A NOTE ON STRATIFIABLE SPACES AND ℵ-SPACES

by Sang Hun Rhee.

Contents

1. Introduction

Stratifiable spaces have been introduced by Borges [1] and [4] CEDER proved that any ℵ-space is k-semistratifiable spaces. In this note, we give a simple characterization of ℵ-space. We show that the image of Nagata space under a closed pseudo-open finite to one compact mapping is stratifiable and that the image of a compact and ℵ-space under a k-mapping is an ℵ-space. In the end we investigates the properties of the image of ℵ-spaces and stratifiable spaces under N-mapping.

2. Definitions and elementary properties

DEFINITION 2.1. [1]. A topological space X is a *stratifiable* space if X is T_1 and, to each open $U \subset X$, one can assign a sequence $\{U_n\}_{n=1}^{\infty}$ of open subsets of X such that

(a) $\bar{U}_n \subset U$,

(b) $\bigcup_{n=1}^{\infty} U_n = U$,

(c) $U_n \subset V_n$ whenever $U \subset V$.

This correspondence $U \to \{U_n\}_{n=1}^{\infty}$ is *a stratification of* X whenever the U_n satisfy (a), (b) and (c) of DEFINITION 2.1.

DEFINITION 2.2. [5]. A topological space X is a *semi-stratifiable* space if, to each open set $U\subset X$, one can assign a sequence $\{U_n\}_{n=1}^{\infty}$ of closed subset of X such that

(a) $\bigcup_{n=1}^{\infty} U_n = U$,

(b) $U_n \subset V_n$ whenever $U \subset V$.

The correspondence $U\rightarrow\{U_n\}_{n=1}^{\infty}$ is called *a semi-stratification* for the space X. M. Henry showed, in [5], that stratifiable spaces are semi-stratifiable spaces, but these implication cannot be reversed.

DEFINITION 2.3. [2]. A regular T_1 space with a σ-locally finite k-network is called *an* $\aleph$*-space*.

DEFINITION 2.4. [2]. *A k-network* $\mathscr{P}$ for a space X is a family of subsets of X such that if $C\subset U$, with C compact and U open in X, then there is a finite uion R of members of $\mathscr{P}$ such that $C\subset R\subset U$.

A network $\mathscr{P}$ for a space X is a family of subsets of X such that if $x\in U$, with U open, then there is a $P\in\mathscr{P}$ such that $x\in P\subset U$.

DEFINITION 2.5. [2]. *A k-semistratification* of a space X is a semistratifiable $U\rightarrow\{U_n\}_{n=1}^{\infty}$ for the space X such that given any compact subset K with $K\subset U$, there is a natural number n with $K\subset U_n$.

M. Henry. [5], obtained the following.

LEMMA 2.5. *A space X is k- semistratifiable if and only if to each closed set $F\subset X$, one can assign a sequence $\{U_n\}_{n=1}^{\infty}$ of open subsets of X such that*

(a) $\bigcap_{n=1}^{\infty} U_n = F$

(b) $U_n \subset V_n$ *whenever* $U\subset V$,

(c) If $F\cap K=\phi$ *with K compact in X,*

then there is open set U_n *with* $U_n\cap K=\phi$.

Clearly stratifiable spaces are k-semistratifiable and k-semistratifiable spaces are semi-stratifiable, but these implications cannot be reversed.

Notation and terminology will follow that of J. L. Kelley [11] and all mappings will be continuous and subjective, and N is the set of natural numbers. we denote the interior of a subset A of a topological space by Int (A).

3. Main theorems

For this section, we consider the following terminologies. A collection $\mathscr{B}$ of subsets of X is said to be *a pseudo base* if for each compact subset K of X and each open subset U of X containing K there is a $B\in\mathscr{B}$ such that $K\subset B\subset U$. Let $\mathscr{S}$ be a subbase for a space X and let $\mathscr{P}$ be a σ-locally finite family of subsets of X such that if $C\subset U\in\mathscr{S}$ with C compact, then $\exists R=\bigcup_{n=1}^{\infty} P_i\in\mathscr{P}$ such that $C\subset R\subset U$. We call such a family $\mathscr{P}$ (after Michael's $\mathscr{S}$-k-pseudo base) *an $\mathscr{S}$-k-network*.

