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Design of Multistage Optimal Controller for Nonlinear Systems Based on Multilayer Neural Networks

Byeong-woo Bae* and Kyung-youun Kim**

ABSTRACT

In this paper we describe a method to solve the optimal control problem of nonlinear systems using neural networks which is called here as a two-level multilayer neural network (TLMNN). The TLMNN has internal networks which consist of an upper-level neural network (UNN) for modeling and a lower-level neural network (LNN) for control. The weights of each network is optimized by introducing an augmented function so that a performance measure is minimized. Some examples were used to illustrate the characteristics of the proposed algorithm.

1. Introduction

There exist many methods, including classical frequency-domain techiques, for designing control laws for linear systems. In contrast, there exist relatively few methods for nonlinear systems. Also, when optimal control is employed, determination of the optimal control law requires solution of a partial differential equation in x and κ (where x is the state and κ is the time); unfortunately, when the order of the system is large the procedure to obtain optimal control law becomes a tedious and time comsuming task. Futhermore, optimization techniques generally consist of a mathematical procedure and for this procedure it is necessary to have a mathematical representation of a system. When there exist system uncertainties and system variations by environments around, the given mathematical model is not efficient for a fine control performance. For that reason, the modeling task is the first important step before achieving a control law

* : Department of Electronic Engineering, Kyungpook National University
** : Department of Electronic Engineering, Cheju National University

since the results of the optimization will be critically dependent upon the validity of the model.

Our study is motivated from appearance of neural networks, specifically multilayer neural network(MLNN) based on back-propagation learning algorithm,[1] which recently have studied by control community.[2-5] The MLNN has the potential to solve many nonlinear control problems that cannot be answered by conventional analytic approaches owing to its capability to learn system characteristics through nonlinear mapping.

Iiguni et al.[2] designed a controller that integrates an classical optimal regulator and backpropagation neural networks to handle system uncertainties. Kawato et al.[3] proposed a control method using both a feedback error scheme and a inverse dynamic model using neural network : as learning proceeded using output of feedback error scheme, the inverse dynamic model gradually took the place of the external feedback as a main controller which may cope with system uncertainties. However these control systems have a disadvantage that require a basic system model. Nguyen et al.[4] presented a self-learning control system which is based on a conventional multistage system and uses a neural emulator trained as system model.

In this paper, a neural network-based multistage control system is developed that we employ formal tools from classical optimal control theory[6-8] to determine optimal weights for a neural network whose task is defined as the optimization of a specific performance index. First of all, the strategy of our algorithm is to tune parameters of a controller and to obtain a control law by feedforward computing. Secondly, the method for modeling system dynamics is introduced to obtain a robust control performance for system uncertainty. This is very important problem in the control of the real process since the optimal control law can be obtained for the given system dynamics.

2. Optimization Problem of Nonlinear Control Systems

2.1 Overview of multistage systems

Consider a problem of minimizing a performance measure of the general form

$$J = h[x(k_f)] + \sum_{k=0}^{k_f-1} g^k [x(k), u(k)] \tag{1}$$

subject to a sequential set of equality constraints

$$x(k+1) = \phi^k [x(k), u(k)], \quad x(0) : \text{given}, \quad k = 0, 1, \cdots, k_f-1 \tag{2}$$

where $x(k) \in R^p$ and $u(k) \in R^m$ are the state of the discrete-time nonlinear dynamical system and the control input, respectively. It is assumed that the final time k_f is fixed and the final state $x(k_f)$ is free. Essentially, the problem is to find the control law u(k) that minimizes the performance measure J.

Now if we include (2) by introducing the Lagrange multiplier p(k), the performance measure can be augmented by

$$L = h[x(k_f)] + \sum_{k=0}^{k_f-1} \{g^k[x(k), u(k)] + p^T(k+1) [\phi^k[x(k), u(k)] - x(k+1)]\}$$
$$= h[x(k_f)] - p^T(k_f) + \sum_{k=1}^{k_f-1} [H^k - p^T(k) x(k)] + H^0 \qquad (3)$$

where the function H^k called the Hamiltonian is defined as follows :

$$H^k = g^k[x(k), u(k)] + P^T(k+1) \phi^k[x(k), u(k)] \qquad (4)$$

Here, it is assumed that the dynamics of the system (2) is to be unceretain or unknown. Hence the conventional optimal control algorithm can not cope with the problem stated above. Now we describe the TLMNN to obtain optimal control law for the given uncertain or unknown system.

