Free Shelf Waves over a Linear Double Shelf as in the Yellow Sea*

*Pang Ig-chan***

황해와 같은 선형 양향성 대륙붕에서의 자유 대륙붕파*

방 의 찬**

Introduction

The theories of coastally trapped waves have gradually been established and now there is little doubt about that coastally trapped waves play an important role in the coastal ocean response to applied wind streses. The theories have been first developed for the single shelf case (Buchwald & Adams, 1968; Gill & Schumann, 1974; Huthnance, 1975, 1978; Brink & Allen, 1978; LeBlond & Mysak, 1978; Clarke and VanGorder, 1986; etc.) and thereafter for the various cases of different shelf such as submarine banks and trenches (Louis, 1978; Mysak etc., 1979, 1980, 1981; Brink, 1983). Recently, the theory for the double shelf case has been reported. Hsueh and Pang(1989) have developed the theory

and shown a good assesment of its application to the Yellow Sea. Pang (1992) has shown the general properties of coastally trapped waves over an exponential double shelf topography. An exponential topography allows analytical solutions in the whole ranges of wave number for non-divergence case. Over a double shelf topography, two sets of waves propagate in opposite directions, with the shallow waters to the right in the northern hemisphere. The gropu velocities of shelf waves have the same direction as the phase velocities in the long waves, but the opposite direction in the short waves.

An exponential topography, however, does not allow analytical solutions for divergence case. The horizontal flow divergence is essential for explaining a major part of the ocean response on a double shelf (Hsueh & Pang, 1989), while it is ignored on a single

** Dept.of Oceonography, College of Ocean Sciences

shelf when the cross-shelf dimension is much smaller than Rossby deformation radius. Analytical solutions for divergence case could be obtained by a linear topography. Hsueh & Pang (1989) have obtained them for long waves. However, in spite of the basic establishment, the theory should be further developed to include short waves and to prove the orthogonality of the bases. The purpose of this paper is to develop the theory of coastally trapped free waves over a double shelf topography for divergence cases. It is extended to short waves, which is necessary for small scale coastal ocean dynamics sch as reflecting or scattering, and the orthogonality is proved for eigenfunction expansions.

Field Equation and Boundary conditions

Small perturbations to a barotropic ocean satisfy the equation :

$$Hp_{xxt} + H_x p_{xt} + Hp_{yyt} + fH_x p_y + (rp_x)_x$$
$$- ((f^2 - \omega^2)/g) p_t$$
$$= - ((f^2 - \omega^2)/g) \ p_{at} + f(Y_x - X_y) \cdots\cdots (1)$$

In this equation, x, y, t, p, g, f, r, H, p_a, X and Y refer respectively to cross-shelf distance, alongshore distance, time, perturbation pressure divided by mean water density, acceleration due to gravity, Coriolis parameter, bottom resistance coefficient, water depth, atmospheric perturbation pressure divided by mean water density, kinematic stresses in x and y direction at surface (the wind stresses divided by mean water density). Subscripts indicated drivatives.
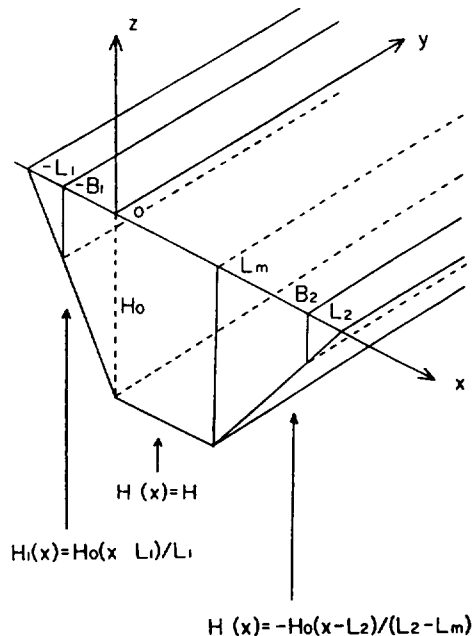
Fig.1 shows a schematic representation of

$$H_1(x) = H_0(x \ L_1)/L_1$$

$$H(x) = -H_0(x-L_2)/(L_2-L_m)$$

Fig. 1. Schematic representation of the coordinate system and geometry of two shelves of linear depth profile and a level intervening region. The coordinates x, y, and z refer to the cross-shelf, alongshore, and vertical directions and are oriented eastward, northward, and upward, respectively.

the coordinates system and geometry of two shelves of linear depth profile and a level intervening region. To begin with, an intervening region is put between the two shelves so that shelf 1, intervening region, and shelf 2 are placed in $-B_1 \le x \le 0$, $0 \le x \le L_m$, and $L_m \le x \le B_2$, respectively. So, the linear bottom topography (H) can be set as follows :

$$H(x) = \begin{cases} H_1 = H_0 (x + L_1) L_1 \\ \quad -L_1 \le x \le 0 \quad \text{in shelf 1} \\ H_m = H_0. \\ \quad 0 \le x \le L_m \quad \text{in middle area} \cdots (2) \\ H_2 = -H_0 (x - L_2) / (L_2 - L_m), \\ \quad L_m \le x \le L_2 \quad \text{in shelf 2} \end{cases}$$

