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General Human Capital and Wage Differentials

*Pil-Soo Ko**

Contents

Ⅰ. Introduction

In this study we consider the effect of general human capital, i.e., differences in employees' initial skill (or schooling) levels, on the decision of hiring. We first derive a search model which explicitly incorporates these differences as well as search costs, unemployment compensation, and layoff possibilities. This model permits the development of a "willing applicants constraint" which determines a firm's possibilities of controlling the hiring flow by means of wage offers and skill requirements. By using this constraint as a point of reference, we study the determination of the firm's optimal job offers. Our analysis can be considered as an extension of that of Mortensen (1970). In our analysis we study the effects of a firm's growth rates (of employment) and other relevant characteristics of a firm together with the effects of minimum wage legislation on these optimal job offers. Furthermore, we examine the consequences for a firm's behavior with respect to job offers when the incumbent employees resist all wage cuts but demand at

* 경상대학 경제학과(Dept. of Economics, Cheju Univ., Cheju-do, 690-756, Korea)

least as good wages as the new entrants.

II. A Job-Search Model

In this section we develop a job search model, and in so doing we pay special attention to the skill dimension, i.e., to employees' skill endowments and employers' skill requirements. As for skill endowment, we assume that an employee looking for a job cannot change it. On the other hand, it is assumed that all skills can be measured in a single dimension.[1]

The term "skill requirement", in turn, indicates the minimum amount of skills required in a certain job. In this connection, it is appropriate to ask whether skill requirement is an instrument for the firm. Obviously, two extreme cases in this respect are : On the one hand, that the firm may have no skill requirements: and on the other hand, that the firm may have nothing but the skill requirement as an instrument. In the former case (which we call the "flexwage" case after Pissarides (1976, 1984) the firm in question makes a job offer to everybody irrespective of their skill endowment, that is, the firm first checks the employee's skill endowment and then make a wage offer according to the observed amount of skills. In the latter case (analogously called the "fixwage" case), the wage rate is given to the firm (for example, because of some institutional reasons) and the firm changes its skill requirement so that the required amount of labor can be hired in different periods.[2]

Job search models are typically related to the flexwage case, even though it would also be possible to develop a job search model based on the fixwage case, as done by Pissarides (1976). The intention here is to consider a general case in which a firm has both wage offers and skill requirements as variables.

Before presenting the search model, the hiring process is briefly described. First, we assume that labor input is heterogeneous with repect to skills, different skills being, however, substitutable. Workers do not know the wage offered or the skills required by any particular employer, although they are assumed to know the relevant distribution of

1) Instead of speaking about skills, we could also use the term "general human capital". In contrat, "firm-specific human capital" plays no role in this connection.
2) A further discussion on the relevance of these labor market cases is provided by Hicks (1974), p. 74.

wage offers. Since a worker looking for a job does not know whether his skills, i.e., general human capital, meet the requirements for a particular vacancy, and in order to ascertain the wage offered for filling the opening, he must search for it. Prior to each vacancy sampled, the worker must determine the terms which he would willing to accept. For these terms, the critical element is the reservation wage, i.e., the lowest acceptable wage offer.

If now a worker searches a firm, the employer first checks whether the worker's skill endowment equals or exceeds the skill requirement for a particular vacancy, and if it does, he will then make a wage offer to the worker. If this wage offer equals or exceeds the worker's reservation wage, hiring takes place. There is no possibility of recall if the worker turns down the wage offer.[3]

The following basic notation is adopted : A worker has a skill endowment, denoted by x_0. He faces a known distribution of wage offers with respect to the jobs for which he is qualified, denoted by $f(w|x_0)$. The probability density function $f(w|x_0)$ can now be defined to be :

$$(1)\quad f(w|x_0) = m(w)\theta(w|x \leq x_0)$$

where $m(w)$ stands for the distribution of firms with respect to wage offers and the conditional (cumulative) distribution, $\theta(w|x \leq x_0)$, the probability that a worker with x_0 is qualified for a job paying w.

Implicitly (1) contains both fixwage and flexwage cases: in a pure flexwage case, $f(w|x_0)$ would coincide with $m(w)$, whereas in a pure fixwage case, $f(w|x_0)$ would coincide with $\theta(w|x \leq x_0)$.[4]

(1) now represents a general form of a job offer, i.e., a relationship between a wage offer, w, and the respective skill requirement, x. Here we may refer to Mortensen(1970), who utilizes a simpler framework. Thus Mortensen postulates a deterministic relationship between w and x over all firms, so that $w = w(x)$, $w' > 0$. Hence a worker is qualified for all job openings which offer a wage rate equal to $w(x_0)$ or less. Consequently the probability that a randomly selected vacancy is one for which the worker is qualified is

3) In this connection, we concentrate on the reservation wage only. Clearly, a more general index of job characteristics should be used, as, for example, in Mortensen(1975).

4) (1) is a form proposed by Pissarides(1976), p.87 (even though he does not utilize this form).

simply $(P(w \leq w(x_0)) = F(x_0)$, where $F(w)$ denotes the cumulative PDF with respect to w, $w_0 = w(x_0)$.[5]

Of course, it is not stated that a worker receives a job offer in every period; instead, we must specify a corresponding measure. Now let η denote the probability of a getting a job offer. Clearly η is not invariant over time; if, for example, unemployment rises, competition for available vacancies increases, diminishing the probability of an offer.[6]

In this connection, we assume simply that $\eta=1$, mainly because the economy is assumed to be in a stationary state.[7]

Before we consider the actual search model, we should still discuss the proper search strategy. Standard job-search models assume more or less implicitly that a participant either ignores the possibility of searching after becoming employed, or considers such search to be prohibitively expensive. Thus, the search strategy is characterized by a once-for-all decision of a life-time job, that is, a worker does not search for a temporary job. Clearly this strategy is non-optimal. Burdett (1978) offers more realistic search strategies. Their models implies that if a participant receives a very good offer, he finishes searching for good; alternatively he may accept the offer and continue searching on-the-job.[8]

5) In this case, a worker hunting a job faces uncertainty only in one dimension, i.e., with respect to the skill requirement of the particular job opening. This is so, because, for any particular vacancy, the wage offer is known once the skill requirement is known, see Mortensen (1970), p. 849.

6) Of course η also depends on the intensity of search, but here it is simply assumed that the search intensity is a given constant.

7) In fact, our analysis contains two very strong assumptions, i.e., that, on the one hand, there is no learning, and that on the other hand, the search process is completely random. As for learning, it was stated above that the worker knows the PDF $f(w|x_0)$, and thus new information does not arise regarding this PDF as the search goes on. A more realistic analysis is provided in, for example, Kohn & Shavell (1974) and Porter (1985).

The assumption that the search process is completely random is not very realistic, either, because individuals are able to distinguish among firms ex ante, and they sample some specific firms in a systematic fashion rather than just sampling the job market in general. This fact is incorporated in the analysis of Salop (1973), which Salop calls a "systematic search model". In this model, a worker selects a firm to sample in addition to the reservation wage. The main implication of this model is that the reservation wage declines as search goes on because the worker searches his best opportunities first and the poorer ones later.

