J. of Basic Sciences, Cheju Nat'l Univ.
20(1), 125~133, 2007

기초과학연구 제주대학교
20(1), 125~133, 2007

Bloch oscillations in the miniband transport in semiconductor superlattices

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A new theory of electron drift velocity in a miniband of a semiconductor superlattice is presented and is compared with the work of Esaki and Tsu [1]. Our results are in good agreement with the experimental valuse of Artaki *et al* [4].

PACS numbers: 73.21.Cd, 72.20.Ht, 72.10.Bg, 73.40.-c

Keywords: Bloch Oscillation; Negative Differential Conductivity; Semiconductor superlattice

Esaki and Tsu (ET) [1] proposed that two different regimes can be identified when an electric field E is applied to a semiconductor superlattice. At low fields, the current increases linearly with field. In the high field regime, where the electric field exceeds some critical value E_c , the current is expect to decrease with increasing field, due to the electron reflection at the minizone boundary (Bloch oscillation). Thus, ET [1] concluded that electrons in a miniband of a semiconductor superlattice, accelerated by an electric field perpendicular to the superlattice layers can have negative differential conductivity (NDC) versus electric field behavior. Recently, these predictions were clearly demonstrated experimentally [2, 3] in semiconductor superlattices under the condition of wide minibands. Nevertheless, the experimentally deduced peak velocity v_d and the threshold electric field E_c (at which v_d peaks) as functions of miniband width Δ, differ significantly from the predictions of that of the ET theory [2]. Also, sophisticated Monte Carlo [4] computations for a GaAs/GaAlAs superlattice yield a peak velocity smaller than that of the ET result by a factor of 3 to 6. A detailed theory, which is based on a nonlinear balance-equation approach has been developed by Lei *et al* [5]. Their theory is in remarkably good agreement with experimental data, and it uses impurity scattering rate as an adjustable parameter to perform numerical calculations. However, since the ET theory is very powerful in dealing with the basic physics of electronic transport in semiconductor superlattice and since it is still in a rather simplified form, it is of great interest to develop it into a more sophisticated form. In this paper we put the ET theory into a new framework and we show that the extended theory not only provides a concrete picture for the electronic transport in semiconductor superlattices, but also it agrees with the experimental data very well.

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According to ET [1], the energy band of a superlattice with period d can be approximated by a sinusoidal form

$$\varepsilon_k = \frac{\Delta}{2}\ (1 - \cos kd) \tag{1}$$

where k is the wavevector along the superlattice axis, and Δ is the miniband width. In the presence of a static field E and with the energy and momentum relaxation being neglected, the equations of motion are

$$v = \frac{1}{\hbar}\frac{\partial \varepsilon_k}{\partial k} = \frac{\Delta d}{2\hbar}\ \sin kd \text{ and } \hbar\ \dot{k} = eE \tag{2}$$

where v is the electronic velocity. The above equations yield a real space localized oscillatory motion, the Bloch oscillation of electrons with velocity

$$v(t) = v_0 \sin \omega_B t, \tag{3}$$

where $\omega_B = eEd/\hbar$ is the Bloch frequency due to Bragg diffraction, and

$$v_0 = \frac{\Delta d}{2\hbar} = \frac{\hbar k_d}{m(0)}\ \text{, and } m(0) = \frac{2\hbar^2}{\Delta d^2}. \tag{4}$$

Taking account of the effects of relaxation, ET used Pippard's impulse method [6] to obtain the drift velocity as [1]

$$v_d = v_0 \frac{\pi\xi}{1 + \pi^2\xi^2}\ , \tag{5}$$

where

$$\xi = \frac{\omega_B \tau}{\pi} = \frac{eEd\tau}{\hbar\pi} \tag{6}$$

and τ is the relaxation time. From Eq. (5) it is clear that v_d has a peak value, v_p , equal to $v_0/2$ at $\omega_B\tau = 1$. In other words, in the ET theory, the critical electric field is $E = \hbar/e\tau d$ and the peak drift velocity $v_p = v_0/2$. In addition, one can easily see from Eq. (5) that ET conductivity σ_0 is $ne^2\tau/m(0)$ in the low field limit and $\sigma_0/\omega_B^2\tau^2$ in the high field limit, respectively.