As the coastal boundaries, the no-flux boundary condition is applied, which means that the depth integrated offshore velocity vanishes. For most cases of coastal ocean dynamics, wind stresses are applied through the coastal boundary condition. Near coast, the Ekman flux produced by alongshore wind stress gives rise to the convergence and divergence fluxes, which dreives the interior flow. Consequently, the interior flows are driven by the wind stress applied through the coastal boundary condition. In the case of a linear depth profile, Mitchum and Clarke (1986) have concluded that the place where the water depth is about 3 times the Ekman layer thickness is the best place for the imposition of the no-flux condition. At $x=0$, L_m, the 'continuous pressure' and the 'continuous transverse velocity' boundary conditions are applied, as follows :

$$P_{1xt} + (r/h) p_{1x} + fP_{1y} = fY/h,$$
$$\text{at } x = -B_1 \cdots\cdots\cdots (3\text{-}1)$$
$$P_1 = P_m, \quad \text{at } x = 0 \cdots\cdots\cdots (3\text{-}2)$$
$$P_{1xt} + fP_{1y} = P_{mxt} + fP_{my},$$
$$\text{at } x = 0 \cdots\cdots\cdots (3\text{-}3)$$
$$P_m = P_2, \quad \text{at } x = L_m \cdots\cdots\cdots (3\text{-}4)$$
$$P_{mxt} + fP_{my} = P_{2xt} + fP_{2y},$$
$$\text{at } x = L_m \cdots\cdots\cdots (3\text{-}5)$$
$$P_{2xt} + (r/h) p_{2x} + fP_{2y} = fY/h,$$
$$\text{at } x = B_2 \cdots\cdots\cdots (3\text{-}6)$$

To solve the above eigenvalue problem, either frictionless eigenfunction or frictional eigenfunction can be used. Frictionless eigenfunction has been used conventionally, but Webster (1985) has started to use frictional eigenfunction. In this work, frictionless eigenfunction is used.

Dispersion Relations

1. Low-frequency and Long-wave Case

The usual low-frequency and long-wave approximation will be invoked in this section. The field equation (1) yields for divergent, inviscid, low-frequency, long free waves

$$(Hp_{xt})_x + fH_x P_y - (f^2/g) p_t = 0 \cdots\cdots\cdots (4)$$

Upon substituting for the pressure, $p = F(x) \phi(y + ct)$, (4) yields

$$(HF')' + (f/c) H'F - (f^2/g) F = 0 \cdots\cdots\cdots (5)$$

where the 'prime' means the derivative with respective to x and c is the phase speed. Equation (5) with the depth profiles given by (2) yields the following eigen value problem for the frictionless eigenfunction $F(x)$:

$$[\alpha(x+L_1) F_1']' + \mu_1 F_1 = 0 \qquad -L_1 \leq x \leq 0 \qquad (6\text{-}1)$$
$$F_m'' - \lambda^2 F_m = 0 \qquad 0 \leq x \leq L_m \qquad (6\text{-}2)$$
$$[\beta(x-L^2) F_2']' + \mu_2 F_2 = 0 \qquad L_m \leq x \leq L_2 \qquad (6\text{-}3)$$

$$F_1' + (f/c) f_1 = 0 \qquad \text{at } x = -B_1 \qquad (7\text{-}1)$$
$$F_1 = F_m \qquad \text{at } x = 0 \qquad (7\text{-}2)$$
$$F_1' = F_m' \qquad \text{at } x = 0 \qquad (7\text{-}3)$$
$$F_m = F_2 \qquad \text{at } x = L_m \qquad (7\text{-}4)$$
$$F_m' = F_2' \qquad \text{at } x + L_m \qquad (7\text{-}5)$$
$$F_2' + (f/c) F_2 = 0 \qquad \text{at } x = B_2 \qquad (7\text{-}6)$$

where $\mu_1 = f/c - f^2/g\alpha$, $\mu_2 = f/c - f^2/g\beta$, $\lambda^2 = f^2/gH_4$, $\alpha = H_4/L_1$, $\beta = -H_4/(L_2 - L_m)$. Here, F_1, F_m and F_2 represent the eigenfunctions over, respectively, the shelf 1, intervening region, and shelf 2. λ is the reciprocal of barotropic

deformation radius, and α and β are the slope coefficients of the shelves 1 and shelf 2, respectively.