8) Clearly, if we allow voluntary quits to appear, we should have this kind of model which knows quits. The problem with these models, especially with that of Burdett, lies in the explanation of frictional unemployment. In Burdett's model, the optimal search strategy is formulated so that (if search costs are ignored) a participant always accepts an offer which exceeds the present wage or unemployment compensation. It is difficult to believe that frictional unemployment could exist to a marked degree in such an environment.

However, in our analysis, the myopic rule of the once-for-all search is followed. This is mainly because we are not interested here in the life-cycle properties of an employee's behavior.

Now we derive the optimal reservation wage, denoted by y^*, for a worker with skill endowment x_o. It is assumed that he maximizes the present discounted value of expected wealth, EW, with respect to reservation wages, y_t, (the worker is thus assumed to be risk-neutral). First, the conditional probability that randomly selected vacancy is one for which a participant is qualified and is at the same time one which is acceptable to him is :

$$(2)\quad a_t(y_t|x_o) = P(w_t \geq y_t | x_t \leq x_o) = \int_{y_t}^{\infty} f(w_t|x_o)\,dw_t$$

where $f(w_t|x_o)$ is defined above in (1), t refers to the respective period. Now the expected wage offer, given that an acceptable offer is made by a firm, is :

$$(3)\quad h_t(y_t|x_o) = E(w_t|w_t \geq y_t|x_t \leq x_o) = \frac{\int_{y_t}^{\infty} w_t f(w_t|x_o)\,dw_t}{\int_{y_t}^{\infty} f(w_t|x_o)\,dw_t}$$

Before we write out the maximization problem, we (re)write the main assumptions which characterize the job-search process :

Assumption 1 : A participant searches until an offer, $w_t \geq y_t$, is found (then $x_t \leq x_o$ also holds). The worker intends to work an infinite number of periods in the job after hiring takes place.

Assumption 2 : The number of search periods is finite, say N, and is given to the participant.[9]

Assumption 3 : The participant's rate of time preference, r, as well as the PDF $f(w_t|x_o)$ are assumed to be invariant over time.

Assumption 4 : There are no discharges and layoffs after hiring.

Assumption 5 : If the worker searches for a job, he receives a periodic unemployment compensation, denoted by e, if he is unemployed. On the other hand, the direct search costs are c per period. If the worker is searching on-the-job, it is

9) Assumptions 1 and 2 seem to contradict each other. However, these somewhat unrealistic assumptions make the subsequent analysis much more straightforward, and hence we start with them. The case in which N approaches infinity will also be studied.

assumed that his wage rate is w_1, the search costs again being c.

Assumption 6 : The employer can observe immediately the true value of x_o.

Assumption 7 : The participant can search only one vacancy in a period, but on the other hand, he always receives a job offer.[10]

Now we write out the present discounted value of expected wealth, EW, with respect to an unemployed searcher, by using (2) and (3), that is :[11]

$$(4)\ EW=a_1h_1+(1-a_1)(e-c)+(1-a_1)a_2h_2R-(1-a_1)(1-a_2)(e-c)R+a_1h_1R+(1-a_1)(1-a_2)a_3h_3R^2+(1-a_1)(1-a_2)(1-a_3)(e-c)R^2+(1-a_1)a_2h_2R^2+a_1h_1R^2+\cdots,$$

where superscripts refer to periods, $R^t=(1+r)^{-t}$. Because there is an infinite number of terms in (4) due to Assumption 1, we sum the outcomes corresponding to N search periods by using the fact that

$$\sum_{t=0}^{\infty} x(1+r)^t = x(1+r)/r. \quad \text{So we obtain :}$$

$$(5)\ EW=((1+r)/r)a_1h_1+(1-a_1)(e-c)+(1-a_1)/r)a_2h_2+(1-a_1)(1-a_2)(e-c)R+\cdots+(\prod_{t=1}^{N-1}(1-a_t)a_Nh_N)((1+r)^{N-2})r+(\prod_{t=1}^{N}(1-a_t)e)/((1+r)^{N-2})r$$

Prior to every period the searcher must determine the reservation wage, y_t. The respective optimal vlues of y_t, $t=1,2,\cdots,N$, maximize EW, thus we can write out the following first-order conditions, $\partial EW/\partial y_t=0$, i.e.,

$$(6)\ \begin{cases} \bar{y}_{t-1}=(1+r)^{-1}\bar{a}_t\bar{h}_t+(r/(1+r))(e-c)+((1-\bar{a}_t)/(1+r)^2)\bar{a}_{t+1}\bar{h}_{t+1} \\ \quad +(r(1-\bar{a}_t)/(1+r)^2)(e-c)+(\prod_{j=t}^{N-1}(1-\bar{a}_j)\bar{a}_N\bar{h}_N)/(1+r)^{N-t})r \\ \quad +(\prod_{j=t}^{N}(1-\bar{a}_j)e)/(1+r)^{N-t})r \qquad \text{for } t=2,3,\cdots,N \\ \bar{y}_N=e, \end{cases}$$

10) The question of whether the participant searches a firm or a single vacancy still remains unanswered.

11) The basic maximization problem is a common folklore in job-search theory, see, for example, Salop(1973), p.194. We have, however, introduced search costs, an unemployment compensation and (later) the probability of layoffs into the basic model.

where the bars indicate the corresponding optimal values. We cannot solve directly the explicit values of $\bar{y}_t$ for $t=1, 2, \cdots, N$. However, it is possible by successive displacements to solve the following recursive equation :

$$(7)\quad \bar{y}_{t-1}=\frac{1}{1+r}\,[\bar{a}_t\bar{h}_t+r(e-c)+(1-\bar{a}_t)\,\bar{y}_t]$$

Using (7) we could now define the time path of $\bar{y}_t$, for the search period; since in this case the search horizon is finite, the reservation wage is decreasing over time.[12]
If, however, the search horizon were also infinite (as the working horizon already is), the reservation wage would stay constant, i.e., $\bar{y}_{t-1}=\bar{y}_t$, hence :

$$(8)\quad y^*=\frac{a^*h^*}{r+a^*}+\frac{r(e-c)}{r+a^*}$$

follows. (8) is the result also obtained by Mortensen (1970b), p. 851, with the aid of a slightly different model. As Mortensen points out, the interpretation of (8) is straightforward. That is, the reservation wage equates the marginal cost of search, $y-e+c$, with the present value of marginal expected returns from searching, i.e., $(a(h-y))/r$.[13]

In the next section, we use (8) as a base when deriving the willing applicants constraint facing a firm. Before this, however, we briefly examine the case in which layoffs can appear. We assume simply that there is a constant probability, say u, that an employee is laid off (i.e., discharged) in a period. Analogously to (4), we can now write the present discounted value of expected wealth, EW, in the form :

$$\begin{aligned}(9)\quad EW=&a_1h_1+(1-a_1)(e-c)+R[a_1h_1(1-u)+(1-a_1)a_2h_2+((1-a_1)(1-a_2)+ua_1)(e-c)]\\&+R^2[a_1h_1(1-u)^2+(1-a_1)a_2h_2(1-u)+((1-a_1)(1-a_2)+ua_1)a_3h_3+(((1-a_1)(1-a_2)\\&+ua_1)(1-a_3)+a_1(1-u)u+(1-a_1)a_2u)a_2u)(e-c)]+R^3[\cdots\ .\end{aligned}$$

Now only the following recursive equation can be solved :

12) For a formla proof, see Pissarides (1976), pp. 128~141. We might also mention here that there is empirical evidence which clearly shows that y_t is declining over time, cf., for example, Holt (1970), pp. 96~101.