The ET theory is elegant and powerful but it is clear that a modification is necessary because it does not totally agree with experimental results. In our opinion, the crux of the matter is that the direct application of the Pippard method [6] by ET to the calculation of electron conduction in the minibands of superlattices may not be totally valid. This is because of the fact that while the start point of the Pippard impulse method is a free electron, an electron in the minibands of a superlattice is best described by a sinusoidal miniband and thus is not free. As a result, one would expect that the ET theory has a tendency to overestimate the conductive behavior of the electrons in the minibands of the superlattice. This is really the case when one compares theory with experiment, where one finds that the experimental values of v_d and E_c are much smaller than that of the corresponding values given by the ET theory. Based on the above observation, our plan to improve the ET theory is to discard the use of the Pippard's impulse method and, instead, incorporate the sinusoidal miniband nature of the electrons.

We start from the equation of motion for an electron in a sinusoidal miniband in the relaxation time approximation [6–8]

$$\frac{dv}{dt} + \frac{v}{\tau} = v_0 \omega_B \cos kd \tag{7}$$

where τ is the relaxation time, v_0 and ω_B are given by Eq. (3). It is clear that in the no dissipaion limit ($1/\tau \to 0$), Eq. (7) is consistent with Eq. (2) and Eq. (4).

Eq. (7) is a first order linear differential equation, the exact solution of which can be readily obtained as

$$v(t) = v_0 \cos\theta \left\{ \sin(\theta + \omega_B t) - \sin\theta e^{-t/\tau} \right\}, \tag{8}$$

where we have taken $v(0) = 0$, and

$$\sin\theta = \left(1 + \omega_B^2 \tau^2\right)^{-1/2}. \tag{9}$$

We note that in the free electron limit ($d \to 0$), it is easy to see that Eq. (8) reduces to

$$v(t) = \frac{eE\tau}{m^*}\left(1 - e^{-t/\tau}\right), \tag{10}$$

where we have used $m(0) \to m^*$, and m^* is the effective mass of the electron in the bulk semiconductor. Also, in the long time limit ($t/\tau >> 1$), the last term in the bracket of Eq. (8) vanishes and Eq. (8) reduces to

$$v(t) = v_0 \cos\theta \sin(\theta + \omega_B t), \tag{11}$$

which is a pure periodic motion. Another important case of Eq. (8) is the weak dissipation limit where $t/\tau \to 0$. In this case, Eq. (7) is consistent with Eq. (2) and Eq. (4), and the velocity $v(t)$ in Eq. (7) takes the Bloch oscillation form Eq. (3). This implies that in addition to the well known fact that Eq. (7) is a good approximation in the long time limit, it is also a good first approximation in the weak dissipation limit. Finally, the time average of velocity $v(t)$ of Eq. (8) is an useful quantity, and for later discussion, we present here as

$$\bar{v}(t) = \frac{1}{t}\int_0^t v(t')dt' = v_0 \frac{\tau}{t} \frac{\omega_B \tau}{1 + \omega_B^2 \tau^2} \left[e^{-t/\tau} - \cos\omega_B t + \frac{1}{\omega_B \tau} \sin\omega_B t \right]. \tag{12}$$