The solution of the above eigenvalue problem leads to a dispersion relation that aloows the determination of phase speed c, as follows :

$$\exp(-m_1L_2) \cdot [J_0(2b_1) J_0(2b_2)/J_0(2a_1) J_0(2a_2)]$$
$$\cdot [f/c-\{\mu_1/b_1\} \cdot \{J_1(2b_1)/J_0(2b_1)\}]$$
$$\cdot [f/c-\{\mu_2/b_2\} \cdot \{J_1(2b_2)/J_0(2b_2)\}]$$
$$\cdot [\lambda+\{\mu_1/a_1\} \cdot \{Y_1(2a_1)/Y_0(2a_1)\}]$$
$$\cdot [\lambda-\{\mu_2/a_2\} \cdot \{Y_1(2a_2)/Y_0(2a_2)\}]$$
$$-\exp(-m_1L_2) \cdot [J_0(2b_1) Y_0(2b_2)/J_0(2a_1) Y_0(2a_2)]$$
$$\cdot [f/c-\{\mu_1/b_1\} \cdot \{J_1(2b_1)/J_0(2b_1)\}]$$
$$\cdot [f/c-\{\mu_2/b_2\} \cdot \{Y_1(2b_2)/Y_0(2b_2)\}]$$
$$\cdot [\lambda+\{\mu_1/a_1\} \cdot \{Y_1(2a_1)/Y_0(2a_1)\}]$$
$$\cdot [\lambda-\{\mu_2/a_2\} \cdot \{J_1(2a_2)/J_0(2a_2)\}]$$
$$-\exp(-m_1L_2) \cdot [Y_0(2b_1) J_0(2b_2)/Y_0(2a_1) J_0(2a_2)]$$
$$\cdot [f/c-\{\mu_1/b_1\} \cdot \{Y_1(2b_1)/Y_0(2b_1)\}]$$
$$\cdot [f/c-\{\mu_2/b_2\} \cdot \{J_1(2b_2)/J_0(2b_2)\}]$$
$$\cdot [\lambda+\{\mu_1/a_1\} \cdot \{J_1(2a_1)/J_0(2a_1)\}]$$
$$\cdot [\lambda-\{\mu_2/a_2\} \cdot \{Y_1(2a_2)/Y_0(2a_2)\}]$$
$$+\exp(-m_1L_2) \cdot [Y_0(2b_1) Y_0(2b_2)/Y_0(2a_1) Y_0(2a_2)]$$
$$\cdot [f/c-\{\mu_1/b_1\} \cdot \{Y_1(2b_1)/Y_0(2b_1)\}]$$
$$\cdot [f/c-\{\mu_0/b_2\} \cdot \{Y_1(2b_2)/Y_0(2b_2)\}]$$
$$\cdot [\lambda+\{\mu_1/a_1\} \cdot \{J_1(2a_1)/J_0(2a_1)\}]$$
$$\cdot [\lambda-\{\mu_2/a_2\} \cdot \{J_1(2a_2)/J_0(2a_2)\}]$$
$$-\exp(m_1L_2) \cdot [J_0(2b_1) J_0(2b_2) J_0(2a_1) J_0(2a_2)]$$
$$\cdot [f/c-\{\mu_1/b_1\} \cdot \{J_1(2b_1)/J_0(2b_1)\}]$$
$$\cdot [f/c-\{\mu_2/b_2\} \cdot \{J_1(2b_2)/J_0(2b_2)\}]$$
$$\cdot [\lambda-\{\mu_1/a_1\} \cdot \{Y_1(2a_1/Y_0(2a_1)\}]$$
$$\cdot [\lambda+\{\mu_2/a_2\} \cdot \{Y_1(2a_2)/Y_0(2a_2)\}]$$
$$+\exp(m_1L_2) \cdot [J_0(2b_1) Y_0(2b_2)/J_0(2a_1) Y_0(2a_2)]$$
$$\cdot [f/c-\{\mu_1/b_1\} \cdot \{J_1(2b_1)/J_0(2b_1)\}]$$
$$\cdot [f/c-\{\mu_2/b_2\} \cdot \{Y_1(2b_2)/Y_0(2b_2)\}]$$
$$\cdot [\lambda-\{\mu_1/a_1\} \cdot \{Y_1(2a_1)/Y_0(2a_1)\}]$$
$$\cdot [\lambda+\{\mu_2/a_2\} \cdot \{J_1(2a_2)/J_0(2a_2)\}]$$
$$+\exp(m_1L_2) \cdot [Y_0(2b_1) J_0(2b_2)/Y_0(2a_1) J_0(2a_2)]$$
$$\cdot [f/c-\{\mu_1/b_1\} \cdot \{Y_1(2b_1)/Y_0(2b_1)\}]$$

$$\cdot [f/c-\{\mu_2/b_2\} \cdot \{J_1(2b_2)/J_0(2b_2)\}]$$
$$\cdot [\lambda-\{\mu_1/a_1\} \cdot \{J_1(2a_1)/J_0(2a_1)\}]$$
$$\cdot [\lambda+\{\mu_2/a_2\} \cdot \{Y_1(2a_2)/Y_0(2a_2)\}]$$
$$-\exp(m_1L_2) \cdot [Y_0(2b_1) Y_0(2b_2)/Y_0(2a_1) Y_0(2a_2)]$$
$$\cdot [f/c-\{\mu_1/b_1\} \cdot \{Y_1(2b_1)/Y_0(2b_1)\}]$$
$$\cdot [f/c-\{\mu_2/b_2\} \cdot \{Y_1(2b_2)/Y_0(2b_2)\}]$$
$$\cdot [\lambda-\{\mu_1/a_1\} \cdot \{J_1(2a_1)/J_0(2a_1)\}]$$
$$\cdot [\lambda+\{\mu_2/a_2\} \cdot \{J_1(2a_2)/J_0(2a_2)\}]=0 \cdots (8)$$

