Estimation of Thrust Forces by High Energy Pipe Rupture

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고온고압의 배관파열에 따른 유체분출력의 계산

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Summary

A proper determination of the fluid jet thrust forces through the ruptured area is critical in the design of pipe restraints against the possible failure of major equipment due to pipe whipping when rupture occurs. In this paper, a static analysis has been made which can be used as a static design guide. In the development of equations, emphasis has been laid on actual credible circumstances rather than on the severest case to avoid excessive design margins and at the same time, the margins included in equations has been discussed qualitatively.

1. Introduction

In a nuclear power plant, the most critical accident following rupture of the high energy pipe is the loss of coolant due to the fluid loss. But in addition to this kind of thermal hydraulic accident, another type of accident might be considered, that is, the failure of the major equipment by pipe whipping at the moment of pipe rupture. The easiest and the most efficient way to prevent this accident from occurring is the proper layout of piping and major equipment at the stage of plant design. But when the layout does not permit the effective means against the possible pipe whipping, particular consideration should be given to the design of restraints or hangers by the determination of thrust forces. This, in turn, requires detail analyses of the mechanism of pipe crack and fluid loss plus the mutual relations between crack and fluid jet as well. In other words, the flaws which are present in the pipe or on the surface may trigger the gradual crack growth by plastic deformation and finally the instantaneous crack. The crack opening and propagation depend strongly on the spatial distribution of the fluid pressure and arrest with the depressurization of the system by fluid loss.

Presently the thrust forces are estimated under the assumption of the hypothetical circumferential guillotine break (ASCE, 1980), which results in over-margins in design. Further, the thermal hydraulic analyses show the trend of transition from the hypothetical guillotine break LOCAs to the more realistic small LOCAs after TMI accident and it is Author's opinion that the estimation of thrust forces against pipe whipping design be made in parallel with this trend. However in this paper, circumferential large ruptures as well as axial ruptures are discussed for the sake of the directions of the moment and force, whichever is applicable to the design of pipe restraints and displacements.

2. Axial Crack

For the case of axial crack, assumptions are made that the crack is one dimensional and of cusp tip, then,

$$W = \frac{W_0}{2}\left(1 - \cos\frac{\pi x}{l}\right) \quad \cdots\cdots (1)$$

where W = effective width of crack
W_0 = maximum half crack opening
l = crack transition length
x = distance behind crack tip.

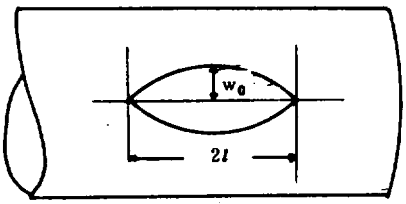

Figure 1. Axial cusp crack

By integrating Eq. (1), the crack opening area is obtained as

$$A_{ax} = 2 W_0 l \quad \cdots\cdots (2)$$

It should be noted that in Eq. (2), crack opening history is not considered. That is, the crack opening is assumed to occur at time $t = t_0$, (highest system pressure) and at the same time, the crack closure due to the hoop stress decrease by the system pressure drop is excluded. Though the axial crack area is expressed simply as in Eq.(2), W_0 and l should be determined. For the case of the crack of $2l'$ on the infinite plane, the stress intensity factor, k, is

$$k = \sigma(\pi l')^{\frac{1}{2}} \quad \cdots\cdots (3)$$

where σ is the stress perpendicular to the crack.