2.2 Optimization problem based on the TLMNN structure

In this section we present a new control algorithm which embeds parameter optimization and derivation of the optimal control law based on the same sort of a basic idea as the previous a multistage control system.

First of all, we consider an architecture of the optimal control system using the MNN's which in this paper is called a two-level MNN (TLMNN). TLMNN is composed of an upper-level MNN (UNN) and a lower-level MNN (LNN). Fig. 1 shows the architecture of the proposed TLMNN.

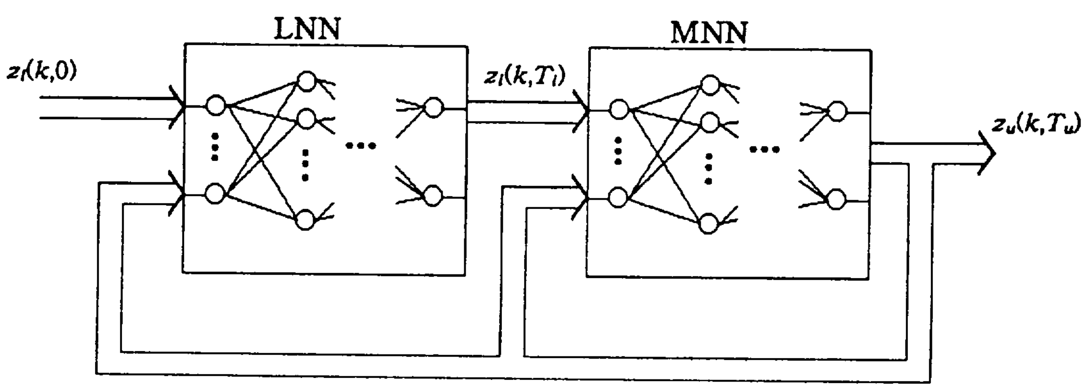

Fig. 1 Architecture of the TLMNN.

The above figure shows a kth stage of a multistage system using the TLMNN's, which has the same structure as Fig. 1. As shown in Fig. 2, the dynamic equation of the UNN can be written as follows :

$$z_u(k,t+1)=f_u[W_u(k,t),z_u(k,t)] \quad \forall\ t\in[0,\ T_u-1] \tag{5}$$

where the subscript u denotes upper-level and $f_u(\cdot)$ is an activation function which is of a linear function type at last layer and a sigmoid function type otherwise. And $z_u(k,t)$, $t=1,\cdots,T_u$, represents the node output of the t-th layer of the UNN at kth stage and $W_u(k,t)$ is weight matrix between the t-th and (t+1)-th layers. When $t=T_u-1$, the equation (5) corresponds to the output of the last layer of the UNN, $x(k+1)$, which must be desired to be the state, (2). Also, at t=0, $z_u(k,t)$ is the input of the UNN, which is composed of the output of the LNN and the external input, i.e., the delayed state. Often the LNN is called a feedforward controller the output of which is obtained by feedforward computing not by feedback and is used as the control input.

The dynamic equation of the LNN is

$$z_\ell(k,t+1)=f_\ell[W_\ell(k,t),z_\ell(k,t)] \quad \forall\ t\in[0,\ T_\ell-1] \tag{6}$$

where the subscript ℓ represents lower-level, $f_\ell(\cdot)$ equals with $f_u(\cdot)$, and $W_u(k,t)$ is weight matrix between the t-th and (t+1)-th layers. At the last layer of the LNN the node output (6) corresponds to the control input at kth stage and at t=0 is the external input, i.e., the desired state and the delayed state.