where $a_1=(\mu_1L_1)^{1/2}$, $a_2=(-\mu_2(L_2-L_m))^{1/2}$, $b_1=(\mu_1(-B_1+L_1))^{1/2}$, $b_2(\mu_2(B_2-L_2))^{1/2}$, and J_m and Y_m refer to the mth order Bessel Functions of the 1st and 2nd kind, and λ, μ_1, μ_2 are given above.

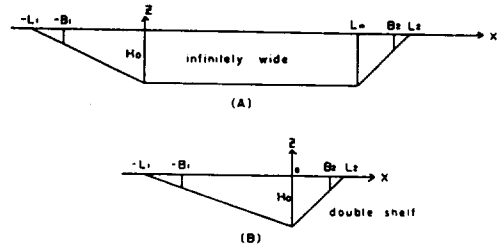When L_m goes to infinity as shown in Fig.2 (a), the dispersion relation (8) reduces to

Fig. 2. Schematic representations of the cross-shelf sections for (A) two shelves with an infinitely wide intervening region, and (B) a double shelf (two adjoining shelves).

$$[[J_0(2b_1)/J_0(2a_1)]$$
$$\cdot [f/c-(\mu_1/b_1) \cdot \{J_1(2b_1)/J_0(2b_1)\}]$$
$$\cdot [\lambda-\{\mu_1/a_1\} \cdot \{Y_1(2a_1)/Y_0(2a_1)\}]$$
$$-[Y_0(2b_1) Y_0(2a_1)]$$
$$\cdot [f/c-(\mu_1/b_1) \cdot \{Y_1(2b_1)/Y_0(2b_1)\}]$$
$$\cdot [\lambda-\{\mu_1/a_1\} \cdot \{J_1(2a_1)/J_0(2a_1)\}]]$$
$$\times[[J_0(2b_2)/J_0(2a_2)]$$
$$\cdot [f/c-(\mu_2/b_2) \cdot \{J_1(2b_2)/J_0(2b_2)\}]$$
$$\cdot [\lambda-\{\mu_2/a_2\} \cdot \{Y_1(2a_2)/Y_0(2a_2)\}]$$
$$-[Y_0(2b_2)/Y_0(2a_2)]$$
$$\cdot [f/c-(\mu_2/b_2) \cdot \{Y_1(2b_2)/Y_0(2b_2)\}]$$

$$\cdot [\lambda - \{\mu_2/a_2\} \cdot \{J_1(2a_2)/J_0(2a_2)\}]] = 0 \cdots (9)$$

This shows two independent sets of coastally trapped wave, which means that if the shelves are apart sufficiently enough, two sets of coastally trapped waves do not interact with each other, as for an exponential topography (Pang, 1992).

When L_m goes to zero (double shelf case) as shown in Fig.2(b), it yields

$$a_1\mu_2 J_0(2b_1) J_0(2b_2) Y_0(2a_1) Y_1(2a_2)$$
$$\cdot [f/c - \{\mu_1/b_1\} \{J_1(2b_1)/J_0(2b_1)\}] \cdot [f/c - \{\mu_2/b_2\}$$
$$\{J_1(2b_2)/J_0(2b_2)\}]$$
$$-a_1\mu_2 J_0(2b_1) Y_0(2b_2) Y_0(2a_1) J_1(2a_2)$$
$$\cdot [f/c - \{\mu_1/b_1\} \{J_1(2b_1)/J_0(2b_1)\}] \cdot [f/c - \{\mu_2/b_2\}$$
$$\{Y_1(2b_2)/Y_0(2b_2)\}]$$
$$-a_1\mu_2 Y_0(2b_1) J_0(2b_2) J_0(2a_1) Y_1(2a_2)$$
$$\cdot [f/c - \{\mu_1/b_1\} \{Y_1(2b_1)/Y_0(2b_1)\}] \cdot [f/c - \{\mu_2/b_2\}$$
$$\{J_1(2b_2)/J_0(2b_2)\}]$$
$$+a_1\mu_2 Y_0(2b_1) Y_0(2b_2) J_0(2a_1) J_1(2a_2)$$
$$\cdot [f/c - \{\mu_1/b_1\} \{Y_1(2b_1)/Y_0(2b_1)\}] \cdot [f/c - \{\mu_2/b_2\}$$
$$\{Y_1(2b_2)/Y_0(2b_2)\}]$$
$$-a_2\mu_1 J_0(2b_1) J_0(2b_2) Y_1(2a_1) Y_0(2a_2)$$
$$\cdot [f/c - \{\mu_1/b_1\} \{J_1(2b_1)/J_0(2b_1)\}] \cdot [f/c - \{\mu_2/b_2\}$$
$$\{J_1(2b_2)/J_0(2b_2)\}]$$
$$+a_2\mu_1 J_0(2b_1) Y_0(2b_2) Y_1(2a_1) J_0(2a_2)$$
$$\cdot [f/c - \{\mu_1/b_1\} \{J_1(2b_1)/J_0(2b_1)\}] \cdot [f/c - \{\mu_2/b_2\}$$
$$\{Y_1(2b_2)/Y_0(2b_2)\}]$$
$$+a_2\mu_1 Y_0(2b_1) J_0(2b_2) J_1(2a_1) Y_0(2a_2)$$
$$\cdot [f/c - \{\mu_1/b_1\} \{Y_1(2b_1)/Y_0(2b_1)\}] \cdot [f/c - \{\mu_2/b_2\}$$
$$\{J_1(2b_2)/J_0(2b_2)\}]$$
$$-a_2\mu_1 Y_0(2b_1) Y_0(2b_2) J_1(2a_1) J_0(2a_2)$$
$$\cdot [f/c - \{\mu_1/b_1\} \{Y_1(2b_1)/Y_0(2b_1)\}] \cdot [f/c - \{\mu_2/b_2\}$$
$$\{Y_1(2b_2)/Y_0(2b_2)\}] = 0 \cdots\cdots\cdots\cdots (10)$$