13) Instead of using a discrete time model, the same result, (8), could also be obtained with a continuous time model, see Chow & Robbins & Siegmund (171), p. 113.

(10) $y_{t-1}=1/(1+r)[(r+u)(e-c)+(1-u)a_t h_t+((1-a_t)-u)y_t]$.

If $y_{t-1}=y_t$, we can solve :

(11) $$y^*=\frac{(1-u)a^*h^*}{r+a^*+u}+\frac{(r+u)(e-c)}{r+a^*+u}$$

Clearly, if u goes to zero, (11) coincides with (8). In this connection, we could study the effects of an increase in the unemployment compensation, e. According to (8) and (11), an increase in e implies a higher value of y^*, implying in turn, a lower value for the probability of acceptance, i. e., (2), which could be interpreted as an increase in frictional unemployment.[14]

However, this result is not wholly true if not all unemployed searchers are eligible for the unemployment compensation. A typical feature of many unemployment compensation systems is that some groups, like quitters, youths, etc., are not eligible for compensation. On the other hand, if they are hired and then laid off, they, by definition, become eligible. To take account of these employees, we should develop an appropriate search model. In this case, the present discounted value of expected wealth, EW, is of the form :

(12) $$\begin{aligned}EW=&a_1h_1-c+R[a_1h_1(1-u)+(1-a_1)a_2h_2+ua_1e+((1-a_1)(1-a_2)+ua_1)c]+\\&R^2[a_1h_1(1-u)^2+(1-a_1)(1-a_2)a_2h_2+((1-a_1)(1-a_2)+ua_1)a_3h_3-(((1-a_1)(1-a_2)\\&+ua_1)(1-a_3)+a_1(1-u)u+(1-a_1)a_2u)c+a_1u(1-a_3)+a_1(1-u)u)e+R^3[\cdots\ .\end{aligned}$$

By using the same procedure as above, we can solve in the case of $y_{t-1}=y_t$

(13) $y^*=1/(r+a^*+u)[(1-u)a^*h^*-(a^*u(r+u))/(1+r)e-(r+u)c]$.

Clearly, if e increases, the workers not eligible for the unemployment compensation, should decrease their reservation wage (and get a job sooner and thus benefit (indirectly) from the increased e).

Summing up, it can be seen that if only a fraction of unemployed searchers are eligible

14) The frictional unemployment rate could be defined as : $\mu=v/(\alpha+v)$, where v denotes the rate new entrants enter the labor force and α the (average) probability that a rendomly selected wage offer is accepted (on condition that searchers are qualified for the jobs offered). For the derivation of μ, see, for example, Mortensen (1970), p. 860.

for the unemployment compensation, an increase in this compensation can either increase or decrease the reservation wages (and thereby the level of frictional unemployment). The direction of the change does, of course, crucially depend on the relative magnitude of eligible and noneligible job searchers. The same result is obtained by Mortensen (1977) - even though he uses a more sophisticated model.[15]

Ⅲ. The Willing-Applicants Constraint Facing a Firm

In this section we derive a function, $s(w, x)$, which shows the number of workers who are, on the one hand, willing to be hired at wage rate w, and, on the other hand, qualified with respect to the skill requirement, x. In order to do so, we must first derive the distribution of reservation wages and skill endowments.

The previous section concerned a worker with a skill endowment, x_0. Of course, not all workers with this skill endowment are alike. Instead they differ, for example, according to the following characteristics : Attitude toward risk, search horizon, search intensity, knowledge of the PDF $f(w|x_0)$, search costs, etc.[16]

Clearly, it would be difficult to derive the distribution of reservation wages (and skill endowments) by referring to all these differences. In the following, only search costs will be considered. Consequently, the derivation is not very rigorous, but it will serve to illustrate the main idea of the reservation wage (and skill endowment) distribution.

Our derivation concerns the distibution of reservation wages with the skill endowment x_0, denoted by the PDF $g(y|x_0)$; we use a distribution of direct search costs, the corresponding PDF being denoted by $s(c)$. Now if we use (8) as a reference, we can show that the reservation wage, y, it the solution to :[17]

$$(14)\quad \int_y^{\infty}(w-y)\,dF(w) = r(y-e+c)$$

If we solve this quantity with respect to c, we can write simply $c=C(y)$ with nonzero C'

15) There is really a vast literature concerning the role of the unemployment compensation in the labor market. Here we refer only to a special issue of *Industrial and Labor Relations Review* Vol. 30, No. 4 (1977).

16) Needless to say, all these factors affect the level of reservation wages. For a further analysis, the reader may consult Pissarides (1976), and Lippman & McCall (1976).

17) For simplicity, here the stars are omitted from y^*, etc.

existing. This, in fact, implies :[18]

(15) $c(y) = \bar{c}$

$\bar{y} \geq y$ implies $c \geq \bar{c}$

Therefore we can write :

(16) $P(y \leq \bar{y}) = \int_{\bar{c}}^{\infty} s(c)\,dc$

This defines the distribution of reservation wages, i.e.,

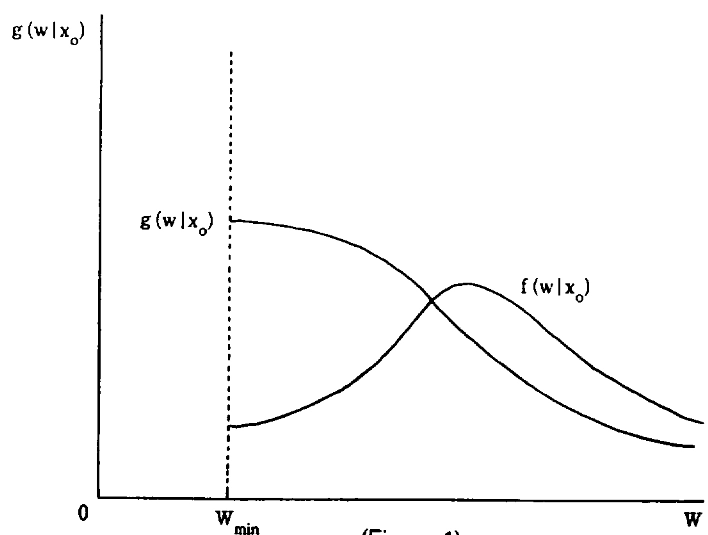
(17) $g(\bar{y}|x_o) = -s(\bar{c})\,d\bar{c}/d\bar{y}$

By using the fact that $c = \bar{C}(y)$, we get the following value for the derivative dc/dy :

(18) $d\bar{c}/d\bar{y} = c'(\bar{y}) = [-1 + F(\bar{y})]/r - 1$

Hence, (17) can be written in the form :

(19) $g(\bar{y}|x_o) = s(C(\bar{y}))[1 + r - F(\bar{y})]/r$



(Figure 1)

18) This convention is analogous to the one of Axell (1986), pp. 83~84 regarding reservation price.