THEOREM 3.1. *Let X be a regular T_1-space and $\mathscr{S}$ be a subbase for X. Then X is ℵ-space iff it has a σ-locally finite $\mathscr{S}$-k-network.*

PROOF. The necessity is trivial. To prove the condition sufficient, Suppose that $\mathscr{P}=\bigcup_{n=1}\mathscr{P}_n$ is a σ-locally finite $\mathscr{S}$-k-network for X.

Let $\mathscr{M}$ be the class of all finite subsets of N and for each $E\in\mathscr{M}$ put $\mathscr{F}(E)$ is the class of all finite intersections of members of $\bigcup\{\mathscr{P}_n : n\in E\}$. Since $\bigcup\{\mathscr{P}_n : n\in E\}$ is a subset of $\mathscr{F}(E)$ and each $\mathscr{P}_n$ is locally finite. Then each $\mathscr{F}(E)$ is locally finite so that $\mathscr{F}=\bigcup\{\mathscr{F}(E) : E\in\mathscr{M}\}$ is σ-locally finite. We shall show that $\mathscr{F}$ is a k-network for X. First suppose that $C\subset U\in\mathscr{B}$, where C is compact $\mathscr{B}$ is a base for X consisting of all finite intersections of members of $\mathscr{S}$. Then $U=\bigcap_{n=1}^{\infty}\{S_i : S_i\in\mathscr{S}\}$ and for each i $(i=1,2,\cdots n)$, there is a finite union R_i of members of $\mathscr{P}$ such that $C\subset R_i\subset S_i$. Then $C\subset\bigcap_{i=1}^{n} R_i\subset U=\bigcap_{i=1}^{n} S_i$

and $\bigcap_{i=1}^{n} R_i$ can be expressed as a finite union of members of $\mathscr{F}$. For by constract and $\mathscr{P}$ is a σ-locally finite $\mathscr{S}$-k-network for X. Now let U be an arbitrary open set, and $C\subset U=\bigcup B_i$ $(B_i\in\mathscr{B})$ (C is compact). Then $C\subset\bigcup_{i=1}^{n} B_i=U(B_i\in\mathscr{B})$ and $B_i\subset U$ for each i. Since C is normal. Let $\mathscr{U}=\{B_i : i=1,2,3,\cdots n\}$. Then $\mathscr{U}$ is a point finite open cover of a normal C. Then it is possible to select an open set C_i for each B_i in $\mathscr{U}$ in such a way that $\bar{C_i}\subset B_i$ and the family of all sets C_i is a cover of C. Hence for each i we have $C_i\subset\bar{C_i}\subset B_i$. Therefore $C_i\subset R_i\subset B_i$ for each i. Applying the result of the previous paragraph, we can find $R_1, R_2, R_3,\cdots R_n$ in $\mathscr{F}$ such that $C_i\subset R_i\subset B_i$ for all i. Now if $R=\bigcup_{i=1}^{n} R_i$, then $R\in\mathscr{F}$ and $C\subset R\subset U$.

DEFINITION 3.2. [9]. Let $(X,\mathscr{T})$ be topological space and let $g: N\times X\rightarrow\mathscr{T}$ such that $x\in\bigcap_{n=1}^{\infty} g(n,x)$ for each $x\in X$ if $y_n\in g(n,x)$ for each $n\in N$ implies that the sequence $\langle y_n\rangle$ has x as a cluster point. Then $(X,\mathscr{T})$ is called *a first countable space.*

DEFINITION 3.3. [9]. Let $(X,\mathscr{T})$ be a topological space and let $g: N\times X\rightarrow\mathscr{T}$ such that (1) $x\in\bigcap_{n=1}^{\infty} g(n,x)$ for each $x\in X$.