The dispersion relation implies that the two sets of coastally trapped wave shown in (9) are dependent on each other. Therefore, the two sets of waves are dependent on each other if two shelves are close while they become independent single shelf waves if two shelves are separated far enough. The phase speeds c of two sets of waves range in $g\alpha/f \rangle c \rangle 0$ and $g\beta/f \langle c \langle 0$, respectively. For the range of $g\alpha/f \rangle c \rangle 0$, the set of waves propagate southward while for the range $g\beta/f \langle c \langle 0$, the set of waves propagate northward.

The dispersion relation looks complicated but its essential characteristics are the same as for a non-divergent case (Pang, 1992), except the 1st modes. It should be noted that a linear topography is adapted in this paper for analytic solutions to a divergent case. The (exponential or linear) bottom shape itself does not make any basic difference. In the divergence case, the phase speed of first modes shows different characteristics from that of the rest modes. It is comparable to the phase speed of gravity waves.

Table 1 shows phase speeds for 3 different cases of bottom topography : (A) a single shelf adjacent to an infinitely deep water region, (B) a single shelf adjacent to a region of the finite depth equal to the greatest depth of shelf, and (C) a double shlef. The greatest depth of shelf are $100m$ for all cases and the cases (A) and (C) have 2 subcases : a non-divergence case and a divergence case. The case (B) does not include a non-divergent case, since the divergence effect is essential to allow sea level fluctuations at the edge of shelf. Fig. 3 shows schematic representations of the cross-shelf bottom topography for the cases. From table 1, we can see 2 divergence effects.

One is for the 1st modes. For non-divergence cases, the phase speeds of the 1st modes are propotional to shelf width, which is a characteristics of continental shelf waves. However, For divergence cases, the phase speeds of the 1st modes vary inversely with

which yields two independent sets of waves as follows :

$$M_1/M_{1x} = |\ell|/L_1 (|\ell|-m) \text{ or } M_3 M_{3x} = |\ell|/ (L_2-L_m)(|\ell|-m) , \cdots\cdots\cdots\cdots\cdots\cdots (19)$$

since $M_{1x} = 2L_1 a_1 M_2$ and $M_{3x} = 2(L_2-L_m)a_3 M_4$. For non-trivial, a_1 and a_3 must be negative, which give the positive and negative phase speeds, respectively.

When L_m goes to zero (double shelf case) as shown in Fig.2(b), ity yields

$$M_1 M_3 - a_3 M_1 M_4 - a_1 M_2 M_3 = 0 . \cdots\cdots\cdots\cdots (20)$$

This can be changed as $1-a_3 M_4/M_3 = a_1 M_2/M_1 \equiv p$, therefore,

$$M_1 M_{1x} = p_1/2L_1 \qquad \text{if } a_1 \langle 0, \ a_3 \rangle 0 \quad (21\text{-}1)$$
$$M_3 M_{3x} = p_2/2(L_2-L_m)(p_2-1) \text{ if } a_1 \rangle 0, \ a_3 \langle 0 \quad (21\text{-}2)$$

where $p_1 = 1-a_3 M_4 M_3$ and $p_2 = a_1 M_2/M_1$. They are dependent on the both shelves through p.

In this case, the dispersion relations (17), (18), (21) include continental shelf waves, Kelvin waves, and Poincare waves, while in the preceeding case of low-frequency, only continental shelf waves and Kelvin waves are included. The confluent hypergeometric function is converted to various different functions, such as Bessel Function and Legendre Function, for some limiting cases. It can cover whole ranges of coastally trapped waves. However, since confluent hypergeometric function is still under development and computer programming of this function is extremely complicated, computation using the above equation is not practical yet.