The crack propagation can be determined by considering the local plastic deformation in the vicinity of the crack tip. Let r be the radius of the plastic zone with its center on the tip, then the local stress in the region of plastic zone is given by

$$\sigma = k(2\pi r)^{-\frac{1}{2}} \quad \cdots\cdots (4)$$

where k is the value obtained from Eq. (3). If the local stress, Eq. (4), is larger than the yield stress σ_y, the propagation starts to occur (Knott, J.F., 1979) and from Eq. (4), the propagation length is

$$r_y = \left(\frac{k^2}{2\pi\sigma_y^2}\right)$$

$$l = l' + r_y = l' + \left(\frac{k^2}{2\pi\sigma_y^2}\right) \quad \cdots\cdots (5)$$

In Eq. (3), the crack length can be regarded as the flaw size instead of crack size, and the local stress σ can be approximated by the pipe hoop stress due to internal pressure. Also Eqs. (3) and (5) show that the final crack length is proportional to the flaw size and that the crack length becomes shorter for larger yield strength material as expected. As in the same way, when the plastic zone near the tip is regarded as a circle of radius r_y, the strain at the point $r = r_y$ is $\varepsilon_y = \frac{\sigma_y}{E}$ (E; Young's modulus), and the COD (crack opening displacement) at the crack tip is derived as follow by Buderkin and Stone (1966).

$$\sigma = \left(\frac{8\sigma_y}{\pi E}\right) l' \cdot \ln\left(\sec(\pi\sigma/2\sigma_y)\right) = \frac{k^2}{\sigma_y E} \quad (6)$$

For the estimation of W_0, the Dugdale modeling as shown in Figure 2 is used and the final crack shape can be thought as the cosine shape of

$$W = W_0 \cdot \cos\left(\frac{\pi x}{2(l' + r_y)}\right)$$

Then since

$\delta/2 = W_0 \cos\left(\frac{\pi l'}{2(l' + r_y)}\right)$, W_0 is found to be:

$$W_0 = \frac{\delta}{2\cos\left(\frac{\pi l'}{2(l'+r_y)}\right)} = \frac{\delta}{2\cos\left(\frac{\pi l'}{2l}\right)} = \frac{k^2}{2\sigma_y E\cos\left(\frac{\pi l'}{2l}\right)}$$

$$= \frac{k^2}{2\sigma_y E\cos\left[\pi l'/(2l' + k^2/\pi\sigma_y^2)\right]} \quad \cdots\cdots (7)$$

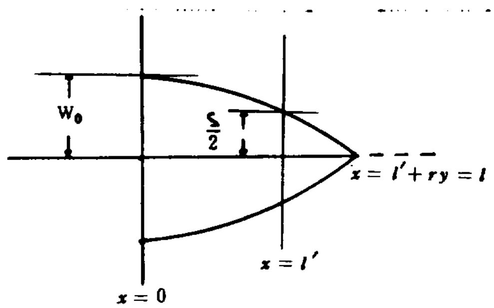

Figure 2. Dugdale model

By inserting Eqs. (7) and (5) into Eq. (2), the axial crack area can be determined and it is found that the crack area is the function of the material properties (σ_y, E) and geometric factors (k,l'). Together with this, the size, orientation and distribution of the flaws can be found by various methods such as NDT but the probabilistic approach (Carlsson. J., 1979) utilizing experential data can render the reasonable results under relevant operating conditions.

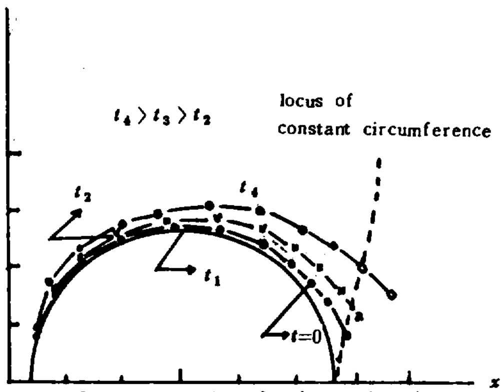

Figure 3. Cross section of a fracturing pipe (Emery, A. F. *et al*, 1981)