(2) If $y_n\in g(n,x)$ and $P_n\in g(n,y_n)$ for each $n\in N$, implies that the sequence $\langle p_n\rangle$ has x as a cluster point. Then $(X,\mathscr{T})$ is called *a γ-space.*

DEFINITION 3.4. Let $(X,\mathscr{T})$ be topological space, let $g: N\times X\rightarrow\mathscr{T}$ such that (1) $x\in\bigcap_{n=1}^{\infty} g(n,x)$ for each $x\in X$.

(2) If $x_n\in g(n,x)$ for each $n\in N$, implies that the sequence $\langle x_n\rangle$ has a cluster point. Then $(X,\mathscr{T})$ is called *a q-space.*

LEMMA 3.5. *Let $(X,\mathscr{T})$ be a regular space in which points are G_δ's. Then $(X,\mathscr{T})$ is a first countable space iff $(X,\mathscr{T})$ is a q-space.*

PROOF : The necessity is trivial.

To converse, let X is a regular q-space and let a point $x\in X$ is a G_δ-subset of X. Then there is a function $g: N\times X\to \mathcal{T}$ such that $\{x\}=\bigcap_{n=1}^{\infty} g(n,x)$ and $x_n\in g(n,x)$ for each $n\in N$. Then the sequence $\langle x_n\rangle$ has a cluster point in X. It follows that if $x_n\in g(n,x)$ for each $n\in N$, then every subsequence of $\langle x_n\rangle$ has x as its unigue cluster point, since $\bigcap_{n=1}^{\infty} g(n,x)=\{x\}$. Hence the sequence $\langle x_n\rangle$ has a cluster point x.

The following LEMMA is obvious from the DEFINITIONS.

LEMMA 3.6. *Suppose a topological space X has a semi-stratification* $U\to\{U_n\}_{n=1}^{\infty}$ *with the property that if U is an open in X and* $P\in U$, *then* $P\in Int(U_n)$ *for some* $n\in N$. *Then X is a stratifiable.*

By [4] LEMMA 3.5, and LEMMA 3.6. we obtain the following COROLLARY.

COROLLARY 3.7. *Let X be a k-semistratifiable q-space in which points are* G_δ*'s. Then X is stratifiable.*

PROOF. Let U be an open set in a k-semistratifiable and q-space X and $U\to\{U_n\}_{n=1}^{\infty}$ is an increasing k-semistratification for the space X, and $P\in U$. Assume that $P\in X-Int(U_n)$ for each $n\in N$. Since X is q-space, there exists decreasing sequence $\langle V_{(n)}\rangle$ of neighborhoods of P such that if $x\in V_{(n)}$ for each $n\in N$, then $\langle x_n\rangle$ has a cluster point in X. We may assume that each point x_n is in the open set U and $\{P\}=\bigcap_{n=1}^{\infty} V_{(n)}$. It follows that if $x_{(n)}\in V_{(n)}$ for each $n\in N$, then every subsequence of $\langle x_n\rangle$ has P as its unique cluster point, so $\langle x_{(n)}\rangle$ converges to P. Thus $\{x_n : n\in N\}\cup\{P\}$ is compact subset of U. Therefore exists a positive integer m such that $\{x_n : n\in N\}\cup\{P\}\subset U_n$ for each $n\geqq m$. which is contradict to choicing x_n. Thus by LEMMA 3.6, X is a stratifiable.

4. Properties by mappings

DEFINITION. 4.1. [3] A mapping $f: X\to Y$ is *compact* if $f^{-1}(y)$ is compact for each $y\in Y$.

DEFINITION. 4.2. [3] A mapping $f: X \to Y$ is ***a ka-mpping*** if $f^{-1}(K)$ is a compact subset of X whenever K is compact set in Y.

DEFINITION. 4.3. A mapping $f: X \to Y$ is called ***compact covering*** if every compact subset of Y is the image of some campact subset of X.

EDWIN HALFAR [3] showed that if a mapping $f: X \to Y$ is closed and compact, then f is k-mapping.

DEFINITION 4.4. A mapping $f: X \to Y$ is ***pseudo-open*** if for each $y \in Y$ and any neighbourhood U of $f^{-1}(y)$. it follows that $y \in Int[f(U)]$.