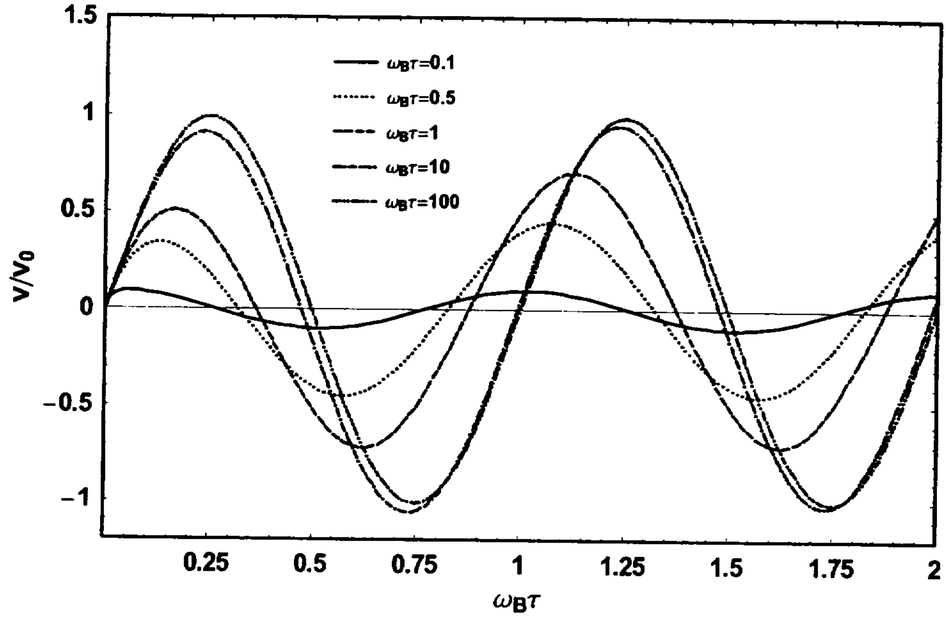

FIG. 1: Velocity v(t) (in units of $v_0 = \Delta d/2\hbar$) of an electron in a sinusoidal miniband of a superlattice with period d and external field E, as a function of time t (in units of $2\pi/\omega_B$), as calculated by (8), for variety values of $\omega_B\tau$= 0.5, 1, 5, 10, 100, where $\omega_B = eEd/\hbar$ is the Bloch frequency, τ is the relaxation time, and Δ is the miniband width.

Equation (8) is a key results and it will form the basis of our further analysis. In Fig.1, we plot $v(t)$ (in units of v_0) as a function of t (in units of $2\pi/\omega_B$) for different values of $\omega_B\tau = 0.1, 0.5, 1.0, 10, 100$. As can be seen from the figure, in the presence of a static electric field an electron in a sinusoidal band with dissipation undergoes oscillating motion. When $\omega_B\tau >> 1$, (and from Eq. (9) one has $\theta \rightarrow 0$), one has the high field (weak dissipation and strong field) limit , Eq. (8) reduces to Eq. (3), and the periods of the oscillation approach close values to $2\pi/\omega_B$. As dissipation becomes stronger ($\omega_B\tau$ decreases), two main changes occur. First, the oscillation period becomes much smaller than $2\pi/\omega_B$ when $\omega_B\tau < 1$ (for example, at $\omega_B\tau = 0.5$, the first oscillation period is about $3\pi/2\omega_B$). The other change is that the two sector within an oscillation period become more uneven, i.e., an electron spends more time in one direction than the other. As a result, with a static electric field the superlattice will show a net flow of electrons (the current). Thus, Eq. (8) provides an concrete picture for the electronic motion in a superlattice with static electric field. It tells us that an electron in a superlattice tends to undergo an oscillating motion. In the high field limit ($\omega_B\tau >> 1$), the oscillation is more or less periodic, while in the low field limit ($\omega_B\tau << 1$), the oscillation becomes non-periodic. Therefore, it is necessary to analyze Eq. (8) and Eq. (12) in the two limits, the high field limit and the low field limit, separately..

Since in the high field limit, one expects an electron undergoes Bloch oscillation in the superlattice, it is reasonable to take the first period of the electronic oscillation as identified by Eq. (8) as the period of that Bloch oscillation. Based on this assumption, we now proceed to calculate the actual value of the period T for the electronic Bloch oscillation in the high field limit. This can be readily done by applying the condition $v(T) = v(0)$ to Eq. (8), which results

$$\omega_B\tau = \frac{e^{-T/\tau} - \cos\omega_B T}{\sin\omega_B T}. \tag{13}$$

Eq. (13) shows that T can take only some limited values. This is further illustrated in Fig. 2, where we use Eq. (13) to plot T (in units of $1/\omega_B$) as a function of τ (in units of $1/\omega_B$). As can be seen from the figure, in the extreme high field limit ($\omega_B\tau \to \infty$), $\omega_B T$ tends to 2π. In fact, from Eq. (13) after some straightforward algebra, one can derive the following analytical expression

$$\omega_B T = 2\pi\left(1 - \frac{1}{\omega_B^2\tau^2}\right) \text{ for } \omega_B\tau >> 1 \text{ and } \omega_B T = \frac{3\pi}{2} + \omega_B\tau \text{ for } \omega_B\tau << 1. \tag{14}$$

Adopting the physical picture that the Bloch electron in superlattices in high field limit undergoes periodic motion with a period defined by Eq. (13), the electronic drift velocity v_d can be evaluated as an average quantity over the time interval T. Thus, we obtain from Eq. (12)

$$v_d = \bar{v}(T) = \frac{1}{T}\int_0^T v(t')dt' = v_0\frac{\tau}{T}\sin\omega_B T, \text{ for } \omega_B\tau > 1 \tag{15}$$

where T is determined by Eq. (13).