Eigenfunctions

The eigenfunctions for the double shelf are as follows :

$$F = \begin{cases} F_1 & \text{in the shelf 1} \\ F_2 & \text{in the shelf 2} \end{cases}$$

$$F_1 = \begin{cases} A \cdot [u_2/a_1 T_1] \cdot [J_0(2a_2)Y_1(2a_2)-J_1(2a_2)Y_0(2a_2)] \\ \times[-G_{12}J_0(2\{u_1(x+L_1)\}^{\frac{1}{2}})+G_{11}Y_0(2\{u_1(x+L_1)\}^{\frac{1}{2}})] \text{ for } g\beta/f\langle c\langle 0 \\ A \cdot [J_0(2\{u_1(x+L_1)\}^{\frac{1}{2}})-(T_4/T_3)\cdot Y_0(2\{u_1(x+L_1)\}^{\frac{1}{2}})] \text{ for } g\alpha/f\rangle c\rangle 0 \end{cases}$$

$$F_2 = \begin{cases} A \cdot [J_0(2\{u_2(x-L_2)\}^{\frac{1}{2}})-(T_2/T_1)\cdot Y_0(2\{u_2(x-L_2)\}^{\frac{1}{2}}] \text{ for } g\beta/f\langle c\langle 0 \\ A \cdot [u_1/a_1 T_3] \cdot [J_0(2a_1)Y_1(2a_1)-J_1(2a_1)Y_0(2a_1)] \\ \times[-G_{22}J_0(2\{u_2(x-L_2)\}^{\frac{1}{2}})+G_{21}Y_0(2\{u_2(x-L_2)\}^{\frac{1}{2}})] \text{ for } g\alpha/f\rangle c\rangle 0 \end{cases}$$

where A is a arbitrary constant and
$$G_{11} = (f/c) \cdot J_0(2b_1)-(u_1/b_1) \cdot J_1(2b_1)$$
$$G_{12} = (f/c) \cdot Y_0(2b_1)-(u_1/b_1) \cdot Y_1(2b_1)$$
$$G_{21} = (f/c) \cdot J_0(2b_2)-(u_2/b_2) \cdot J_1(2b_2)$$
$$G_{22} = (f/c) \cdot Y_0(2b_2)-(u_2/b_2) \cdot Y_1(2b_2)$$

$$T_1 = [(u_1/a_1) \cdot J_1(2a_1)Y_0(2a_2)-(u_2/a_2) \cdot J_0(2a_1)Y_1(2a_2)] \cdot G_{12}$$
$$-[(u_1/a_1) \cdot Y_1(2a_1)Y_0(2a_2)-(u_2/a_2) \cdot Y_0(2a_1)Y_1(2a_2)] \cdot G_{11}$$
$$T_2 = [(u_1/a_1) \cdot J_1(2a_1)J_0(2a_2)-(u/_2a_2) \cdot J_0(2a_1)J_1(2a_2)] \cdot G_{12}$$
$$-[(u_1/a_1) \cdot Y_1(2a_1)J_0(2a_2)-(u_2/a_2) \cdot Y_0(2a_1)J_1(2a_2)] \cdot G_{11}$$
$$T_3 = [-(u_1/a_1) \cdot Y_1(2a_1)J_0(2a_2)+(u_2/a_2) \cdot Y_0(2a_1)J_1(2a_2)] \cdot G_{22}$$
$$-[-(u_1/a_1) \cdot Y_1(2a_1)Y_0(2a_2)+(u_2/a_2) \cdot Y_0(2a_1)Y_1(2a_2)] \cdot G_{21}$$
$$T_4 = [-(u_1/a_1) \cdot J_1(2a_1)J_0(2a_2)+(u_2/a_2) \cdot J_0(2a_1)J_1(2a_2)] \cdot G_{22}$$
$$-[-(u_1/a_1) \cdot J_1(2a_1)Y_0(2a_2)+(u_2/a_2) \cdot J_0(2a_1)Y_1(2a_2)] \cdot G_{21}$$
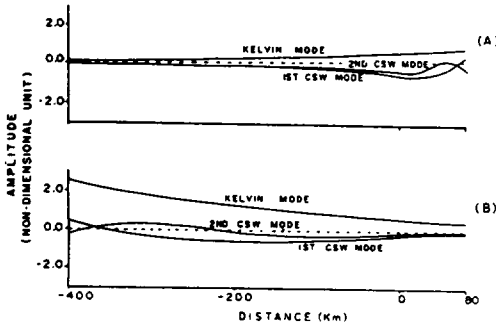
Fig. 4. The cross-shelf amplitudes of the first 3 eigenfunctions of shelf waves propagating (A) into paper (northward) and (B) out of paper (southward).

