> - N_<)}(t). \tag{A1.8}$$

Let us define K and K' functions as

$$K(N, N'; t) \equiv K_1(N, N'; t) - \left(\frac{N' + 1}{N + 1}\right)^{1/2} K_2(N, N'; t) \tag{A1.9}$$

$$K'(N, N'; t) \equiv K_1(N + 1, N' + 1; t) - \left(\frac{N' + 1}{N + 1}\right)^{1/2} K_2(N, N'; t). \tag{A1.10}$$

Then using

$$L_{N+1}^{(N'-N)}(t) - L_N^{(N'-N)}(t) = L_{N+1}^{(N'-N-1)}(t) \tag{A1.11}$$

and

$$(N+1)L_{N'}^{(N-N')}(t) - (N'+1)L_{N'+1}^{(N-N')}(t) = tL_N^{(N-N'+1)}(t) \tag{A1.12}$$

(Gradshteyn and Ryzhik 1963) we obtain

$$K(N, N'; t) = K'(N, N'-1; t)$$

$$= \begin{cases} -\dfrac{N!}{N'!}\, t^{(N'-N)} \exp(-t) L_N^{(N'-N)}(t) L_{N+1}^{(N'-N-1)}(t) & N' > N \\ \dfrac{N'!}{(N+1)!}\, t^{(N-N'+1)} \exp(-t) L_{N'}^{(N-N')}(t) L_N^{(N-N'+1)}(t) & N \geqslant N'. \end{cases} \tag{A1.13}$$

Appendix 2. *K*-function representation of equation (2.34)

The matrix elements in the part of equation (2.34) can be evaluated from equations (2.13), (2.14) and (2.10) as

$$(\gamma_q)_{\alpha+1,\mu} = \langle N_\alpha + 1, \boldsymbol{k}_\alpha | C_q \exp(\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{r}) | N_\mu, \boldsymbol{k}_\mu \rangle$$

$$= C_q J_{N_\alpha+1, N_\mu}(X_\alpha, q_x, X_\mu)\delta_{k_{\mu y}, k_{\alpha y} - q_y}\delta_{k_{\mu z}, k_{\alpha z} - q_z} \tag{A2.1}$$

$$(\gamma_q^+)_{\mu,\alpha+1} = C_q^* J_{N_\mu, N_\alpha+1}(X_\mu, -q_x, X_\alpha)\delta_{k_{\mu y}, k_{\alpha y} - q_y}\delta_{k_{\mu z}, k_{\alpha z} - q_z} \tag{A2.2}$$

$$(\gamma_q^+)_{\mu-1,\alpha} = C_q^* J_{N_\mu - 1, N_\alpha}(X_\mu, -q_x, X_\alpha)\delta_{k_{\mu y}, k_{\alpha y} - q_y}\delta_{k_{\mu z}, k_{\alpha z} - q_z} \tag{A2.3}$$

where $J_{NN'}(X_\alpha, q_x, X_\mu)$ is defined by equation (A1.1). X_α and X_μ are, respectively, given by $X_\alpha = -r_0^2 k_{\alpha y}$ and $X_\mu = -r_0^2 k_{\mu y} = -r_0^2(k_{\alpha y} - q_y) = X_\alpha + r_0^2 q_y$. By taking into account the relation $j_{\mu-1}^+ / j_\alpha^+ = [N_\mu/(N_\alpha+1)]^{1/2}$ and equations (A2.1), (A2.2) and (A2.3), the numerator of the first part of equation (2.34) is reduced to

$$(\gamma_q)_{\alpha+1,\mu}[(\gamma_q^+)_{\mu,\alpha+1} - (\gamma_q^+)_{\mu-1,\alpha} j_{\mu-1}^+ / j_\alpha^+]$$

$$= |C_q|^2 \left[K_1(N_\alpha + 1, N_\mu; t) - \left(\frac{N_\mu}{N_\alpha + 1}\right)^{1/2} K_2(N_\mu - 1, N_\alpha; t) \right]$$

$$\times \delta_{k_{\mu y}, k_{\alpha y} - q_y}\delta_{k_{\mu z}, k_{\alpha z} - q_z}$$

$$= |C_q|^2 K(N_\alpha, N_\mu; t)\delta_{k_{\mu y}, k_{\alpha y} - q_y}\delta_{k_{\mu z}, k_{\alpha z} - q_z} \tag{A2.4}$$

where we have used equations (A1.5), (A1.6), (A1.10) and (A1.13). The second, third and fourth parts of equation (2.34) can be calculated in a similar manner. Adding up all the parts leads to equations (2.38) and (2.39).

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