Now we can draw the PDF $g(.)$: figure 1 illustrates a case in which $s(c)$ is rectangular and $f(w|x_0)$ normal with a minimum wage rate w_{min}.

Hwoever, we are not interested in the conditional PDF $g(y|x_0)$ but rather the joint PDF of y and x_0. This PDF of employees' skill endowments, denoted by $\psi(x_0)$. Now the probability that a (random) searcher is hired by the firm is :

$$(20)\quad r(w, x) = P(w>y, x<x_0) = \int_x^\infty \int_0^w g(y, x_0)\,dy\ dx_0$$

In this connection, we must disaggregate the workers searching the firm according to their employment status, that is, into employed searchers and unemployed searchers. Using subscripts e and u respectively for these groups, we can write out the function $s(w, x)$, which shows the number of workers willing to be hired while at the same time being qualified with respect to the firm's skill requirement, that is :

$$(21)\quad s(w, x) = N_u r_u(w, x) + N_e r_e(w, x)$$

where N_u denotes the number of search contacts made to the firm by unemployed job searchers and $r_u(w, x)$ the probability that such a searcher accepts the wage offer w while simultaneously being qualified for the firm. N_e and $r_e(w, x)$ are defined in a corresponding way in relation to employed searchers. The crucial question is how N_u and N_e are determined. The simplest answer is that all firms receive the same number of contacts, thus N_u would simply be the number of unemployed job searchers divided by the number of firms and N_e a corresponding measue concerning those searching on-the-job. Here we assume only that N_u and N_e are given constants for the firm.

Now we should discover the properties of $s(w, x)$, especially the appropriate signs of the partial derivatives. Differentiation gives :

$$(22)\quad S_w = N_u g_{u\bar{x}}(w) + N_e g_{e\bar{x}}(w)$$

$$(23)\quad S_{ww} = N_u g_{u\bar{x}}(w) + N_e g'_{u\bar{x}}(w)$$

$$(24)\quad S_x = -N_u g_{u\bar{w}}(x) - N_e g_{e\bar{w}}(x)$$

$$(25)\quad S_{xx} = -N_u g'_{u\bar{w}}(x) - N_e g'_{e\bar{w}}(x)$$

$$(26)\quad S_{xw} = -N_u g_u(w, x) - N_e g_e(w, x),$$

where $s_w = \partial s(w, x)/\partial w$, $g_{u\bar{x}}(w) = g_u(x \leq x | w)$, i.e., the conditional (marginal) PDF of w (corresponding to the joint PDF $g(w, x)$ defined in (20); $g_{uw}(x)$ is defined in an analogous way.

The signs of s_w and s_x are unambiguously positive and negative, respectively. Similarly, $s_{xw} = s_{wx}$ is unambiguously negative. In contrast, the signs of s_{ww} and s_{xx} are ambiguous. In order to discover these signs we should know the joint PDF $g_e(y, x_o)$ and $g_u(y, x_o)$, as well as the (relative) magnitude of N_e and N_u. However, we do not possess this information.

Nevertheless, the following facts can be stated : If both $g_{ex}(w)$ and $g_{ux}(w)$ are decreasing with w, i.e., "if there are more low reservation wage searchers (at each skill category) than high reservation wage searchers", s_{ww} is negative. The same analogy implies that s_{xx} is positive.

IV. The Determination of a Firm's Job Offers

In this section we consider the hiring process of a firm, especially the determination of the optimal job offers in relation to wage offers and skill requirements. Only new employees (entrants) are considered here, the incumbent employees playing only a secondary role in this connection; for example, we ignore voluntary quits. In fact, we proceed as if there were two separate problem facing a firm : the first (considered here) concerning the hiring of new employees and the second concerning such things as (firm-specific) training, upgrading, and the long-run wage policy of the incumbent employees.

A firm can influence the number of workers hired within a period by changing the wage offer, w, and the skill requirement, x. If the firm in question increases its stock of labor at a rate g, and if the withdrawal rate of labor is q (here assumed given to the firm), the firm faces the following constraint with respect to hiring :

$$(27) \quad s(w, x) - qL - gL \geq 0$$

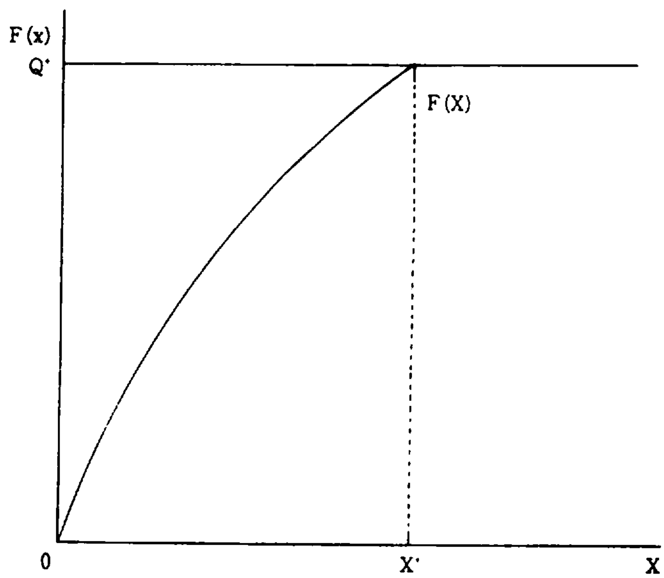
where $s(w, x)$ is defined in (21), and L denotes the stock of labor in the firm. As for the optimal values of w and x, Mortensen suggests that the combination of w and x which minimizes the current outlay is selected. Clearly, both w and x involve costs, that is, a higher wage offer implies a larger wage bill. On the other hand, given the wage offer, either in the sense that productivity falls or that expenditure is required to train the new employees so that they attain the same level of skill as the existing employees.

Next we develop a model along these lines and employ it to derive the optimal job offer(s) of the firm. There are, in addition, two specific problems to be analyzed when using this model as a framework of reference; first, the question of what happens if the job offers cannot be changed without adjusting the benefits of the incumbent employees, and secondly how the minimum wage legislation affects employment, in particular, in this framework.

As stated earlier, a firm tries to minimize its current outlay caused by new employees. There are two costs : wage costs and the (opportunity) costs due to an insufficient skill level. In order to be able to consider the latter type of cost, we should introduce the relevant cost function. To do so we first postulate the following production function :

$$(28)\quad Q=F(x)$$

where Q denotes output pre employee, the production function, $F(x)$, is assumed to be additive over employees, and it is assumed that $F(x)$ is concave with $F'>0$ and $F''<0$ for $x<x^*$ and $F'=0$ for $x\leq x^*$ (see figure 2).[19]



(Figure 2)

19) The assumption that $F'=0$ for $x\geq x^*$ is not of decisive importance; it only makes our analysis more accessible. On the other hand, we know that there are not any employees with $x=\infty$.

