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내부 원통이 급격히 회전하는 동심 원통 사이의 액체에서 Taylor 와류의 발생 조건

김 민 찬* · 이 상 백*

The Onset Condition of Taylor Vortex in Liquid Layer by an Impulsively Started Rotating Inner Cylinder

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ABSTRACT

The onset of instability in the flow by an impulsively started rotating cylinder is analyzed under linear theory. It is well-known that at the critical Taylor number $T_c = 1695$ instability motion sets in under the narrow-gap approximation. Here the dimensionless critical time τ_c to mark the onset of instability motion for $T \gg T_c$ is presented as a function of the Taylor number T. Available experimental data of water indicates that deviation of the velocity profiles from their momentum diffusion occurs starting from a certain time $\tau \cong 4\tau_c$. It seems evident that during $\tau_c \le \tau \le 4\tau_c$ convective motion is very weak and the laminar diffusive momentum transfer is dominant.

Key Words : Convective Instability, Taylor-Like Vortex, Propagation Theory

Ⅰ. Introduction

The onset of instability induced by an impulsively started rotating cylinder was first investigated experimentally by Chen and Christensen[1]. The initial laminar flow evolves into a secondary flow pattern which consists of a series of Taylor-like vortices. In this transient boundary-layer flow system the critical time t_c to mark the onset of secondary motion becomes an important question. This problem may be called an extension of Taylor instability. The related instability analysis has been conducted by using theamplification theory[2], the frozen-time model [2], and the maximum-Taylor-number criterion [3]. The first model requires the initial conditions and the criterion to define manifest convection. The second model is based on linear theory and yields the critical time as the parameter. The third model is the simplest one, which is based on the transient,

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developing Couette flow only. These models take advantage of the similarity between Taylor instability and Rayleigh-Bnard instability.

Here we will extend the propagation theory, which has been employed to analyze time-dependent Rayleigh-Benard problem, into the instability problem of flow induced by an impulsively started rotating cylinder. The resulting theoretical results will be compared with available experimental data.

II. Stability Analysis

The system considered here is a Newtonian fluid confined between concentric cylinders of radii R_1 and $R_2(>R_1)$. Let the axis of inner cylinder be along the z' axis of a cylindrical coordinate system (r', θ, z'). For time $t \geq 0$, the inner cylinder is impulsively started and maintained at a constant surface speed $V_0'(=R_1\Omega_1)$ and outer cylinder is kept stationary $\Omega_2=0$, where Ω_1 and Ω_2 are the angular velocities of inner and outer cylinder, respectively. The schematic diagram of the basic system is shown in Fig. 1.

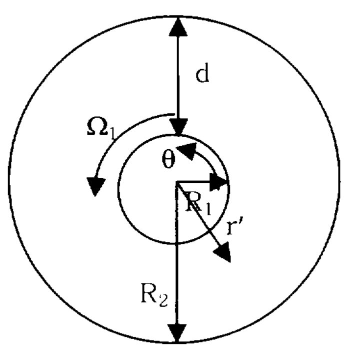


Fig. 1. Schematic diagram of system considered here.

For a high V_0', secondary motion will set at a certain time and the governing equations of flow field is expressed as

$$\nabla \cdot \vec{U}=0, \tag{1}$$

$$\left\{\frac{\partial}{\partial t}+\vec{U}\cdot\nabla\right\}\vec{U}=-\frac{1}{\rho}\nabla P+\nu\nabla^2\vec{U} \tag{2}$$

where $\vec{U}$, P, ν and ρ represent the velocity vector, the dynamic pressure, the kinematic viscosity and the density respectively. For small t, the basic velocity field is represented by

$$V_0=\operatorname{erfc}\left\{\frac{y}{\sqrt{4\kappa t}}\right\} \tag{3}$$

For small t, by neglecting the effect of curvature, i.e. employing narrow-gap approximation, the above Eqs. (1) and (2) can be linearized and the resulting dimensionless disturbance equations of tow-dimensional flow using Eq. (3) are represented by

$$\left(\frac{\partial^2}{\partial y^2}-a^2-\frac{\partial}{\partial\tau}\right)\left(\frac{\partial^2}{\partial y^2}-a^2\right)u=2V_0a^2v \tag{4}$$

