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Pole Placement of Linear Parameter Dependent System

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ABSTRACT

In this paper, we studied the linear parameter dependent (LPD) systems. Pole sensitivity is defined by the variation of the pole location with respect to the parameter variation, and a control algorithm which is based on the well-known pole-placement state feedback control and minimizes the pole-sensitivity is presented.

Key Words : Linear parameter dependent system, pole placement, pole sensitivity, eigenstructure

I. Introduction

Many of the engineering systems can be modeled by linear parameter dependent systems which had been controlled by gain-scheduling algorithm until late 1980s. Recently, some results about stability, performance and robustness for gain scheduling control have been reported [1]. By G.D. Wood[2], a mechanical system is modeled by LPD system and H_∞ control algorithm is suggested. S.G. Scott[3] shows quadratic stability performance for LPD system by using the well known Lyapunov equation and presents a H_∞ controller design algorithm. The LQG design for LPD system is studied by W. Fen[4] and he shows that the unstable system by using constant feedback can be stabilized by parameter dependent feedback. But, all these algorithms have computational difficulties because the system has multiple operating points or calculation of integral with respect to parameter values in solving LMI.

In this paper, we studied the linear parameter dependent (LPD) systems. Pole sensitivity is defined by the variation of the pole location with respect to the parameter variation, and a control algorithm which is based on the well known pole-placement state feedback control and minimizes pole- sensitivity is presented.

II. Problem formulation and pole sensitivity

2.1 LPD System

Many of the physical system can be modeled by linear parameter dependent system. Before introducing the LPD system, we need to define the set of all admissible parameter trajectories.

Definition 1[4]. Given a compact set $P \subset R^S$, the parameter set F_P denote the set of all piecewise continuous functions mapping R^+ into P with finite number of discontinuities in any interval.

By the definition 1, the parameter value $\rho_i \in F_P$

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are differentiable with respect to time

A state space realization of LPD system is

$$\dot{x}(t) = A(\rho)x(t) + Bu(t) \quad (1)$$

where. $\rho \in F_P, x(t) \in R^n, u(t) \in R^{n_u}$ and $y(t) \in R^{n_y}$. For the notational purpose. the time variable is deleted for all equations remainder of this paper. i.e. $A(\rho(t))$ is abbreviated by $A(\rho)$

As the matrix functions. The system matrices $A(\rho), B(\rho)$ are dependent on the parameter values. $A(\rho), B(\rho)$ are continuous functions of a parameter ρ. and they are norm bounded on the compact set $P \subset R^S$. Without loss of generality. system matrix is assumed in this paper as

$$A(\rho) = A_0 + A_i(\rho)$$
$$B(\rho) = B \quad (2)$$

In equation (2). the input matrix $B(\rho)$ is assumed to be constant which can be obtained by addition of integrator in to the input channel and simple coordinate transformation of the LPD system equation.

2.2 Pole Sensitivity

In the LPD system. the pole locations are not fixed and vary with small change of parameter. The relationship between the i-th eigenvalue and eigenvector is

$$A(\rho)v_i(\rho) = v_i(\rho)\lambda_i(\rho) \quad (3)$$

where. $v_i(\cdot)$is the i-th right eigenvector and $\lambda_i(\cdot)$ is the system pole. The equation (3) is differentiable with respect to the parameter value ρ because it is piece-wise continuous. Thus the i-th pole displacement with respect to the j-th parameter variation can be obtained by

$$\frac{\partial A(\rho)}{\partial \rho_j} v_i(\rho) + A(\rho)\frac{\partial v_i(\rho)}{\partial \rho_j}$$
$$= v_i(\rho)\frac{\partial \lambda_i(\rho)}{\partial \rho_j} + \lambda_i(\rho)\frac{\partial v_i(\rho)}{\partial \rho_j} \quad (4)$$

By pre-multiplying the left eigenvector $u_i(\rho(t))$ to both side of the equation (4). we obtain

$$u_i(\rho)\frac{\partial A(\rho)}{\partial \rho_j} + u_i(\rho)A(\rho)\frac{\partial v_i(\rho)}{\partial \rho_j}$$
$$= u_i(\rho)v_i(\rho)\frac{\partial \lambda_i(\rho)}{\partial \rho_j} + u_i(\rho)\lambda_i(\rho)\frac{\partial v_i(\rho)}{\partial \rho_j} \quad (5)$$

Equation (5) becomes as follows because of the property of the left eigenvectors and eigen-values

$$u_i(\rho)\frac{\partial A(\rho)}{\partial \rho_j} v_i(\rho) = u_i(\rho)A(\rho)\frac{\partial v_i(\rho)}{\partial \rho_j} \quad (6)$$

Definition 2. The pole sensitivity. defined as the ratio of pole displacement with respect to the parameter variation. is described by

$$S_{ij} := \frac{\partial \lambda_i}{\partial \rho_j} = \frac{u_i \frac{\partial A(\rho)}{\partial \rho_j} v_i}{u_i v_i} \quad (7)$$

where. u_i, v_i is left and right eigen-vector of i-th system pole respectively.