In figure 2, Q^* now simply represents the maximum output per employee, i.e., $Q^* = F(x^*)$. x^* can now be used as a scale in the appropriate cost function, $h(x^*-x) = Q^*-F(x)$. Clearly, if $F(x)$ is concave, $h(.)$ must be convex in x.[20]
Now we can write out the cost minimization problem, that is :

(29) $$\min_{w,x} C = w + h(x^*-x)$$

subject to $s(w,x)-qL-gL \geq 0$, $w \geq w_{min}$, $x \geq 0$

where the notation has the following interpretation :

- w is the wage offer
- x is the skill requirement, within one period all workers (searching the firm) receives the same job offer
- $s(w,x)$ is the willing applicants constraint defined in (21)
- q is the withdrawl rate of labor in the firm
- g is the (given) growth rate of employment in the firm
- w_{min} is the minimum wage rate
- $h(x^*-x)$ is the cost function defined above in (28).

The Lagrangean corresponding to (29) is :[21]

(30) $$\wedge(w, x, \lambda, a_1, a_2) = -w \; -h(x^*-x) + \lambda(s(w,x)-qL-gL) + a_1(w+w_{min}) + a_2 x$$

The necessary conditions are :[22]

(31) $$\partial\wedge/\partial w = -1 + \lambda s_w + a_1 = 0$$

(32) $$\partial\wedge/\partial x = h'(x^*-x) + \lambda s_x + a_2 = 0$$

(33) $$\partial\wedge/\partial\lambda = s(w,x)-qL-gL \geq 0 \text{ and } \lambda(s(w,x)-qL-gL) = 0$$

20) If we postulated a training cost function, it would be analogous to $h(x^*-x)$. In this case, we would only require some average, long-run value of skills as a scale. In addition, In addition, here the price of output is simply unity.

21) Here we study, in fact, the problem : [max −c]. We could also introduce a discount term into the cost minimization problem, but this would not affect the subsequent results because it would concern both w and h(.).

22) By definition, the gross hiring rate, q+g, must also be strictly positive.

(34) $\partial\Lambda/\partial a_1 = w - w_{min} \geq 0$ and $a_1(w - w_{min}) = 0$

(35) $\partial\Lambda/\partial a_2 = x \geq 0$ and $a_2 x = 0, a_1, a_2, \lambda \geq 0$.

If the willing-applicants constraint, (27) is not binding, $\lambda = 0$, which, in turn, implies $a_1 > 0$, and thus $w = w_{min}$. On the other hand, $\lambda = 0$ implies that $h' = 0$ (because $a_2 \geq 0$), which, in turn, takes place if $x = x^*$, which, of course, also satisfies (35).

Next we study the case in which (27) is binding, i. e., $\lambda > 0$. First, we consider the case $x > 0$ & $w > w_{min}$, i. e., $a_1 = a_2 = 0$.

From (31) and (32) we can solve :

(36) $s_x/s_w = -h'(x^* - x)$

which can be interpreted to mean that in optimum, w^*, x^*, the slope of the $s(w, x)$-qL-gL locus must be equal to that of the $C = w + h(.)$ locus. Now the expansion path, i. e., a locus of w^*, x^*, corresponding to different values of $(q+g)L$ is simply :

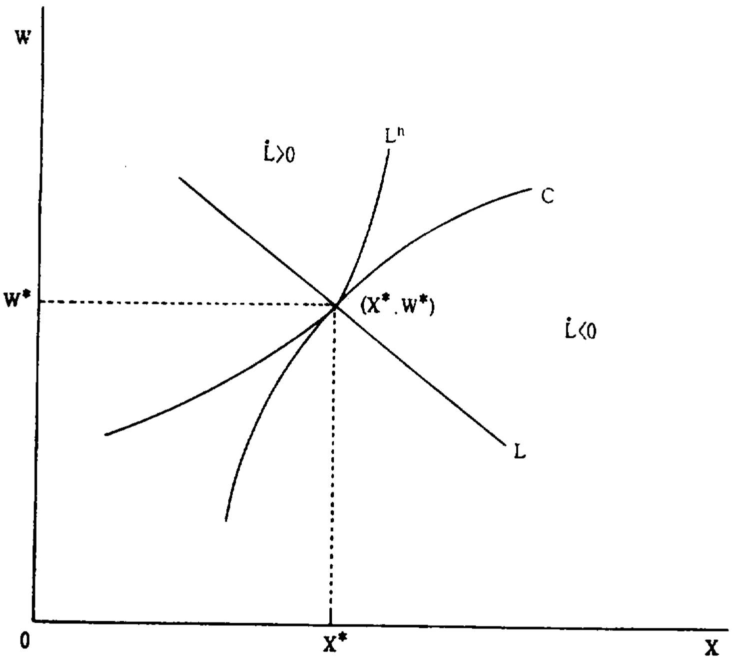
(37) $s_x + h' s_w = 0$.

The corresponding slope is :

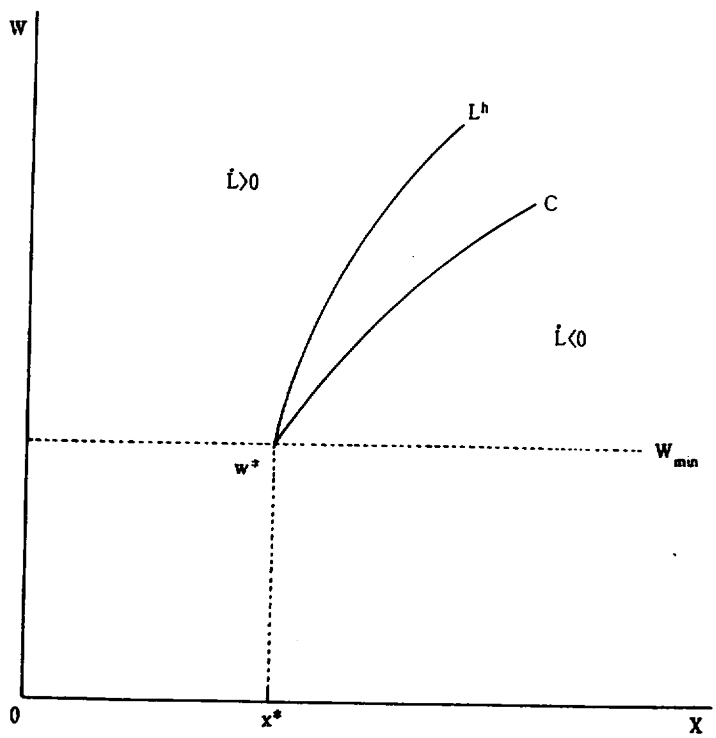
(38) $dw/dx = \dfrac{-s_{xx} + h'' s_w - h' s_{wx}}{s_{xw} + h' s_{ww}}$

Presumably its sign is negative. What is, hwoever, needed is that $s_{xx} < 0$ and $s_{ww} < 0$. The previous analysis does not unambiguously support these assumptions. This is especially true with respect to the sign of s_{xx}. In order to obtain $s_{xx} < 0$, the conditional PDF $g_{ux}(x)$ (as well as $g_{ew}(x)$) should be increasing with x, cf. (25). If, instead, s_{xx} were positive, a firm would have increasing returns from lowering the skill requirement, x. This could make the $s(w, x)$-qL-gL locus concave and this, in turn, would imply a corner solution with respect to the optimal job offer, w^*, x^*.