Fig.4 shows the amplitudes of the first 3 eifenfunctions across the shelf : (A) for $g\beta/g$ $\langle c \langle 0$ and (B) for $g\alpha/f \rangle c \rangle 0$. Thus the sets of waves in Fig.4 (A) and (B) propagate into paper along shelf 2 and out of paper along shelf 1, respectively. The 1st eigenfunctions have their maximum amplitudes along coast and decay exponentially across the whole shelf. They have no node, like Kelvin wave over flat bottom. They appear only when horizontal divergence is allowed. The rest eigenfunctions are continental shelf waves. The 2nd modes are the 1st modes of continental shelf waves, and so on. Continental shelf waves oscillate over one shelf and extend in an exponential decay over the other shelf. Therefore, they have nodes across the shelf. The 1st continental shelf mode has one node and the next mode has two nodes, and so on.

Orthogonality of eigenfunctions

Upon substituting for the pressure, $p = \sum\limits_{n=-\infty}^{\infty}$ $F_n(x) \phi_n(y+c_n t)$, the governing equation and

boundary conditions for a double shelf topography are as follows :

$$[\alpha(x+L_1)F_{1n}']' + (-\ell^2\alpha(x+L_1) + f\alpha/c_n + (\omega^2-f^2)/g) \cdot F_{1n} = 0$$

$$-L_1 \langle x \langle 0 \qquad (22\text{-}1)$$

$$[\beta(x-L_2)F_{2n}']' + (-\ell^2\beta(x-L_2) + f\beta/c_n + (\omega^2-f^2)g] \cdot F_{2n} = 0$$

$$0 \langle x \langle L_2 \qquad (22\text{-}2)$$

$$F_{1n}' + (f/c_n)F_{1n} = 0 \quad \text{at } x = -b_1 \qquad (23\text{-}1)$$

$$F_{1n} = F_{2n} \quad \text{at } x = 0 \qquad (23\text{-}2)$$

$$F_{1n}' = F_{2n}' \quad \text{at } x = 0 \qquad (23\text{-}3)$$

$$F_{2n}' + (f/c_n)F_{2n} = 0 \quad \text{at } x = B_2 \qquad (23\text{-}4)$$

Multiplying the equations (22-1) & (22-2) by F_m and integrating them across the shelves yields

$$\int_{-B1}^{0} F_{1m}[\alpha(x+L_1)F_{1n}']'dx + \int_{0}^{B2} F_{2m}[\beta(x-L_2)$$

$$F_{2n}']'dx + [f\alpha/c_n + (\omega^2-f^2)/g] \cdot \int_{-B1}^{0} F_{1m}F_{1n}dx + [f$$

$$\beta/c_n + (\omega^2-f^2)g] \cdot \int_{0}^{B2} F_{2m}F_{2n}dx - \ell^2\alpha \cdot \int_{-B1}^{0} (x$$

$$+L_1)F_{1m}F_{1n}dx - \ell^2\beta \cdot \int_{0}^{B2} (x-L_2)F_{1m}F_{1n}dx = 0$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (24)$$

By integration by parts and using the bounary conditions,

$$\alpha L_1 F_{1m}(0)F_{1n}'(0) + (f\alpha/c_n)(-B_1+L_1)F_{1m}(-B_1)$$

$$F_{1n}(-B_1) + \beta L_2 F_{2m}(0)F_{2n}'(0) + (f\beta/c_n)(-B_2+L_2)$$

$$F_{2m}(-B_2)F_{1n}(-B_2) - \int_{-B1}^{0} \alpha(X+L_1)F_{1m}'F_{1n}'dx - \int_{0}^{B2} \beta$$

$$(x-L_2)F_{2m}'F_{2n}'dx + [f\alpha/c_n + (\omega^2-f^2)/g] \cdot \int_{-B1}^{0}$$

$$F_{1m}F_{1n}dx + [f\beta/c_n + (\omega^2-f^2)/g] \cdot \int_{0}^{B2} F_{2m}F_{2n}dx - \ell^2$$

$$\alpha \cdot \int_{-B1}^{0} (x+L_1) F_{1m} F_{1n} dx - \ell^2 \beta \cdot \int_{0}^{B2} (x-L_2)$$

$$F_{1m} F_{1n} dx = 0 \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (25)$$

Interchanging m and n and substracting them yields

$$\alpha L_1 \cdot (F_{1m}(0) F_{1n}'(0) - F_{1m}'(0) F_{1n}(0)) + \beta L_2 \cdot$$

$$(F_{2m}(0) F_{2n}'(0) - F_{2m}(0)' F_{3n}(0)) + (f/c_n - f/c_m \cdot (\alpha$$

$$\int_{-B1}^{0} F_{1m} F_{1n} dx + \beta \int_{0}^{B2} F_{2m} F_{2n} dx + \alpha (L_1 - B_1) F_{1m} (-B$$

$$_1) F_{1n}(-B_1) + \beta (L_2 - L_m) F_{2m}(B_2) F_{2n}(B_2)) = 0. \cdots (26)$$