By the definition 2. S_{ij} means i-th pole displacement with j-th parameter variation. Thus. the i-th pole movement is

$$d\lambda_i = S_{i1}d\rho_1 + S_{i2}d\rho_2 + \cdots + S_{ik}d\rho_k \quad (8)$$

and pole movement for all poles is

$$d\Lambda = Sd\rho \quad (9)$$

where is the pole-sensitivity matrix whose elements are defined by the equation 4.

2.3 Pole-Placement Control Problem

For the LPD system. it is very difficult to stabilize the closed loop system by using constant state feedback gain. But. most of the case. the parameter dependent state feedback gain matrix can stabilize the closed loop system. We now state the pole placement problem for the LPD system by using the parameter dependent state feedback gain matrix. The state feedback gain. used in this paper. is

$$F(\rho)=F_0+F_i(\rho) \tag{10}$$

Then. the closed loop dynamic equation is

$$\dot{x}(t)=[(A_0-BF_0)+(A_i(\rho)-BF_i(\rho))]x(t) \tag{11}$$

The closed loop poles are equal to the eigen-value of the closed loop system matrix $(A_0-BF_0)+$ $+(A_i(\rho)BF_i(\rho))$ and then. the pole-sensitivity is obtained by

$$S_{ij}:=\frac{\partial\lambda_i}{\partial\rho_j}=\frac{u_i\dfrac{\partial A[A_i-BF_i](\rho)}{\partial\rho_j}v_i}{u_iv_i} \tag{12}$$

The control problem is "Find state feedback gain matrix such that".

1) place the closed loop poles on the desired location (or in desired region)
2) make the pole-sensitivity equal to zero(or minimize the pole sensitivity matrix).

In this paper. the state feedback gain matrix F_0 is computed for satisfying the condition 1) and $F_i(\rho)$ is for condition 2).

III. Controller design

There are several ways for the state feedback pole -placement design. In this paper. we use two methods to obtain the state feedback gain matrix. One of them locates the closed loop poles on the desired location and the other is in the desired region. And we suggest two methods for obtaining the assistance state feedback matrix. One of which makes the pole-sensitivity matrix equal to zero and the other minimizes it.

3.1 Pole-Placement

The matrix F_0 is used for the pole-placement which makes the closed loop poles locate on the desired location or lie in the desired region. We. firstly. state a method of finding state feedback gain matrix F_0 which locates the closed loop poles on the desired location. The input matrix B. the rank of which is m. is partitioned as

$$B=[U_1\ U_2]\begin{bmatrix} Z \\ 0\end{bmatrix} \tag{13}$$

where. U_1, U_2 are unitary matrix and Z is nonsingular matrix with rank m. Let Λ_D and V_D be desired closed loop pole matrix and right eigenvector matrix. respectively. Then. the following equation is necessary and sufficient condition for the existence of the state feedback gain matrix which places the closed loop poles on the desired location.

$$U_2^T(A_0V_D-V_D\Lambda_D)=0 \tag{14}$$

If the equation (14) is hold then the state feedback gain matrix F_0 is

$$F_0=Z^{-1}U_1^T(A_0-V_D\Lambda_DV_D^{-1}) \tag{15}$$

The proof of the equation (14) and (15) can be found many books and papers which treat the linear system control. The next is regional pole placement. We. now. introduce the LMI regions and regional pole placements. The LMI regions are convex subset D of the complex plan characterized by

$$D=\{z\in C: L+Mz+M^Tz^*\} \tag{16}$$

where M and L are fixed ream matrices. and z and z^* are complex valued scalar and its complex conjugate pair. The valued function

$$f_D(z):=L+Mz+M^Tz^* \tag{17}$$

is called the characteristic function of the region. For example, a characteristic function of a circle with radius r and center $(-q, 0)$ is

$$f_{circ}(z) = \begin{pmatrix} -r & q+z \\ q+z^* & -r \end{pmatrix} < 0 \tag{18}$$

and LMI representation is

$$\begin{pmatrix} -rX & qX + A_F X \\ qX + XA_F^T & -rX \end{pmatrix} < 0, \qquad X > 0 \tag{19}$$

where, $A_F = A_0 - BF_0$. A complex region where the closed loop poles lie in is assumed to be a conic sector whose characteristic function is

$$f_{sector}(z) = \begin{pmatrix} \sin\theta(z+z^*) & \cos\theta(z-z^*) \\ -\cos\theta(z-z^*) & \sin\theta(z+z^*) \end{pmatrix} \tag{20}$$

and LMI representation is

$$\begin{pmatrix} \sin\theta(A_F X + XA_F^T) & \cos\theta(A_F X - XA_F^T) \\ -\cos\theta(A_F X - XA_F^T) & \sin\theta(A_F X + XA_F^T) \end{pmatrix} < 0, \quad X > 0 \tag{21}$$