Consider the performance measure defined as the following form

$$J=\frac{1}{2}\|x(k_f)-x_d\|_H^2+\frac{1}{2}\sum_{k=0}^{k_f-1}\{\|x(k)-x_d\|_Q^2+\|u(k)-u_d\|_R^2\} \tag{7}$$

where k_f denotes a final stage, and H, Q and R are real symmetric positive semi-definite penalty-weighting matrices.

The Lagrangian is

$$\begin{aligned}L=J+\sum_{k=0}^{k_f-1}\{&\sum_{t=0}^{T_u-1}\gamma^T(k,t+1)[z_u(k,t+1)-f_u(W_u(k,t),z_u(k,t))]\\&+\sum_{t=0}^{T_\ell-1}\beta^T(k,t+1)[z_\ell(k,t+1)-f_\ell(W_\ell(k,t),z_\ell(k,t)]\\&+p^T(k+1)[F(x,u,W_u)-x(k+1)]\end{aligned} \tag{8}$$

where γ and β are referred to as the Lagrange multipliers of the t-th layer, which are associated with the UNN and the LNN, respectively, and p is the costate vector.

Assumed that the penalty-weighting matrix of the last stage, H, equals to Q, let decompose (7) into the additive quadratic form by projection theorem,[8] that is,

$$J=J_u+J_\ell \tag{9}$$

where

$$J_u=\frac{1}{2}\|x(k_f)-\hat{x}(k_f)\|_Q^2$$

$$+\frac{1}{2}\sum_{k=0}^{k_f-1}\{\| x(k)-\hat{x}(k)\|_Q^2 \qquad (10)$$

$$J_l=\frac{1}{2}\| x(k_f)-x_d\|_Q^2+\frac{1}{2}\sum_{k=0}^{k_f-1}\{\| x(k)-x_d\|_Q^2 + \| u(k)-u_d\|_R^2 \qquad (11)$$

When the first term of the right-hand side of (8) is substituted into (10), the optimization problem is to estimate the dynamic characteristics of the system by minimizing the decomposed performance measure (10), which corresponds to changing the weights of the UNN. Accordingly, the solution is split into three necessary conditions :

$$\frac{\partial L}{\partial \gamma}=0\ ;\quad \frac{\partial L}{\partial z_u}=0\ ;\quad \frac{\partial L}{\partial \omega_{upq}}=0 \qquad (12)$$

where ω_{upq} is a weight between the pth node of the t-th layer and the qth node of the (t+1)-th layer of the UNN.

The first condition of (12) produces the dynamic equation of each layer, which is identical with the forward dynamic equation (5). While satisfying (5) and holding $W_u(k, t)$ constant elements, from the second condition, the Lagrange multiplier $\gamma(k, t)$ is obtained as follows :

$$\gamma(k,t)=\left(\frac{\partial f_u[W_u(k,t),z_u(k,t)]}{\partial z_u}\right)^T \gamma((k,t+1) \quad \forall\ t\in[0,\ T_u-1] \qquad (13)$$

with boundary condition

$$\gamma(k,T_u)=-Q_t[x(k+1)-\hat{x}(k+1)] \qquad (14)$$

which is the backward equation of t-th layer of the UNN.

Now consider differential changes in (8) due to differential changes in the weights of the UNN, which for extremum must be zero. When using steepest descent technique, the third condition of (12) is written as follows :

$$\begin{aligned}\Delta\omega_{upq}(k,t) &=\sum_{k=0}^{k}\eta_{uk}\left(\frac{\partial f_u[W_u(k,t),z_u(k,t)]}{\partial \omega_{upq}}\right)^T \gamma(k,t+1)\\ &=\eta_{uk}\left(\frac{\partial f_u[W_u(k,t),z_u(k,t)]}{\partial \omega_{upq}}\right)^T \gamma(k,t+1)\\ &+\sum_{k=0}^{k-1}\eta_{uk}\left(\frac{\partial f_u[W_u(k,t),z_u(k,t)]}{\partial \omega_{upq}}\right)^T \gamma(k,t+1)\\ &=\eta_{uk}\left(\frac{\partial f_u[W_u(k,t),z_u(k,t)]}{\partial \omega_{upq}}\right)^T \gamma(k,t+1)\\ &+\eta_{u(k-1)}\Delta\omega_{upq}(k-1,t)\end{aligned} \qquad (15)$$

where η_{uk} is learning rate which determines learning speed at kth stage and $\eta_{(k-1)}$ is momentum constant. In the right-hand side of the above equation (15) the first term is the weight change term in the kth stage and the second term is the momentum term which equals to weight change of a previous stage.