At this point, it should be pointed out that the crack opening area, Eq. (2), includes an additional margin with respect to the increase of flow area (channel area) near the crack zone (EPRI, NP-763, 1976). The cross sectional area of the rupture zone increases as in Figure 3, and increase in flow channel area leads to the larger system depressurization rate which contributes to margins when the effect is neglected. If the flow channel area, A, is regarded as a function of time, then from continuity,

$$\left(\frac{\partial(A\rho)}{\partial t}+\frac{\partial(GA)}{\partial x}\right)\Delta x=-G_l W\,\Delta x$$

$$\frac{\partial\rho}{\partial t}+\frac{1}{A}\frac{\partial(GA)}{\partial x}=-G_l\frac{W}{A}-\frac{\rho}{A}\frac{\partial A}{\partial t} \quad \cdots\cdots (8)$$

where G = channel flow rate

G_l = leakage flow rate

A = channel area.

The second term on the right hand side of Eq. (8) represents the effect of channel area change. On the other hand, the relationship between the wetted perimeter of the channel area and crack width is

$$D_h = D_{h0}+\frac{W}{\pi} \quad \cdots\cdots (9)$$

where D_{h0} = non-ruptured wetted perimeter

D_h = ruptured wetted perimeter.

From Eqs. (8) and (9),

$$\frac{1}{A}\frac{\partial A}{\partial t}=\frac{2}{D_h}\cdot\frac{\partial D_h}{\partial t}=\frac{2}{\pi D_h}\cdot\frac{\partial W}{\partial t} \quad \cdots\cdots (10)$$

Further from Eq. (1), $W = \frac{W_0}{2}(1-\cos\frac{\pi x}{l})$

$=W_0\sin^2\left(\frac{\pi x}{2l}\right)$ and by letting $l = nD_h$,

$$\frac{\partial W}{\partial t}=\frac{\pi W_0}{2l}\sin\left(\frac{\pi x}{l}\right)\cdot\frac{dx}{dt}=\frac{\pi W_0}{2nD_h}\sin\left(\frac{\pi x}{l}\right)\cdot\frac{dx}{dt}$$

Therefore,

$$\frac{1}{A}\frac{\partial A}{\partial t}=V_c\cdot\frac{2W}{\pi D_h}\cot\left(\frac{\pi x}{2l}\right)\cdot\left(\frac{\pi}{l}\right) \quad \cdots\cdots (11)$$

where $V_c = \frac{dx}{dt}$ = crack velocity.

Comparing $G_l\frac{W}{A}$ with $\frac{\rho}{A}\cdot\frac{\partial A}{\partial t}$ in the second equation of Eq. (8), then

$$\left(\frac{\rho}{A}\cdot\frac{\partial A}{\partial t}\right)/\left(G_l\frac{W}{A}\right) = \left[\rho V_c\cdot\frac{2W}{\pi D_h}\cdot\cot\left(\frac{\pi x}{2l}\right)\cdot\frac{\pi}{l}\cdot\frac{\pi}{4}\cdot D_h^2\right]/\left(\rho V_l W\right)$$

$$=\frac{V_c}{V_l}\cdot\frac{\pi D_h}{2l}\cdot\cot\left(\frac{\pi x}{2l}\right) \quad \cdots\cdots (12)$$

Two effects on the system depressurization (i.e., effects due to leakage and area change) are compared each other as tablized below (EPRI, NP-763, 1976).

Table 1. Effects on depressurization-leakage and area change

Crack type, $\frac{l}{\pi D_h}$ (C_t)	Narrow $C_t=3$	Wide $C_t=1$
Crack Speed, V_c/V_l	High 5 Low 0.5	High 5 Low 0.5
Transition position (which makes Eq. (12) unity), $\frac{x}{D_h}$ ($=\frac{x\pi C_t}{l}$)	High 4.2 Low 0.5	High 2.4 Low 0.5

As can be found in the table above, the transition position should be approximately $4D_h$ to make two effects identical each other for the case of high speed crack. But almost all the practical material is of high ductility, hence the crack speed is relatively low (~300 m/sec) and the crack propagation is comparatively long. Accordingly the effect of the depressurization (i.e., the rate of mass flux decrease) by the area change is not so great, but by excluding the area change term as for the case of static design, an additional margin can be expected.