The LMI of the common region of above two LMIs can be obtained only diagonally, connecting them. Thus, the steps of the regional pole placement is summarized as

1) Select a complex region in which the closed loop poles lie.
2) Characterize the complex region with some LMIs
3) Find state feedback gain matrix by solving these LMIs

This procedure is described very well in the references[6][7].

3.2 Minimization of Pole-Sensitivity

In this section, we present two methods of finding auxiliary state feedback gain matrix which make all elements of the pole sensitivity matrix equal to zero or which minimize the norm of the pole sensitivity matrix.

Now, it is assumed that parameters in the system matrix are first order, i.e., the system matrix is depended by the parameters which are first order. Then the system matrix is

$$A(\rho) = A_0 + \sum_i A_i \rho_i \tag{22}$$

and the pole sensitivity is obtained by

$$S_{ij} = \frac{u_i(A_j - BF_j)v_i}{u_i v_i} \tag{23}$$

In order to make the pole sensitivity S_{ij} equal to zero, one of the following two equations must be hold.

$$I_n - (U_1 Z)(Z^{-1} U_1^T) = 0 \tag{24}$$

$$A_i - BF_i = V_D^\perp, \qquad V_D V_D^\perp = 0 \tag{25}$$

And where, U_1 and Z are defined in the equation (13) and n is the rank of the system, and $V_D^\perp$ is ortho-normal and the rank of matrix $[\, V_D \;\; V_D^\perp]$ is equal to n. Generally, equations (24) and (25) are not hold because the second term of the equation (24) has rank m<n and the rank of the matrix V_D is equal to n. By reasons described above, we use two tricks which guarantee the small pole sensitivity. Rewrite the pole sensitivity equation as

$$S_{ij} = \frac{ui\left(\begin{vmatrix} A_j^{11} & A_j^{12} \\ A_j^{21} & A_j^{22} \end{vmatrix} - [\, U_1 \;\; U_2] \begin{bmatrix} Z \\ 0 \end{bmatrix} [\, F_j^1 \;\; F_j^2] \right) v_i}{u_i v_i} \tag{26}$$

and it can be partitioned by

$$S_{ij} = u_i^1(A_j^{11} - U_1 Z F_j^1)v_i^1 + u_i^1(A_j^{12} - U_1 Z F_j^2)v_i^2 + u_i^2 A_j^{21} v_i^1 + u_i^2 A_j^{21} v_i^2 \tag{27}$$

The first and second terms of equation (27) can be deleted because the sub-matrix $U_1 Z$ is full rank and proper selection of F_j^1 and F_j^2 cancels A_j^{11} and A_j^{12}. Thus the equation (27) becomes

$$S_{ij} = u_i^2 A_j^{21} v_i^1 + u_i^2 A_j^{21} v_i^2 \tag{28}$$

Also, the equation (28) is minimized by selecting eigenvectors u_i, v_i.

The equation (28) can be generalized for the system matrix that is depended by the parameters which are not first order. Then the pole sensitivity is

$$\begin{aligned} S_{ij} = \; & u_i^1 \frac{\partial}{\partial \rho_j}(A^{11} - U_1 Z F^1) v_i^1 + u_i^2 \frac{\partial}{\partial \rho_j} A^{21} v_i^1 \\ & + u_i^1 \frac{\partial}{\partial \rho_j}(A^{12} - U_1 Z F^2) v_i^2 + u_i^2 \frac{\partial}{\partial \rho_j} A^{21} v_i^2 \end{aligned} \tag{29}$$

By same reason discussed above, the equation (29) becomes

$$S_{ij} = u_i^2 \frac{\partial}{\partial \rho_j} A^{21} v_i^1 + u_i^2 \frac{\partial}{\partial \rho_j} A^{21} v_i^2 \tag{30}$$

And, by proper selection of u_i, v_i, the pole sensitivity can be minimized.

Ⅳ. CONCLUSION

In this paper, the linear parameter dependent system is considered. The pole sensitivity is defined as the rate of the pole displacement with respect to the parameter variation. The pole placement algorithm is introduced, and we present two methods for which the pole sensitivity is minimized. One of them is based on the cancellation of parameter values and the other minimizes the effect of parameter variation. The algorithm presented in this paper is applicable to systems which is not highly coupled parameter values. Also, for nonlinear systems which have simple nonlinearity, presented algorithm is applicable.

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