When the first term of the right-hand side of (8) substitutes into (11), the optimization problem corresponds to finding the optimum weights of the LNN so that (11) is minimized.

Necessary conditions for a minimum of (11) are : a sequential set of equality constraints of each layer in the LNN is given as (6), and the backward equation is

$$\beta(k,t)=\left(\frac{\partial f_\ell[W_\ell(k,t),z_\ell(k,t)]}{\partial z_\ell}\right)^T \beta(k,t+1) \quad \forall\ t\in[0,\ T_u-1] \tag{16}$$

and the weight change is

$$\Delta\omega_{\ell pq}(k,t)=\eta_\ell\left(\frac{\partial f_\ell[W_\ell(k,t),z_\ell(k,t)]}{\partial \omega_{\ell pq}}\right)^T \beta(k,t+1)+\Delta\omega_{\ell pq}(k+1,t) \tag{17}$$

When $t=T_\ell$ at kth stage, the boundary condition of the second subcondition (16) is

$$\frac{\partial L}{\partial z_\ell(k,T_\ell}=\frac{\partial L}{\partial u(k,T_\ell}=0 \tag{18}$$

$$\Longrightarrow \beta(k,T_\ell)=R[u(k)-u_d]+\left(\frac{\partial F[x,u,W]}{\partial u}\right)^T p(k\text{-}1) \tag{19}$$

where $p(k+1)$ is costate vector.

Up to now, we derived the equations for parameter optimization of the TLMNN. Now we consider the control problem of the TLMNN, which corresponds to obtaining the boundary condition of the backward dynamic equation (19). For convenience we define a scalar sequence H^k :

$$\begin{aligned}H^k=&\frac{1}{2}\{\|x(k)-x_d\|_Q^2+\|u(k)-u_d\|_R^2\}\\&+r^T(k,T_U)F_u[x(k),u(k),W_u(k,t)]\\&+\beta^T(k,T_L)F_\ell[x(k),x_d(k),W_\ell(k,t)]\\&+p^T(k+1)F_u[x(k),u(k),W_u(k,t)]\end{aligned} \tag{20}$$

A stationary value of H^k with respect to $\beta(k,T_L)$ will occur, where we have the control law

$$\begin{aligned}u(k)=&\frac{\partial H^k}{\partial\beta(k,T_L)}\\=&F_\ell[x(k),x_d(k+2),W_\ell(k,t)]\end{aligned} \tag{21}$$

Hence we obtain

$$\begin{aligned}x(k+1)=&\frac{\partial H^k}{\partial p(k+1)}\\=&F_u[x(k),u(x),W_u(k)]\end{aligned} \tag{22}$$

$$\begin{aligned}p(k)=&\frac{\partial H^k}{\partial x(k)}=Q[x(k)-x_d]\\&+\frac{\partial F_\ell[x(k),x_d(k)]}{\partial x(k)}\beta(k,T_L)\\&+\frac{\partial F_u[x(k),x(k)]}{\partial x(k)}p(k+1)\end{aligned} \tag{23}$$

with boundary condition

$$p(k_f)=Q[x(k_f)-x_d] \tag{24}$$

and $x(0)$ is given.