Eq. (15), supplemented by Eq. (13) gives the drift velocity (and the current) at the high field limit and it should be a good approximation as long as $\omega_B\tau \geq 1$ and system undergoes Bloch oscillations. Some comments are in order: (i) When $T = 2\pi n/\omega_B$, there is no drift current, and this will happen only in the $\omega_B\tau \to \infty$. (ii) In the extrem high field limit, we substitute Eq. (13) into Eq. (15), and our result Eq. (15) becomes the same as the ET formula *Eq.*(5), $v_d = v_0/\omega_B\tau$. It follows that the conductivity in this limit is $\sigma_0/\omega_B^2\tau^2$.

Next, we study the low field case of Eq. (12). In the low field limit ($\omega_B \to 0$), electrons in superlattice tend to perform non-periodic motion, and it is necessary to have $t/\tau >> 1$ in Eq. (12) . In this case, Eq. (12) reduces to

$$v(t) \approx v_0\frac{\omega_B\tau}{1+\omega_B^2\tau^2}\left[1 - \frac{\tau}{t}\right], \text{ for } \omega_B\tau < 1. \tag{16}$$

Eq. (16) is another interesting result. In the zero field limit, one expects that the drift velocity is the long time ($t \to \infty$) quantity of Eq. (16), $v_d = v_0\omega_B\tau$. For low but finite electric field, electrons

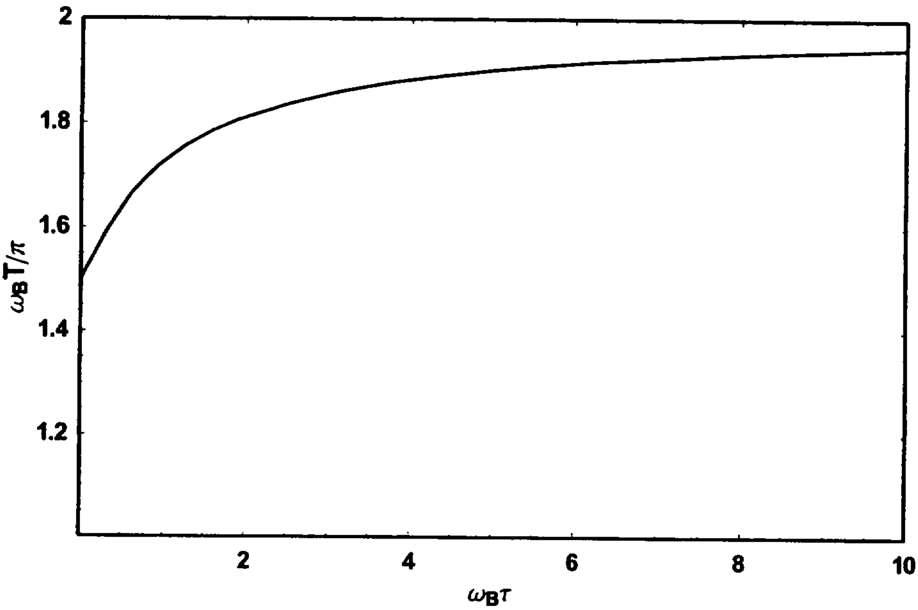

FIG. 2: The period T (in units of π/ω_B) of the electronic motion in a sinusoidal miniband of a superlattice with period d and external field E, as a function of the relaxation time (in units of $1/\omega_B$), as calculated by (12), where $\omega_B = eEd/\hbar$ is the Bloch frequency.

in the miniband of superlattice, may start to have some periodic motion, which reduces the overall value of the drift velocity. The second term in the bracket of Eq. (16) representing such a reduction proportional to $\omega_B^2\tau^2$ (since $t \sim 1/\omega_B$). After incorporating this fact, we propose an expression for the drift velocity in the low field limit as

$$v_d \simeq v_0 \frac{\omega_B \tau}{1+\omega_B^2\tau^2}\left[1-\alpha\omega_B\tau\right], \text{ for } \omega_B\tau < 1 \tag{17}$$

where is α some constant, which will be chosen to make the consistent connection to the high field form Eq. (15).