The first two terms are zero by the boundary conditions. Therefore, when m is not n,

$$\alpha \int_{-B1}^{0} F_{1m} F_{1n} dx + \beta \int_{0}^{B2} F_{2m} F_{2n} dx - HF_m F_n |_{-B1}^{B2}$$

$$= 0. \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (27)$$

To see if (27) does not hold when m=n, suppose to the contrary that

$$\alpha \int_{-B1}^{0} F_{1n}^2 dx + \beta \int_{0}^{B2} F_{2n}^2 dx - HF_n^2 |_{-B1}^{B2}$$

$$= 0. \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (28)$$

Multiplying (22-1) by F_{1n} and (22-2) by F_{2n} and integrating them across the shelf yields the following equation by the relation (28)

$$\int_{-B1}^{0} F_{1n} \cdot (\alpha (\alpha x + L_1) F_{1n}')' dx + \int_{0}^{B2} F_{2n} \cdot (\beta (x-$$

$$L_2) F_{2n}')' dx - (f^2/g) \int_{-B1}^{B2} F_n^2 dx + (f/c_n) \cdot HF_n^2 |_{-B1}^{B2}$$

$$= 0.$$

Integrating by parts and using the boundary conditions (23) leads us to

$$\int_{-B1}^{B2} (HF_n'^2 dx + (f^2/g) \cdot F_n^2) dx = 0. \cdots\cdots (29)$$

Since the integrand $HF_n'^2 dx + (f^2/g) \cdot F_n^2$ is always positive, the assumption (28) creats a contradiction. Therefore, the asumption (28) does not hold when m=n. This leads us, with (27), to the following orthogonality condition.

$$\frac{\alpha \int_{-B1}^{0} F_{1m} F_{1n} dx + \beta \int_{0}^{B2} F_{2m} F_{2n} dx - HF_m F_n |_{-B1}^{B2}}{\alpha \int_{-B1}^{0} F_{1n}^2 dx + \beta \int_{0}^{B2} F_{2n}^2 dx - HF_n^2 |_{-B1}^{B2}} = \delta_{mn}$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (30)$$

This proves that the eigenfunctions of coastally trapped wave over a double shelf are also orthogonal. From the orthogonality of eigenfunction, the method of eigenfunction expansion is possible for the forced problem. (The frictional eigenfunctions are not proved to be orthogonal, which is a difficulty in using the frictional eigenfunctions)

Discussion and conclusion

for a single shelf adjacent to deep open ocean, the divergence effect might not be important. It depends on what kind of phenomenon we are looking at. For wind driven coastal motions, the divergence effect becomes important only when the shelf width is comparable to the Rossby defromation radius. However, for a double shelf, which allows sea level fluctuations at any place across channel, it is always important.

The divergence effect adds Kelvin wave to the solution of coastally trapped waves. As over a flat bottom, the amplitude of Kelvin wave has its maximum and decay exponentially away from coast. Thus it has no node across the shelf. Its phase speed is modified by bottom slope, but essentially comparable to that of gravity wave. As a

shelf is wider, the averaged depth is shallower, and therefore the phase speed of Kelvin wave is slower. This makes that the phase speed of Kelvin wave vary inversely with shelf width. On the other hand, phase speed of continental shelf waves is propotional to shelf width.

As for non-divergence case, there are also two sets of waves for divergence case. Each set include on Kelvin mode and infinite modes of continental shelf waves. The phase speeds of one set are positive and those of the other set are negative. The phase speeds of the Kelvin waves are much faster than that those of the continental shelf waves. The phase speeds of the shelf waves are slower for the higher mode. Thus, the 1st modes, which have the maximum phase speeds, are the Kelvin waves, and the 2nd modes are the 1st modes of continental shelf waves, and so on. The nth modes have n-1 nodes across the

shelf. All the waves propagate with shallow waters to the right in Northern Hemisphere.

The two sets of waves are independent if two shelves are apart sufficiently and dependent on the geometry of both shelves if two shelves are close enough. The frictionless eigenfunctions of coastally trapped waves over a double shelf are proved to be orthogonal. It makes the method of eigenfunction expansion to be possible for the forced problem.

The dispersion relation is extended to high-frequency and short waves. Even for the extended case, the basic characteristics are preserved. The dispersion relation covers some limiting cases, such as low-frequency and long wave case. The extension is necessary for studies of reflection and scattering, etc.. The practical computations, however, are not inaccurate yet since confluent hypergeometric function is not fully developed.

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〈國文抄錄〉

황해와 같은 선형 양향성 대륙붕에서의 자유 대륙붕파

황해의 양향성 대륙붕 해저지형에서 저주파의 분산관계가 우선 장파에 대해 유도되었고, 다음에 단파로 확장되었다. 확장된 경우에는 Hypergeometric 함수가 유도된다. 단일한 대륙붕에서는 해안선과 수직한 방향의 대륙붕규모가 Rossby deformation radius에 비해 많이 작을 때는 수평흐름의 수렴·발산효과가 무시되지만 양향성 대륙붕에서는 수평흐름의 수렴·발산효과가 해안선과 수직한 방향의 대륙붕규모에 관계없이 파동역학에 결정적으로 중요하다. 비마찰 eigenfunction들의 직교가 증명되었다.