In summary, using equations (22) through (24), the optimal control input is obtained from the forward dynamic equation (21) with the states and the weights of the LNN : the weights of the UNN in (22) are changed by (15) and of the LNN in (21) are changed by (17) : the weight changes require the backward dynamic equation; (13) and (16) for parameter optimization and (23) for optimal control. Up to this point we derived the algorithm which embeds the weight changes of the TLMNN and the development of the controller for multistage optimal control, which is summerized as follows :

Step 1 : The sequence $\{[x_0(n),x_d(n)],$ n

$=0, 1, 2, \cdots\}$ are generated by randomly selecting x_0 and x_d on a region of interest. The values of the weights are randomly generated with very small values.

Step 2 : Obtain the state (2) and the output of the UNN of each stage. At each stage update the weights of the UNN using the weight change equation (15) and the backward equation (13).

Step 3 : After obtainig the boudary conditions of all the stages from the costate (23), at each stage change the weights of the LNN using (16) and (17).

Step 4 : Repeat this procedure step 2 to step 3 until the variation of the Lagrangian with respect to the weights,

$$\Delta L=\left(\frac{\partial L}{\partial W_\ell(k,t)}\right)^T\left(\frac{\partial L}{\partial W_\ell(k,t)}\right)$$

is smaller than some pre-chosen small number depending on the accuracy of the control performance required. This procedure can be commonly applied to off-line and on-line operation.

3. Computer simulations

Example 1 :

The problem is to minimize

$$J=\frac{1}{2}\sum_{k=0}^{k_f}\{\|x(k)-x_d\|_Q^2+\|u(k)\|_R^2\}$$

subject to

$$x_1(k+1)=0.9x_1(k)+0.1x_2(k)+0.1u(k)$$
$$x_2(k+1)=0.2x_1(k)+0.1x_2(k)-0.1x^2_2(k)+0.1u(k)$$

where

$$Q_{11}=Q_{22}=0.05,\quad R_{11}=0.1,\quad R_{22}=0.05$$

with

$$x_1(0)=10.0,\quad x_2(0)=4.5$$

The only nonlinearity arises in the term $-0.1x^2_2$ in the equation for x_2.

Figs. 2 through 3 show the optimal trajectories of x_1, x_2, u_1, u_2, where the square-marked responses are trajectories for the given system and the triangle-marked responses are for the system in the presence of the uncertain nonlinear term $0.1x_1(k)x_2(k)$ for each state.

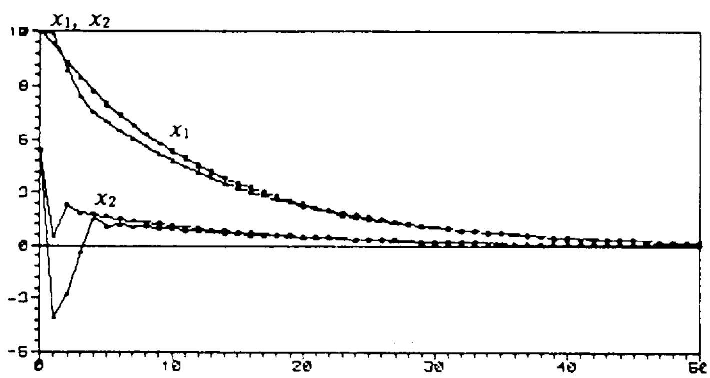

Fig. 2 Optimal states (square-marked) and under system uncertainty (triangle-marked)

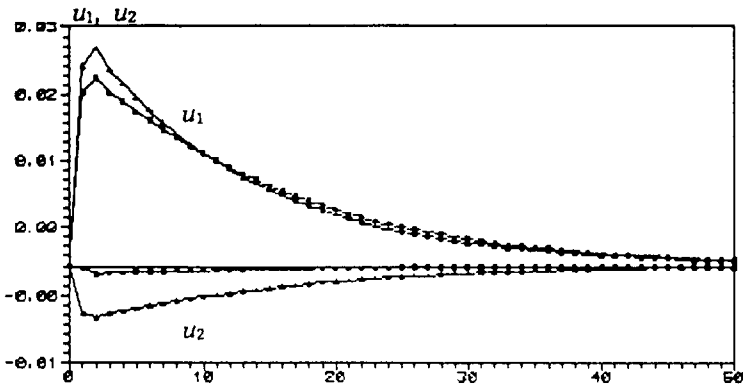

Fig. 3 Optimal controls (square-marked) and under system uncertainty (triangle-marked)