We are now in a position to present the whole picture of the electronic drift velocity in the superlattice. In Fig. 3 we plot v_d (in units of πv_0) as a function of $\omega_B\tau/\pi$ by using Eq. (15) and Eq. (13) for the high field region and Eq. (17) for the low field region, where we take $\alpha = 5$ to make the smooth transition between the two regions. We note that the value of α does not affect the peak value of v_d. For comparison, we have also plotted the ET formula Eq. (5) (dotted line). As can be seen from the figure, while the qualitative feature between our theory and the ET theory are very similar, the v_d in our theory is nevertheless, in general, much smaller than that of the ET

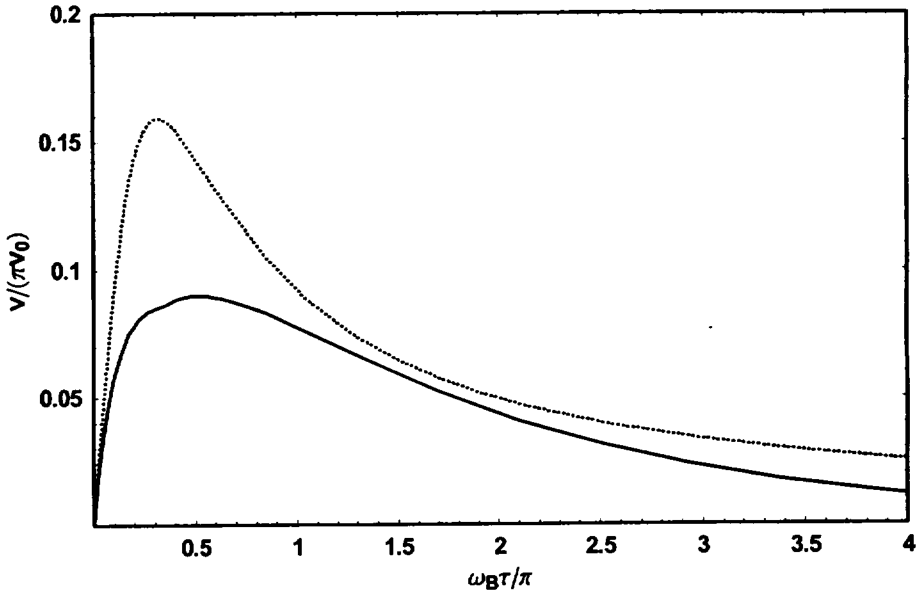

FIG. 3: Drift velocity v_d (in units of πv_0) in a superlattice with period d and external field E, as a function of ω_B (in units of π/τ). Full curve is calculated using (15) and (17). Dotted line is due to Esaki-Tsu formula (5). Here $\omega_B = eEd/\hbar$ is the Bloch frequency, τ is the relaxation time, $v_0 = \Delta d/2\hbar$, and Δ is the miniband width.

result Eq. (5). This is reasonable since we have argued earlier that the ET theory has a trend to overestimate the drift velocity. Only in the high and low field limits, our results approach the ET theory. Also, the figure shows the quantitative differences: v_d peaks at $\omega_B\tau = 2$ with $v_p = 0.15v_0$ in our theory, while in ET theory $v_p = v_0/2$ at $\omega_B\tau = 1$.