2. Circumferential Crack

The guillotine break — the hypothetical condition of the maximum credible accident of a nuclear power plant as mentioned above — assumes the complete cutting of pipe without any deformation (the local area ratio, $\alpha = 1$). But actually the pipe might be considered to be under the bending and/or tension forces and the final shape can be thought as a semi circular rectangular. From Figure 4, the one-dimensional cross sectional area is

$$A = \pi y(D_0 - y) \qquad (13)$$

or by defining the local area ratio $\alpha = A/A_0$ (A_0: original cross sectional area),

$$\alpha = 4\frac{y}{D_0}\left(1-\frac{y}{D_0}\right) \qquad (14)$$

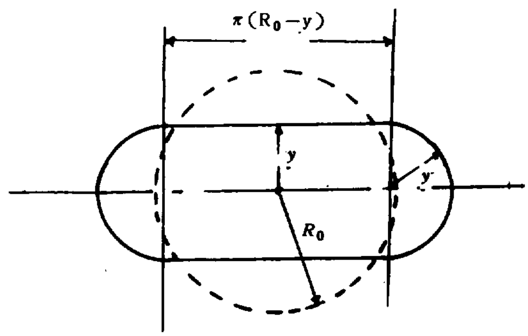

Figure 4. Cross section of a circumferential breaking under tension and bending(McClurken, M. E. *et al*, 1981)

The y value of Eq. (14) is the function of the tension and bending stiffness of the pipe material and the local area ratio α is found to have a value of 0.3-0.8 depending on material properties and load conditions (McClurken, M.E, *et al*, 1981). Eq. (14) is an area of one-dimensional but if a rupture is assumed to occur along the direction which makes shear stress zero, then two-dimensional cross sectional area is

$$A_{cir} = \frac{0.8\,A_0}{\cos(\pi/4)} = 1.13\,A_0 \qquad (15)$$

The area given by Eq. (15), which shows 13% larger than the original cross sectional area includes sufficient margin ($\alpha=0.8$ (max) with two-dimensional area).

3. Fluid Thrust

The linear momentum equation of the fluid escaping through the crack is as below by Reyonold transport theorem.

$$\frac{d}{dt}(m\vec{V}) = \Sigma\vec{F} = \frac{d}{dt}\left(\int_{cv} \rho\vec{V}\,dV\right) + \int_{cs} \rho\vec{V}(\vec{V}\cdot\vec{n})\,dA \quad \cdots\cdots (16)$$

If a control volume is taken around the rupture zone, along with the assumption of that the fluid properties are constant in the control volume, then the one-dimensional linear momentum equation can be written as $F = \rho \cdot$(crack opening area) (velocity)^2. And since the pressure force is $\vec{F}_p = \int_{cs} p(-\vec{n})dA$, the total one-dimensional force by crack leakage is

$$F = A(p_e - p_a) + \rho_e A_e V_e^2 \quad \cdots\cdots (17)$$

where subscript e denotes exit and P_a the atmospheric pressure.

When the system pressure is lower than the critical pressure or when a flashing does not occur, Eq. (17) becomes $F = p_g A + \dot{m}(V_2 - V_1)$ where V_1 is the control volume inlet velocity, V_2 the outlet velocity and p_g is the gauge pressure. Furthermore, from continuity, V_2 is equal to V_1, hence the total thrust force is simply a pressure force of $F = p_g A$ and if the system pressure and crack area are known, the thrust force can be estimated easily. But in reality, the pipe fluid is of high energy (approximately over 160 kg/cm^2, 320°C for primary loop) and with a pipe crack, fluid choking and flashing occur simultaneously.