Example 2 :

We examine the power system which is described as a sixth order non-linear dynamical model.[7]

The system equations can be written as

$x_1(k+1) = x_1(k) + Tx_2(k)$

$x_2(k+1) = (1-TC_1)x_2 - TC_2\sin(x_1(k))x_3$

$\qquad -0.5T\sin(2x_1(k)) + T\frac{x_5}{M}$

$x_3(k+1) = (1-TC_4)x_3(k) + TC_5\cos(x_1(k))$

$\qquad +Tx_6(k)$

$x_4(k+1) = TK_1u_1(k) + TK_2x_2(k)$

$\qquad + (1-T^*K_3)x_4(k)$

$x_5(k+1) = TK_4x_4(k) + (1-TK_5)x_5(k)$

$x_6(k+1) = TK_6u_2(k) + (1-TK_7)x_6(k)$

The parameters in the above equations were calculated to be

$C_1=2.1656$, $C_2=13.997$, $C_3=-55.565$, $C_4=1.020$, $C_5=4.049$,

$K_1=.4429$, $K_2=1.0198$, $K_3=1$, $K_1=5.0$, $K_4=2.0408$,

$K_5=2.0408$, $K_6=1.5$, $K_7=0.5$, $M=0.0338$, $T=0.05$

The physical significance of the problem is that it is assumed that a three phase to ground fault of a short duration occurs on the line side of the transformer and it is desired to compute the optimal machine excitation u_1 and the speeder governor setting u_2 to bring the system back to normal operation whilst minimizing the following performance measure :

$$J=\frac{T}{2}\sum_{k=0}^{k_f}\{\|x(k)-x_d\|_Q^2+\|u(k)\|_R^2\}$$

where

$Q_{11}=4$, $Q_{33}=4$, $R_{11}=R_{22}=1$

with

$x(0)^T=[1.7105\ 0.0\ 5.2\ 0.8\ 0.8\ 0.5]$

For $k_f=40$ (i.e. 2 seconds), Figs. 4 through 9 show the resulting optimal trajectories and Figs. 10 and 11 the optimal controls. In each figures, squre-marked response is control result of the given system and triangle-marked response is control result of the system varied when the parameters, C_1, C_2, C_3, C_4, C_5, of the given system are varied by 10%.

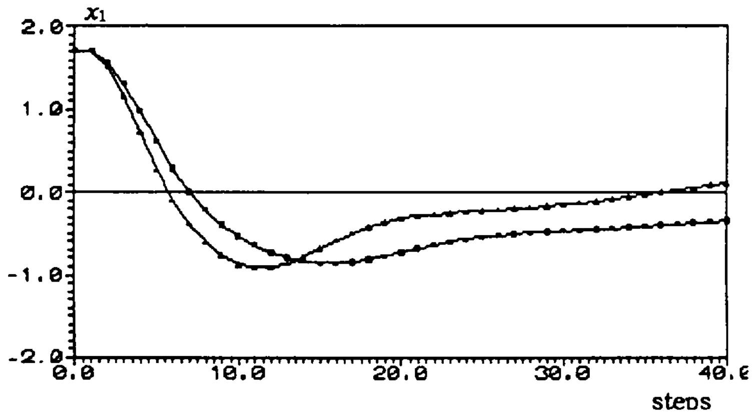

Fig. 4 Optimal state trajectory x_1 (square-marked) and in the presence of parameter variation (triangle-marked).