Figure 3 illustrates an interior Solution, whereas figure 4 depicts a corner solution, L^h denotes the $s(w, x)$-qL-gL locus. The LL locus in figure 3 corresponds to different values of $(q+g)L$, i. e., gross hiring flow to the firm. If that flow increases, we move to the left along the LL locus and vice versa.



(Figure 3)



(Figure 4)

As for the optimal job offer, w^*, x^*, the following comparative statics results can be derived from (31)-(33) by Cramer's rule :[23]

(39) $\partial x^*/\partial g < 0$, $\partial w^*/\partial g > 0$, $\partial x^*/\partial x' > 0$, $\partial w^*/\partial x' > 0$.

That is, the firm raises its wage offer and lowers the skill requirements when it desires to accumulate employees at a more rapid rate.

This result, i.e., (38) and (39), can also be interpreted to mean that firms with a high growth rate of employment, g, pay higher wages and apply looser skill requirements that the low growth rate firms. In other words, wage differentials between firms are due to the differences in frim's growth rates.

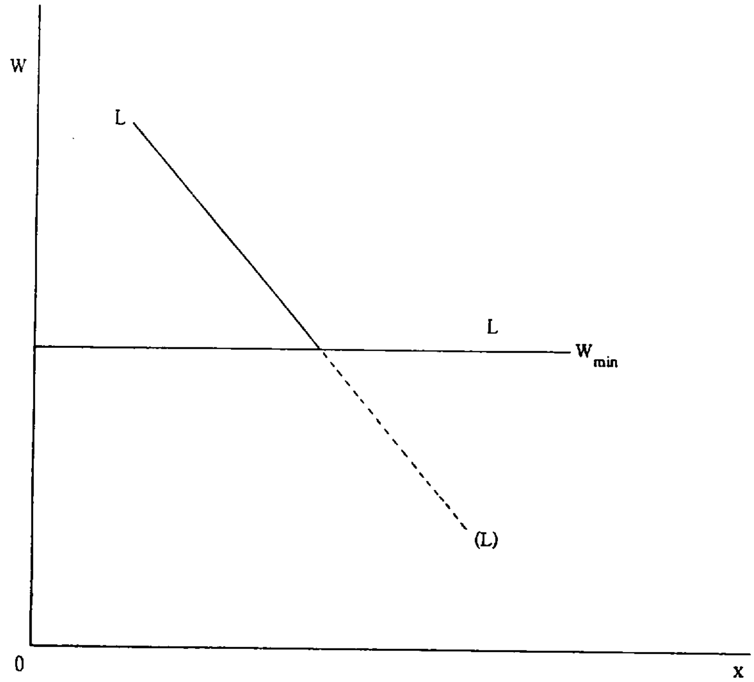
We could also apply this framework in analyzing the cyclical behavior of firms with respect to the whole labor market. Thus, if we assume that changes in employment across firms are correlated in a given period, i.e., if all or most firms desire to accumulate employees at similar rates, the aggregate relationship between wage offers and skill requirements shifts in a more or less consistent manner.

So, if we consider, for example, a temporary depression case with a very low average growth rate of employment, i.e., $-q < g \leq 0$, the job offers would then be characterized by low vaues of w and high values of x (according to the aggregate LL locus). If we consider unemployed job searchers only, it is easy to see that those with a low skill endowment face the worst prospects. Even if they were to lower their reservation wages, this would not improve their situation. Young workers who have just entered the labor force are an example of these "unlucky" employees.

If we consider a single firm again, it is clear that with low values of g the firm may meet the minimum wage constraint, $w \geq w_{min}$. Such a firm has a discontinuous LL locus, as illustrated in figure 5. Thus, in a "depression" case, the firm adjusts only with respect to the skill requirement, whereas in all other cases it adjusts both the wage offer and skill requirement.

Since no firm is able to avoid the minimum wage constraint we would expect that at the aggregate level the adjustment of skill requirements is used more in a depression case

23) By means of envelope theorem, we can also show that $\partial x^*/\partial (q+g)L = \lambda^*$ and $\partial C^*/\partial x' = h'(x'-x^*)$.



(Figure 5)

than other situations.[24]

1. New Entrants VS. Incumbent Employees

The prvious discussion has dealt with the case in which new employees are considered as a separate group in the firm. It is, however, obvious that this assumption is not relevant enough in all situations.

For example, let us assume that there is a long-run equilibrium in the firm, so that the corresponding job offer is w_0*, x_0*. Now, as a result of a shift in output demand, the firm tries to accumulate labor at a faster rate. To do so, the firm must increase its wage offer and lower the skill requirement. However, it cannot be assumed that the incumbent employees simply ignore this change in the job offer. If so, for example, simply assume

24) Of course, the fact that, in general, wages are more or less rigid downwards is of great importance in this respect. In this connection, it is interesting to point out that if we apply a model which has both skill requirements and wage offersas variables, we can produce a cyclical path moving in an anti-clockwise direction in a [dw/dt · w-unemployment] plane by using only the behavior of the firm as a point of reference. For the respective analysis, see Pissarides (1976), p. 239.

that in equilibrium the incumbent employees also had the wage rate w_0^*, they would simply quit and then enter the firm as new employees. The opposite does not, however, hold. If the firm were to try to run down its stock of employment, it would (according to the previous analysis) simply lower the wage offer and increase the skill requirement, (so that labor withdrawal would exceed hiring). Of course, the incumbent employees would not demand that their wages should also be cut; instead, they would 'demand' that their existing wage should be preserved.

Thus, the firm cannot increase the wage offer without increasing the wages of the incumbents, whereas it can decrease the wage offer without any corresponding adjustment. The matter is not completely analogous as far as the skill requirement is concerned. Thus, we might assume that the incumbent employees do not react in the same way with regard to a decrease in the skill requirements; that is, they do not demand compensation in the form of an increase in their wages if, on the other hand, the skill requirement (with respect to the new entrants) is increased.

All in all, a decrease in the growth rate of employment, g, and an increase in g do not have symmetrical effects, especially with respect to wages. We analyze this symmetrical response by using a simple model which is based on the following assumptions :

Assumption 1 : All employees (new entrants and incumbents) have the same wage when $w \leq w_0^*$

Assumption 2 : The skill requirement can be freely adjusted with respect to the new entrants only.