$$\left(\frac{\partial^2}{\partial y^2}-a^2-\frac{\partial}{\partial\tau}\right)v=\mathrm{Tau}\frac{\partial V_0}{\partial y} \tag{5}$$

with proper boundary conditions,

$$u=\partial u/\partial y=v=0 \qquad y=0 \quad \text{and} \quad 1 \tag{6}$$

where $\tau=\nu t/d^2$, $u=d^2u'/(\nu R_i)$, $v=v'/V_0'$, $V_0=V/V_0'$ $y=(r-R_i)/d$ and $d=R_o-R_i$. The subscript '0' denotes the basic state and a represents the dimensionless vertical wavenumber. It should be noted that the radial velocity component u' is nondimensionalized by $\nu R_i/d^2$ rather than V_0'. In the present system the most important parameter is the Taylor number Ta defined as

$$\mathrm{Ta}=\frac{V_0'^2d^3}{\nu^2R_1} \tag{7}$$

Based on the balance between viscous and Coriolis terms, we set $u=\tau u^*(\zeta)$ and $v=v^*(\zeta)$. For a boundary-layer flow system of $\delta\propto\sqrt{\tau}$, the dimensionless time τ plays dual roles of time and length. Here δ denotes the boundary-layer thickness. Now, the self-similar stability equations are obtained from Eqs. (4) and (5) as

$$\left\{(D^2-a^{*2})^2+\frac{1}{2}(\zeta D^3-a^{*2}\zeta D+2a^{*2})\right\}=2V_0 a^{*2}v^* \quad (8)$$

$$\left(D^2+\frac{1}{2}\zeta D-a^{*2}\right)=\mathrm{Ta}^*u^*V_0 \quad (9)$$

where $\zeta=y/\sqrt{\tau}$, $D=d/d\zeta$, $\mathrm{Ta}^*=\tau^{3/2}\,\mathrm{Ta}$ and $a^*=a\sqrt{\tau}$. The proper boundary conditions of no-slip are

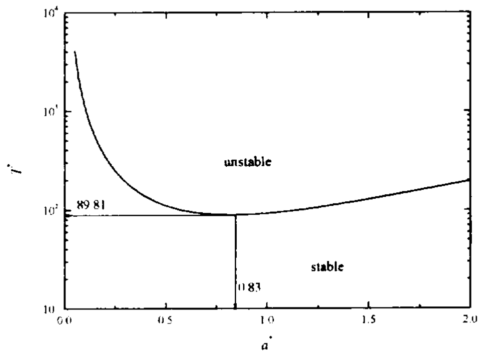
$$u^*=Du^*=v^*=0 \quad \text{at } \zeta=0 \text{ and } \infty \quad (10)$$

For a given τ, Ta^* and a^* are treated as eigenvalues and the minimum value of Ta^* should be found in the plot of Ta^* vs. a^* under the principle of exchange of stabilities.

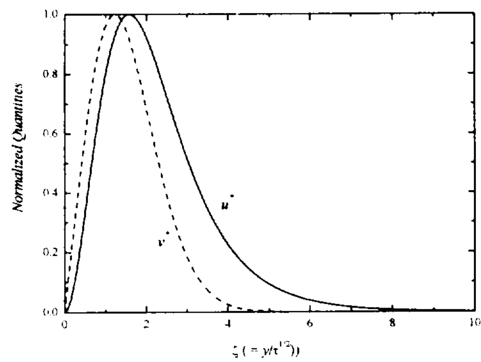
III. Results and Discussions

By using outward shooting method with Newton-Raphson iteration, we solve the above stability equations and obtain the marginal stability curve. Based on the result of Figure 2(a), the critical conditions to mark the onset of secondary motion is given by

$$\tau_c=18.84\ \mathrm{Ta}^{-2/3} \text{ and } a_c=0.19\ \mathrm{Ta}^{1/3} \quad \text{for } \tau\to 0 \quad (11)$$



(a)



(b)