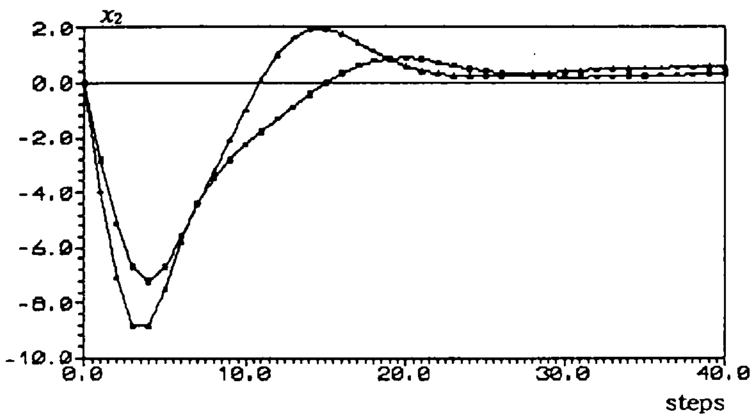

Fig. 5 Optimal state trajectory x_2 (square-marked) and in the presence of parameter variation (triangle-marked).

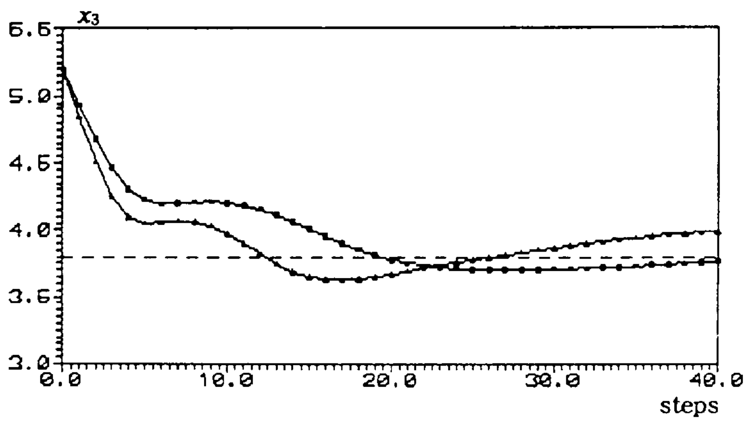

Fig. 6 Optimal state trajectory x_3 (square-marked) and in the presence of parameter variation (triangle-marked).

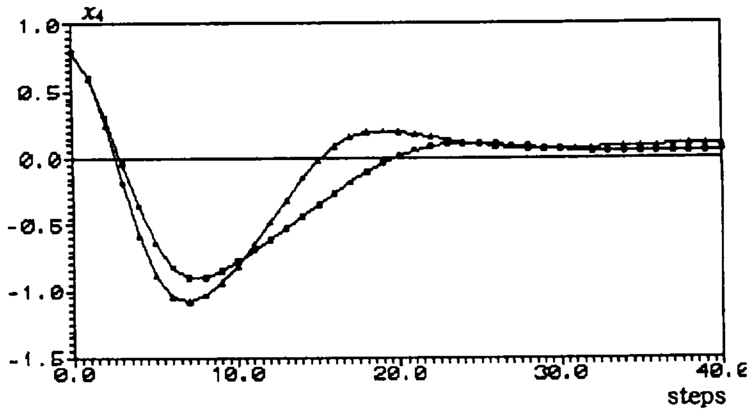

Fig. 7 Optimal state trajectory x_4 (square-marked) and in the presence of parameter variation (triangle-marked).

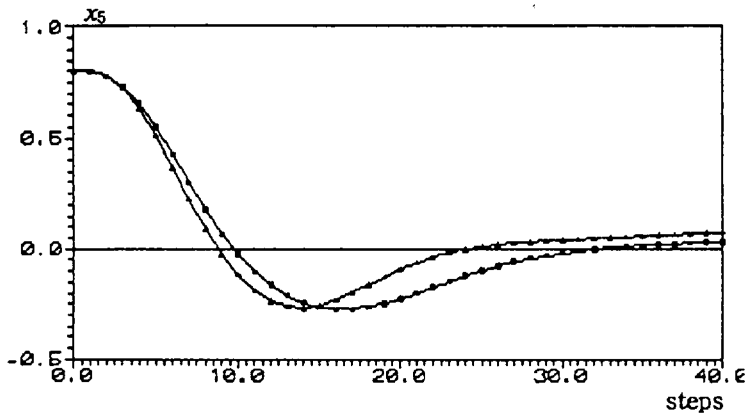

Fig. 8 Optimal state trajectory x_5 (square-marked) and in the presence of parameter variation (triangle-marked).