LEMMA 4.5. ***If $f: X \to Y$ is a pseudo-open finite-to-one mapping and X is a first countable space, then Y is first countable.***

PROOF. Since f is finite to one, we put $f^{-1}(y)=\{x_1, x_2, \cdots x_n\}$ for every $y \in Y$. Then for each x_i, there exists a countable decreasing open neighborhood base $\{U^n_{(x_i)}\}_{n=1}^{\infty}$. Let $U^n = \bigcup_{i=1}^{m} U^n_{(x_i)}$ Then $\{Int[f(U^n)]\}$ is a countable base of y. For, let U be an open neighborhood of y. Then $f^{-1}(U)$ is an open neighborhood of $f^{-1}(y)$. Hence there exists an integer k_i such that $U^{k_i}_{(x_i)} \subset f^{-1}(U)$. Let $k = \max\{k_1, k_2, \cdots k_n\}$. It follows that $y \in Int\ [f(U^k)] \subset U$.

Using an analogue to proof Theorem 2.3. in [5] the following LEMMA 4.6. may be proved.

LEMMA 4.6. ***If a mapping $f: X \to Y$ is a pseudo-open closed compact mapping and X is a k-semistifiable space, then Y is k-semistratifiable.***

PROOF. If $F \subset Y$ be a closed, then $f^{-1}(F)$ is closed in X. For each closed set F of Y and each natural number n, let $F_n = Int[f(f^{-1}(F)_n)]$ where $f^{-1}(F) \to f^{-1}(F)_n$ is a dual k-semistratification for X. we will show that the correspondence $F \to \{F_n\}$ is a dual k-semistratification for Y. Since $f^{-1}(F) \subset f^{-1}(F)_n$ for each $n \in N$, $f^{-1}(F)_n$ is an open neighborhood of $f^{-1}(y)$ for each $y \in F$, and f is a pseudo-open mapping, therefore, we have $F \subset \bigcap_{n=1}^{\infty} Int[f(f^{-1}(F)_n)] = \bigcap_{n=1}^{\infty} F_n$. For the reverse direction, assume $z \notin F$. Then $f^{-1}(z) \cap f^{-1}(F) = \phi$ with $f^{-1}(z)$ compact in X, and therfore there exists a natural number n such that $f^{-1}(z) \cap f^{-1}(F)_n = \phi$. Then $z \notin F_n$ for some n consequently, we have $F = \bigcap_{n=1}^{\infty} F_n$. Next, if F and G are closed subsets of Y such that $F \subset G$, then clearly $Int[f(f^{-1}(F)_n)] \subset Int[f(f^{-1}(G)_n)]$. Finally, et $K \cap F = \phi$ in Y with K compact and F closed in Y. Then $f^{-1}(K) \cap f^{-1}(F) = \phi$, $f^{-1}(K)$ is compact and $f^{-1}(F)$ is closed in X. Hence, $f^{-1}(K) \cap f^{-1}(F)_n = \phi$ for some n. Therefore, $K \cap Int[f(f^{-1}(F)_n)] = \phi$. By LEMMA 2.5., Y is k-semistratifiable.

THEOREM 4.7. *Let X be a Nagata space. If $f : X \to Y$ is closed pseudo-open finite to one compact mapping. Then Y is stratifiable.*

PROOF. By [4]. Since Nagata space are equivalent to the space is first countable and stratifiable. Since first countable and k- semistratifiable is stratifiable. Hence by LEMMA 4.5. and LEMMA 4.6., Y is stratifiable.