For the estimation of critical flow, several models such as isenthalpic, Moody and Extended Henry-Fauske could be used. Among these models, however, the results of the isenthalpic model for the subcooled region always show smaller values than Moody or Henry-Fauske. Moreover, it is desirable to use the Henry-Fauske model to avoid the abrupt discontinuity of phase transition rather than the Moody model which is derived theoritically with taking into account of the slip effect between two phases. And the Extended Henry-Fauske model can be used for the case of that flashing occurs between the center of the control volume and the junction of the control volume (ruptured surface) as well as when the macroscopic averaged fluid property of the control volume is in the subcooled or saturated states (Delhaye, J.M., 1980). With the assumptions of isentropic (no friction) flow and steady state and by applying continuity and momentum equations to each phase, the mass flux is obtained as follows (Elwakil, M.M., 1978).

$$G = \frac{-dp \cdot g_c}{d[(1-x_f)v_l + x_f v_g]} \quad \cdots\cdots (18)$$

where G = mass flux, kg/m^2 sec

v_l, v_g = specific volume (m^3/kg) of liquid and gas

x_f = flowing quality

g_c = conversion factor, constant

Since $\left.\frac{\partial G}{\partial p}\right)_j = 0$ when choking (j = ruptured surface junction), the critical mass flux, G_c, turns out be

$$\frac{G_c^2}{g_c} = -\left(\frac{d}{dp}\left(\frac{x_f S + (1-x_f)}{S}\right) \times [(1-x_f)Sv_l + x_f v_g]\right)^{-1}$$

$$= -\left(\frac{1}{S}\left[(1+x_f(S-1))\,x_f\frac{dv_g}{dp} + (Sv_l[2(x_f-1)+S(1-2x_f)] + v_g[1+2x_f(S-1)])\frac{dx_f}{dp}\right] + f_1\frac{dv_l}{dp} + f_2\frac{dS}{dp}\right)^{-1} \quad \cdots\cdots (19)$$

where S = slip ratio

f_1, f_2 = constants.

Eq. (19) is quite a general equation just derived from continuity and momentum equations. By applying the following assumptions of Henry and Fauske, the critical mass flux is given by Eq. (20).

1) slip equilibrium model : $S = 1$, $dS/dp = 0$

2) $\frac{dv_l}{dp} = 0 : v_l = v_{l0}$ (stagnation specific volume)

3) $\frac{\partial x_f}{\partial p} = N \frac{\partial x_e}{\partial p}$ (x_f = flowing quality, x_e, equilibrium quality)

$$N = x_e/0.14, \quad x_e < 0.14$$
$$= 1, \quad x_e > 0.14$$

4) $x_f = x_0$ (stagnation equilibrium quality)
$= 0$; $1 - x_0 = 1$

5) adiabatic process of steam expansion:
pv_g^n = constant

$$\frac{G_c^2}{g_c} = \left(\frac{x_0 v_g}{np} + (v_g - v_{l0}) N \frac{\partial x_e}{\partial p} \right)^{-1} \quad \ldots\ldots (20)$$

And by the use of the relationship of

$$\frac{d}{dp} = \frac{\partial}{\partial p} + \frac{\partial}{\partial h} \cdot \frac{dh}{dp} = \frac{\partial}{\partial p} + v \frac{\partial}{\partial h},$$

the critical velocity is (Rose, 1967);

$$\frac{g_c}{V^2_{critical}} = \frac{g_c}{a^2} = \frac{\partial \rho}{\partial p} = -\frac{1}{v^2}\left(\frac{dv}{dp}\right)_s$$

$$= \frac{1}{-v^2}\left\{ v\left(\frac{\partial v}{\partial h}\right)_p + \left(\frac{\partial v}{\partial p}\right)_h \right\} = R_p + R_h v \quad \cdots (21)$$

where $R_p = \frac{1}{-v^2}\left(\frac{\partial v}{\partial p}\right)_h$, $R_h = \frac{1}{-v^2}\left(\frac{\partial v}{\partial h}\right)_p$,
a = sonic velocity.