Experimentally, Sibille *et al.* obtained recently, the evidence of miniband negative differential velocity in GaAs/AlAs superlattices. By performing measurements and analysis over a variety of small period superlattice samples, they were able to extract the peak drift velocity v_p and the corresponding E_c as functions of miniband width Δ. Since these quantities are readily calculable by our theory, it is interesting to compare our theory directly with that of the experiments. This is done in Fig. 4, where we plot the dependence on for the peak velocity v_p divided by the superlattice period d. In that figure, dark dots are the experimental results from Ref.2, dotted lines are predictions from the ET theory, and full curves are our theoretical results. From Fig. 3, $v_p = 0.15v_0$ (this value can be determined directly by using Eq. (13) to find out where v_d, given

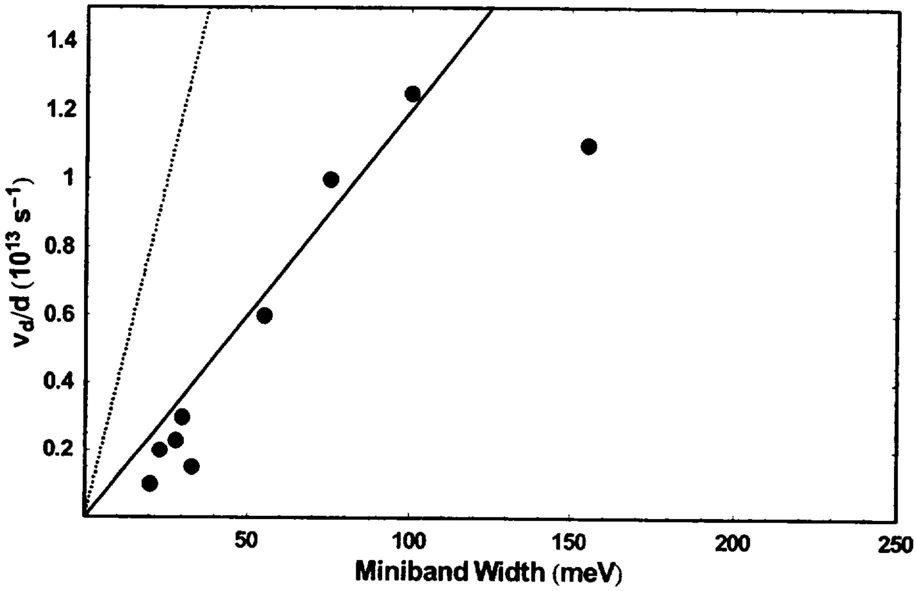

FIG. 4: Peak velocity v_p divided by the superlattice period d (in units of $10^{13}\,s^{-1}$) as a function of the minband width Δ (in units of (meV). dark dots are the experimental results of Ref. 4, dotted lines are the pridictions from the Esaki-Tsu fornula (5), and full curves are our theoretical results as calculated from (15), (13) and (4).

by Eq. (15), reaches its peak value v_p. Using this result in the definition Eq. (4) for v_0, leads to

$$v_p/d = 0.075\Delta/\hbar. \tag{18}$$

We note that the full line in Fig. 4 exactly repeats the suggested guide line for the experimental data in Fig. 3 of Ref. 2. Also, it is interesting to note that Eq. (18) presents a same qualitative feature, i.e., $v_p/d \backsim \Delta/\hbar$, as that of the ET theory, except that it reduces the numerical factor from 0.25 in the ET theory to the number of 0.075, which agrees with experiments.

In summary, in this paper we have presented an analytic dynamic analysis of the electron motion in semiconductor superlattices in an external electric field. By incorporating the sinusoidal miniband nature of the electrons, we obtain the solution Eq. (8) for the velocity $v(t)$, which is applicable in both of the high and low field limit. By analyzing Eq. (8) in detail, it is found that an electron in a superlattice with electric field undergoes oscillating motion. In the high field limit, the oscillation is more or less periodic, while in the low field limit, the oscillation becomes non-periodic. Based on these physical properties, we derive analytical expressions for the electron drift velocity, Eq. (15) (supplemented by Eq. (13)) in high field region and Eq. (17) in the low field region. Our results show that while the qualitative feature between our theory and the ET theory are very similar, the v_d in our theory is nevertheless, in general, much smaller than that of the ET result except in the extreme high and low ends of the field where the two theory agree. Also, we find that the quantitative differences: v_p peaks at $\omega_B\tau = 2$ with $v_p = 0.15v_0$, while in ET theory $v_p = v_0/2$ at $\omega_B\tau = 1$. Our results are shown to agree with the experiments of Ref. 4.

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