COROLLARY 4.8. *Let X be k-semistratifiable and if f is pseudo-open k-mapping and Y is first countable. Then Y is a Nagata space.*

THEOREM. 4.9. *If $f : X \to Y$ is a strongly continuous function. Then f is compact covering mapping.*

PROOF. It is sufficient to show that image of any compact subset of X is compact. Let A be a compact subset of X. Since f is stronly continuous, therefore $f^{-1}(y)$ is open for every y in Y, then clearly $\{f^{-1}(y) : y \in f(A)\}$ in an open covering of A. Hence there are the family many points $y_1, y_2, \cdots y_n \in f(A)$ such that $A \subset \bigcup \{f^{-1}(y_i) : i = 1, 2, \cdots n\}$. If $f(A)$ not compact, there is $z \in f(A)$ such that $z \neq y_i$ for every $i = 1, 2, \cdots n$, therefore there is a element $x \in A$ such that $f(x) = z$. Therefore $x \notin \bigcup f\{f^{-1}(y_i) : i = 1, \cdots n\}$, it is contradict. Hence $f(A)$ is compact.

S. MACDONALD AND S. WILLARD in [10] showed the following THEOREM 4.10.

THEOREM 4.10. *X is compact if and only if every funtion image of X is regular.*

THEOREM 4.11. *If $f : X \to Y$ is k-mapping and X is compact and ℵ-space.*

Then Y is ℵ-space.

PROOF. Since X is ℵ-space, let $\mathscr{V}=\bigcup_{n=1}^{\infty}\mathscr{V}_n$ be a σ-locally finite k-network for X. We shall prove that $\mathscr{W}=\bigcup_{n=1}^{\infty}\mathscr{W}_n$, where $\mathscr{W}_n=\{f(V):V\in\mathscr{V}_n\}$, is a σ-locally finite k-network for Y. Since f is continuous, each $\mathscr{W}_n$ will be locally finite in Y. To prove that $\mathscr{W}$ is a k-network for Y, let K be a compact subset of Y and U be an open subset of Y suh that $K\subset U$. Since f is k-mapping, $f^{-1}(K)\subset f^{-1}(U)$, therefore $f^{-1}(K)$ is compact and $f^{-1}(U)$ is open set. Let R be a finite union of members of such that $f^{-1}(K)\subset R\subset f^{-1}(U)$. Hence $K\subset f(R)\subset U$ and $f(R)$ is a finite union of member of $\mathscr{W}$. since X is compact, by THEOREM 4.10. Y is regular. Y is ℵ-space.

COROLLARY 4.12. *If $f:X\rightarrow Y$ is k-mapping and X is compact and ℵ-space and Y is first countable. Then Y is stratifiable.*

PROOF : By THEOPEM 4.11, Y is ℵ-space and since ℵ-space is k-semistratifiable and Y is first countable, Y is stratifiable.

DEFINITION 4.13. [9] Let X and Y be topological space, let $\Psi:X\rightarrow Y$ be a mapping, and let g be a COC-function for X. Then Ψ is *an N-mapping relative to g* if given any $y\in Y$. and neighborhood W of y, there is a neighborhood V of y and a positive integer n such that if $g(n,x)\cap\Psi^{-1}(V)\neq\phi$ then $\Psi(x)\in W$.

DEFINITION 4.14. [9], Let $(X,\mathscr{T})$ be topological space and let g be a function from $N\times\mathscr{T}$ into $\mathscr{T}$. Then g is called *a COC-function for X* if it satisfies these two conditions : (1) $x\in\bigcap_{n=1}^{\infty}g(n,x)$ for all $x\in X$,

(2) $g(n+1,x)\leq g(n,x)$ for all $n\in N$ and $x\in X$.

By KENNETH ABERNEHY [9], we have the THEOREM.

LEMMA. 4.15. *If Y is a regular space in which points are G_δ's. Then Y is q-space if and only if there is a metrizable space X and a open mapping from*

X onto Y.

PROOF : By THEOREM 2.1 [9] and LEMMA 3.5. is obvious.