Also Eq. (21) can be written, by applying $v = v_f + v_{fg} x$, as

$$\frac{g_c}{V^2_{critical}} = \frac{1}{-v^2}\left[\left(\frac{dv_f}{dp}\right)_s + x \left(v_f \frac{d(v_{fg}/v_f)}{dp} + \frac{v_{fg}}{v_f} \cdot \frac{dv_f}{dp} \right)_s + v_{fg}\left(\frac{dx}{dp}\right)_s \right] \cdots (22)$$

Then, the thrust force is determined by the use of critical mass flux, Eq. (20), and critical velocity, Eq. (22), as

$$F = pA_c + G_c A_c (V_{critical} - V_i), \text{ for circumferential crack}$$
$$= pA_c + G_c A_c V_{critical}, \text{ for axial crack} \quad \ldots\ldots (23)$$

where V_i = control volume inlet velocity
$= \frac{G}{\bar{\rho}}$
$\bar{\rho}$ = control volume averaged density
A_c = crack area.

In the above equation, the fluid property is assumed to be constant in the control volume and as the results of various thermal hydraulic codes show (Tarng, H.J., 1979), if the overall system volume is sufficiently large, the property change of the fluid can be neglected during the crack propagation time, which is very short, and this assumption presents another design margin in that high mass flux is used.

Finally the critical mass flux, Eq. (20) is to be obtained by iterative method which presents an inconvenience. However the critical mass flux can be expanded as a polynomial of stagnation pressure p and stagnation enthalpy h as in Eq. (24) with the appropriate constants and the result shows a good approximation to the theoritical values (Retran-02, 1981).

G (Extended Henry-Fauske critical mass flux, subcooled) = G (p,h), kg/m² sec

$$= 4.93 \sum_{j=0}^{5} \sum_{i=0}^{5} H_{ij} p^j h^i, \quad 20 < p < 210 \ kg/cm^2 \quad \ldots\ldots (24)$$

The constants, H_{ij}, of above equation is described in detail in RETRAN for the numerical calculation.

4. Conclusion and Recommendation

The thrust forces of high energy pipe rupture, Eq. (23) along with Eqs. (24) and (22), can be used as a static design guide with a proper margin (safety factor) with regard to:

1) decoupling of the crack shaping mechanism from the fluid equation and consideration of severer conditions (i.e., maximum final crack area plus maximum initial system pressure)
2) exclusion of the possible crack closure by depressurization
3) exclusion of the effect of the channel area increase
4) ignorance of the skin friction in momentum equations and of the flow separation loss

Among the items listed above, the decoupling of the fluid (fluid loss) from solid (crack) would produce redundant margins since these two factors are coupled strongly during the cracking. This problem could be solved by the dynamic analysis by considering the transient phenomena through the process of crack. But in this case, the shock stress wave should be taken into account which, in turn, presents difficulties in modeling in that the modeling size should be lengthy with many nodal points. To avoid this, the Freund model (Freund, L.B. *et al*, 1976) could be considered and the result by using this model for the simple case has been obtained by Emery *et al* (1981). But this model is difficult to apply to the complex fluid systems and it is suggested to incorporate the crack model with the present various thermal hydraulic codes to describe the more realistic circumstances of cracking along with an additional consideration of the system heat generation and transfer.

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國 文 抄 錄

원자력발전소의 고온고압의 배관이 파열될때, 냉각재 상실에 의한 사고뿐만이 아니라 순간적인 유체의 분출력에 의하여 pipe whipping이 일어나게 되며, 이는 주변 주요기기의 손상을 야기시킬수 있다. 본고에서는 pipe whipping을 감안한 설계치의 기준을 위하여 배관의 파열 및 이에 따른 분출력의 관계를 정적으로 연결시켰으며 동시에 이와 같이 얻어진 값들이 설계치로서의 충분한 마진을 갖고 있음을 보였다.