First, we derive the locus for the optimal job offers with different values of g. Now we need only introduce the costs which result from the extra payment to the incumbent employees into our cost minimization problem; these costs are equal to $(w-w_0^*)L$, so that the total costs due to hiring $(q+g)L$ employees equal $(q+g)L[w+h(.)+(w-w_0^*)(q+g)^{-1}]$. Thus, instead of (29) we can write :

$$(40)\quad \min_{w,\,x}\ C = w + h(x^*-x) + (w-w_0^*)/(q+g)$$

$$\text{subject to } s(w,x) - qL - gL \geq 0 \text{ and } w > w_{min},\ x \geq 0.$$

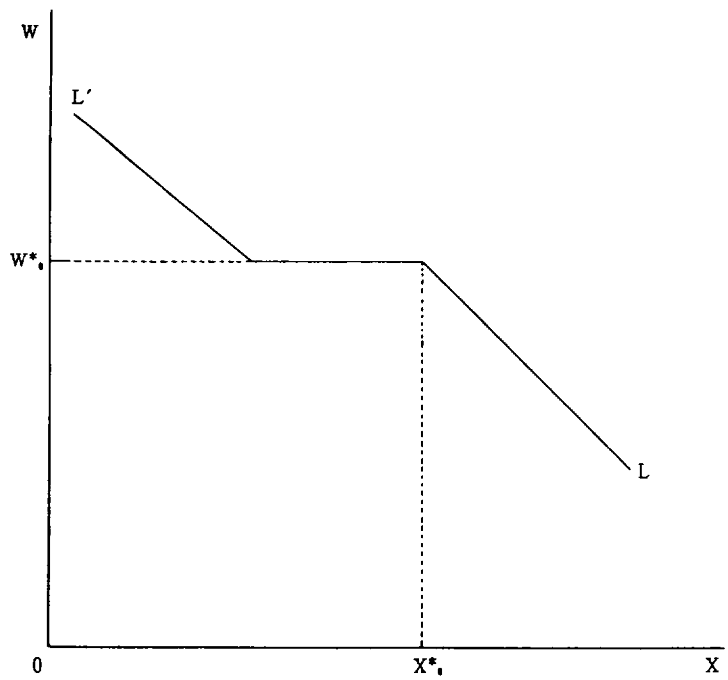
Now, if we consider an interior solution with respect to w and x ($w > w_{min}$ & $x > 0$), we can solve the equation for the LL locus corresponding to (37) :

(41) $s_x + h's_w G = 0$

for $w > w_0^*$, where G denotes $(q+g)/(1+q+g) < 1$. The corresponding slope is :

$$(42)\quad dw/dx = \frac{-s_{xx} + Gh's_w + Gh's_{wx}}{s_{xw} + Gh's_{ww}}$$

It appears that (38) is smaller than (42), that is, (41) is more horizontal than (37).[25]



(Figure 6)

Figure 6 illustrates the LL locus of the optimal job offers. As stated above, the long-run equilibrium is w_0^*. Now, if the growth rate of employment exceeds the corresponding rate, say g_0, the firm lowers only the skill requirement as long as g rises by only a small amount; otherwise, both w and x are adjusted. What is important, however, is that the firm uses the skill requirement more as an instrument for accumulating labor at a faster rate (cf. (42) and (38)).

25) By subtracting (38) from (42), we obtain :
$(1-G)[s_{xx}h's_{ww} - h's_w s_{wx} + s^2 x_{xw} h']/[s_{xw} + h's_{ww}][s_{xw} + Gh's_{ww}]$.
If s_{xx} were positive and very large, a perverse result would again follow.

This result could also be interpreted in terms of (general) training if instead of $h(x^{*}-x)$, we had a cost function of training (for example, so that there would be some constant long run level of siklls per employee). Clearly, our analysis suggests that training might be an appropriate alternative method of acquiring skilled labor for a firm. By means of training, the firm could avoid pressures on its internal wage structure, a possibility which is also noted by Reder (1955), p. 836.[26]

The previous result would not change very much even if there were some premium between the wages of the incumbent employees and those of the new entrants. It is interesting to observe that this simple model could, in this case, generate a countercyclical movement in wage differentials. Thus, if the growth rate of employment decreases, the wages of new entrants fall whereas the wages of incumbent employees remain at the original level. Conversely, if the growth rate of employment increases, both the new entrants' and the incumbents' wages increase, so that the wage premium is not affected. Thus, wage differential between the new entrants and incumbent employees is negatively related to the growth rate of employment.[27]

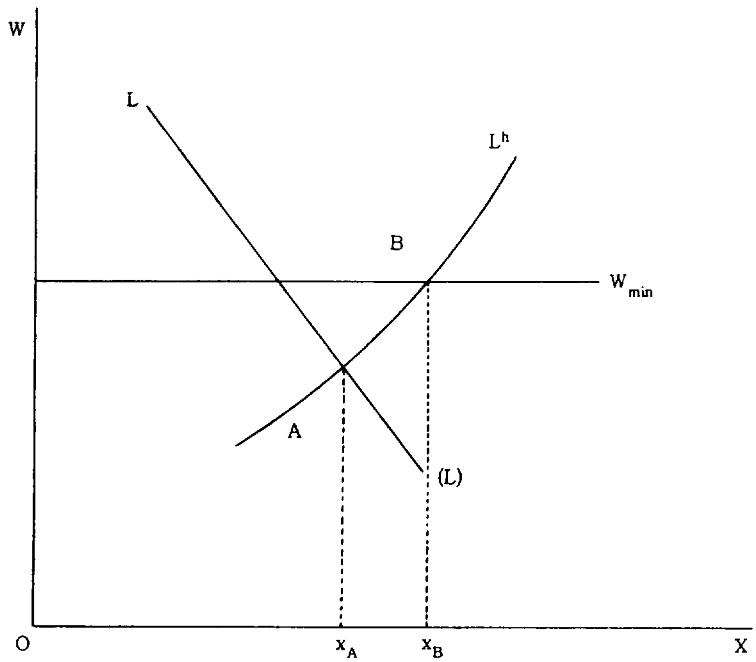
2. The Minimum Wage

Here we study the effects of minimum wages, especially on employment. By using the previous model, (29), as a framework of reference, we can show that an increase in the minimum wage rate, w_{min}, induces an increase in both the wage offers and skill requirements (in the binding case $\partial w/\partial w_{min} = -s_w/s_x$). The effect is illustrated in figure 7.