LEMMA 4.16. *Let $(X, \mathscr{T})$ and Y be topological spaces. If $f : (X, \mathscr{T}) \to Y$ is mapping and Y is q-space and ℵ-space in which points are G_δ's. Then f is an N-mapping.*

PROOF, By LEMMA 3.5, Y is first countable and by [4], Y is k-semistratifiable. Therefore Y is stratifiable. Let h be a stratifiable function for Y, and define $g : N\times \mathscr{T} \to \mathscr{T}$ by $g(n,x)=f^{-1}[h(n,f(x))]$. Since h is COC-function for X, therefore $f(x)\in h[n,f(x)]$ for every n. Hence $x\in f^{-1}f(x)\subset f^{-1}[h(n,f(x))]$ for every n, therefore $x\in g(n,x)$ for every n, and another condition is trivial. Therefore g is a COC-function for X. Now let $y\in Y$, and let W be an open set containing y. Then $Y-W$ is closed and $y\notin Y-W$, hence there exists an $n_0\in N$ such that $y\notin \cup \overline{\{h(n_0,p) : p\in Y-W\}}$. Let $V=Y-\cup \overline{\{h(n_0,p) : p\in Y-W\}}$. Now if $g(n_0,x) \cap f^{-1}(V)\neq\phi$, then $h(n_0, f(x))\cap V\neq\phi$. But this means that $f(x)\notin Y-W$.

THEOREM : 4.17. *Let X and Y be topological spaces. If there is an open N-mapping from X onto Y. Then Y is stratifiable.*

PROOF, Let g be a COC-function for X reltive to which f is an N-mapping. Let $y\in Y$, $n\in N$. Then choose any $s\in f^{-1}(y)$ and define $h(n,y)=f[g(n,s)]$ for every n. We claim that h is a stratifiable for Y. Let H be closed in Y, and suppose that $p\in\cup\overline{\{h(n,z) : z\in H\}}$, for each $n\in N$. Suppose $p\notin H$; then $p\in Y-H=W$, which is open. Thus there exists a neighborhood V of p and an $n_0\in N$ such that if $g(n_0,x)\cap f^{-1}(V)\neq\phi$ then $f(x)\in W$. Now since V is a neighborhood of p, $V\cap(\cup \{h(n,z) : z\in H\})\neq\phi$ for each $n\in N$. Thus there is a $z\in H$ such that $h(n_0,z)\cap V\neq\phi$ Therefore, if t is such that $h(n_0,z)=f[g(n_0,t)]$, we have $g(n_0,t)\cap f^{-1}(V)\neq\phi$. But this implies that $f(t)=z\in W$, an obvious contradition.

5. Conclusions

In a regular T_1-space and if $\mathscr{S}$ is a subbase for the space, we are investigated that the space is $\aleph$-space iff it has a σ-locally finite $\mathscr{S}$-k-network. Also obtained that if a k-semistratifiable q-space in which points are G_{δ}'s, then the space is stratifiable, and that if a regular space in which points are G_{δ}'s, then q-space is a first countable spaces. It is shown that the image of a k-semistratifiable space under a pseudo-open closed compact mapping is k-semistratifiable space and that if X and Y are two space and if there is an open N-mapping from X onto Y, then Y is stratifiable.

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<요 지>

Stratifiable 空間과 ℵ-空間에 관해서

李 尙 憲

Stratifiable 空間과 ℵ-空間에 對해서는 최근 C. J. R Borges와 P. O' Meara에 依해 定義 되었으며 世界各國에서 이들 空間에 對해 硏究되어 오고 있다. 本 論文에서는 ℵ-空間의 定義와 비슷한 性質을 導入하여 同値條件을 구하였으며, 第一可附番 公理의 特性과 pseudo-open finite to one mapping 下에서 第一可附番 公理의 image (像)을 照査한 결과 k-semistratifiable 空間이 stratifiable 空間이 되기위한 條件을 얻었으며, Nagata 空間에서 한 mapping의 image (像)이 stratifiable 空間이 되기 위한 條件들을 照査하였으며, 그리고 어떤 函數가 k-函數이며 한편 그 空間이 긴밀성 (Compact)과 ℵ-空間을 滿足하면 그 函數의 image (像)이 또 ℵ-空間임을 밝혔다. 최근 KENNETH ABERNETHY On characterizing certain classes of first countable spaces by open mappings [9] (1974) 에 依한 N-mapping을 導入, N-mapping의 性質을 얻고 임이의 空間에 對한 N-mapping의 像 (image)이 stratifiable 空間임을 照査하였다.