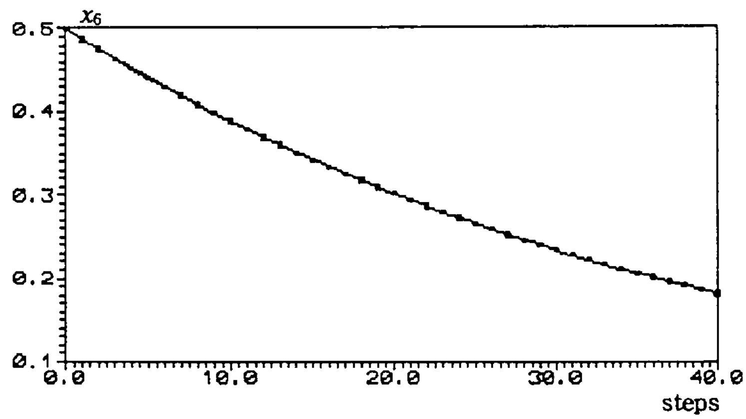

Fig. 9 Optimal state trajectory x_6 (square-marked) and in the presence of parameter variation (triangle-marked).

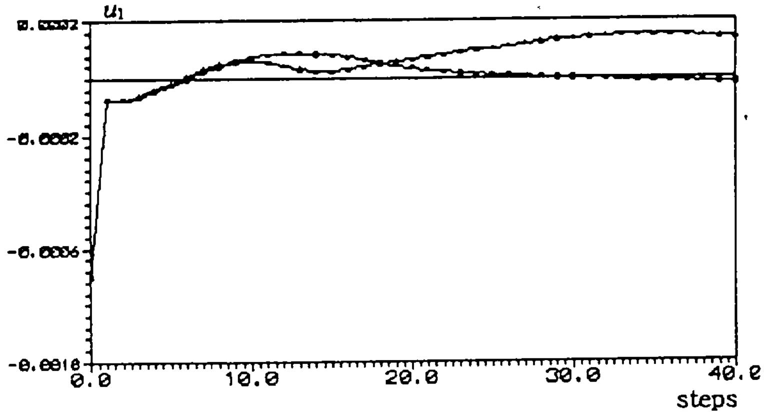

Fig. 10 Optimal contol input u_1 (square-marked) and in the presence of parameter variation (triangle-marked).

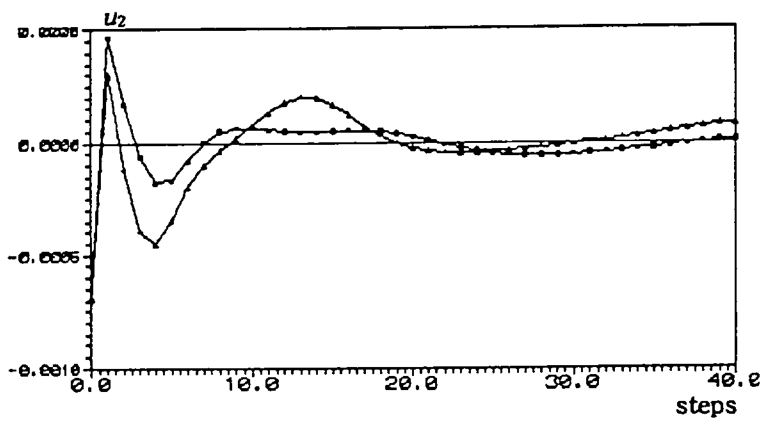

Fig. 11 Optimal contol input u_2 (square-marked) and in the presence of parameter variation (triangle-marked).

4. Conclusions

In this paper, we presents the multistage optimal control algorithm with the TLMNN structure, which was applied to systems under uncertainties and variations. It is not necessary to obtain the mathematical model and can obtain a robust control law which is capable of compensating a modeling error on the on-line operating. The suggested control strategy is shown to be effective by computer simulations for two selected examples.

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