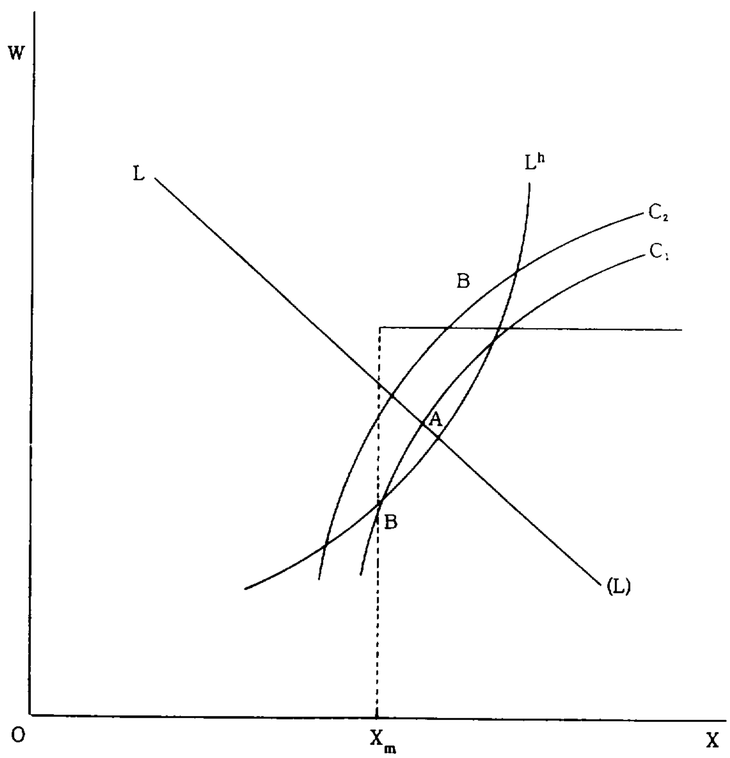
What is particularly interesting in this connection is that the firm reacts to an increase in w_{min} by increasing the skill requirement. This is so because the slope of the $s(w, x)-qL-gL$ locus is always positive. Clearly, those with a skill endowment less than x_B suffer from this increase in w_{min}. It would seem that the public authorities should try to prevent harmful effects of this kind by providing training to those job searchers with small skill

26) It may be worthwhile to quote Reder: he states : "If a firm should need additional workers of a particular grade and cannot obtain them at the going wage rate, it may promote some of its own employees of a lower grade. wage rate, it may orompte some of its own employees of a lower grade. Promoting workers-new or old-has one advantage over bidding up the wage rate: the increased compensation need be paid only to promoted workers. But if a wage increase is granted to one worker, others doing similar work and accustomed to similar rates, must in practice get similar increases", see Reder (1955), p. 836. Cf. also Azariadis (1977) who applies contract theory with this problem.

27) For the relevant empirical evidence, see, for example, Wachter (1974).



(Figure 7)



(Figure 8)

endowments so as to ensure they will not become unqualified when w_{min} is increased. The alternative would be to tie the minimum wage system to workers' skill level so that $w_{min} = w_{min}(x)$. This would, of course, be contrary to the original idea of the whole system.

Figure 8 illustrates such a case. Now it is assumed that the minimum wage covers only those employees with a skill endowment, $x_0 \geq x_m$. Figure 8 shows that in this case a perverse effect is possible; that is, both the firm's wage offer and skill requirement decrease - the firm begins to use "cheap" labor (cheap with respect ot wages only !) instead of more skilled labor at the new minimum wage rate at B.

Minimum wage legislation is generally intended to help the "poor", i. e., the less skilled. Our model shows that minimum wages have more the opposite effect. Figure 5(above), in particular, indicates that in a depression case (i. e., when the growth rate of employment, g, is small), a firm with a binding minimum wage constraint adjusts by increasing or decreeasing only the skill requirement, and this, of course, eliminates the possibilities of an employee with a small endowment finding a job.

V. Conclusion

In this study we analyzed a firm's recruiting process paying special attention to both wage offers ans skill requirements. In section Ⅱ we develop a search model which also includes search costs, unemployment compensation, and a possibility of layoffs. In section Ⅲ we derived the willing applicants constraint facing a firm by referring to this search model, and finally in section Ⅳ we analyzed the determination of a firm's (optimal) job offers by using the analysis of Mortensen (1970) as a point of departure.

The following results are obtained : We show that a firm adjusts both its wage offers, w, and skill requirements, x, to meet its hiring requirements. Furthermore, there is an equilibrium path with respect to w and x over different hiring flows. This relationship explains some structural unemployment phenomena, for example, the high unemployment rate of young employees. If we use this framework as a reference, we can show that minimum wage legislation has harmful effects with regard to unskilled employess.

It is also shown that if a firm cannot discriminate with wages between the new entrants and the incumbent employees, the siill requirement will be use instead of wage offers, especially in the case of economic upturn. it is also worth noting that this model provides an explanation for wage differentials (between firms) and their countercyclical movements.

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〈국문초록〉

일반적 인적자본과 임금격차

고 필 수

인적자본은 기업특수적 인적자본과 일반적 인적자본으로 구분된다는 것을 잘 알려진 사실이다. 전자는 한 기업내에서의 근로자의 행위, 기업의 행위등을 분석할 때 매우 유용한 개념이지만, 한 기업이 근로자를 채용할 때에는 관심의 대상에서 벗어난다. 신규인력의 채용시에는 학력제한, 학력별 모임등 일반적 인적자본의 최저기준(minimum skill requirement)을 설정함으로서 후자의 중요성은 매우 크다.

그러나 일반적 인적자본에 대한 최저기준만을 채용기준으로 삼을 경우 기업은 경기 변동에 따른 적절한 대응을 하지 못함으로써 많은 문제를 야기한다. 이것이 이 논문의 출발점이다. 한 기업은 신규인력을 채용시 임금제의(wage offer)와 최저기준을 동시에 고려함으로써 경직성에서 벗어나 경기변동에 신축적인 대응이 가능하며 기업의 최적직무제의(optimum job offer)를 결정할 수 있기 때문이다. 이를 위하여 Section Ⅱ에서는 탐색비용, 실업보상금 그리고 해고의 가능성을 함께 고려한 탐색모형이 개발된다. Section Ⅲ에서는 위에서 도출된 탐색모형을 이용하여 기업이 직면하는 잠재적 지원자의 제약조건이 도출된다. 그리고 Section Ⅳ에서는 Mortensen의 분석을 출발점으로 이용함으로써 기업의 최적직무제의의 결정을 분석하였다.

위의 분석과정에서 아래와 같은 함의를 도출할 수 있었다.

첫째, 기업은 채용기준을 만족시키기 위하여 임금제의와 인적자본의 최저기준을 모두 변동시킨다. 더욱 더 흥미로운 것은 각각의 채용의 흐름(hiring flow)에 따라 임금제의(w)와 인적자본의 최저기준(x)에 관한 균형경로가 존재한다는 것이다. 이 관계는 약간의 구조적인 실업현상—예를 들면, 젊은 근로자들의 높은 실업율—을 잘 설명해 주고 있다.

둘째, 이 기본적인 틀을 하나의 참고로 사용한다면, 최저임금법은 비숙련근로자들에게 오히려 해로운 효과를 갖는다는 것을 보여준다.

셋째, 기업이 신규채용자와 기존의 근로자들을 임금으로 차별화를 할 수 없다면, 특히 호경기에는 최저기준을 임금제의 대신에 사용할 수 있다는 결론이 도출된다.

넷째, 이 모형 또한 기업들간의 임금격차와 경기변동에 대응하는 움직임에 대한 설명을 제공해 준다.