Fig. 2. Instability conditions for small time of $\tau_c\to 0$ from propagation theory; (a) marginal stabilitycurve and (b) amplitude profiles at $\tau=\tau_c$

The resulting normalized amplitude functions of u^* and v^* are shown in Figure 2(b). For a given Ta, a fastest growing mode of infinitesimal disturbances would be set in at $\tau=\tau_c$ with $a=a_c$. The above equations show that τ_c decreases with an increase in Ta. Figure 3 illustrates that the present predictions of $4\tau_c$ ($\eta\to 1$) compares well with Liu's [5] experimental data ($\eta=0.2$) marking the detection of manifest motion. Here η represents the ratio (R_i/R_o). The agreement of experimental data with the amplification theory and Tan and Thorpe's model[3] is also good but the latter model requires further justification.

Shen[6] suggested the momentary instability condition: the temporal growth rate of the perturbation quantity (r_1) should exceed that of the base flow (r_0). In the present system the dimensionless growth rates are defined as the root-mean-squared quantities of angular velocity components:

$$r_0 = \frac{1}{\langle V_0 \rangle} \frac{d\langle V_0 \rangle}{d\tau} \quad (12a)$$

$$r_1 = \frac{1}{\langle v' \rangle} \frac{d\langle v' \rangle}{d\tau} \quad (12b)$$

where $\langle quantity \rangle \geq \sqrt{\left(\int_A (quantity)^2 dA\right)/A}$ and $A = Sdr'$ with $S = \pi d / a_c$. From the distributions of the base flow(Eq. (3)) and the perturbation quantities, we can obtain the following relation:

$$r_0 = r_1 = \frac{1}{4\tau_c} \text{ for } \tau \to 0 \quad (13)$$

The above equation indicates that propagation theory bounds the momentary stability conception. Foster[7] commented that $\tau_o \cong 4\tau_c$ for the time-dependent Rayleigh-Benard problem. This means that a fastest growing mode of instabilities, which set in at $\tau = \tau_c$, will grow with time until manifest convection is detected at $\tau = \tau_c$. Chen and Kirchner[2] reported similar trend for the present time-dependent flow system. According to their results, the time of intrinsic instability ($\tau = \tau_i$), i.e. the time at which the disturbances first tend to grow, is about one-fourth of the time at which the instability motion is clearly observable experimentally. A growth period will be required, as illustrated in Fig. 3. This scenario is supported by the results from the amplification theory (τ_i and $\tau_3 (= \tau_o)$). A more refined study including η-effect is now in progress.

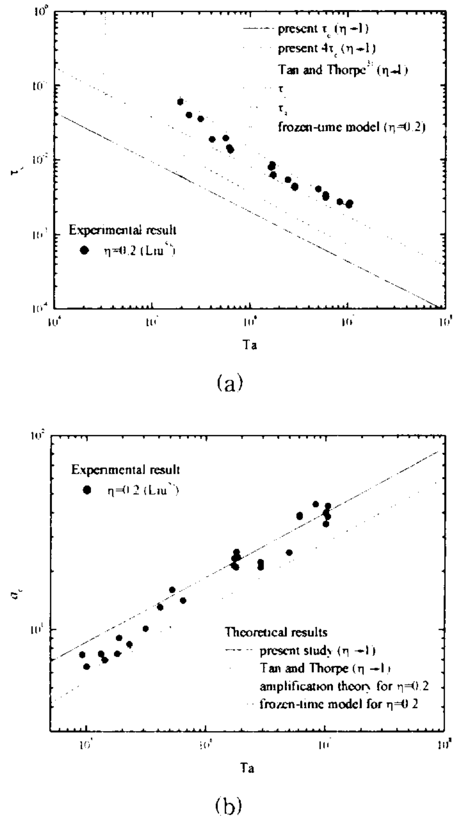


Fig. 3. Comparison of predictions with experimental data of $\eta = 0.2$: For $\eta = 0.1$, τ_i and τ_3 from Chen and Kirchner2): (a) onset time and (b) critical wave number.

III. Conclusions

The onset of secondary motion in the flow by an impulsively started rotating cylinder has been analyzed by using linear stability theory. The propagation theory has been employed to predict the critical time τ_c to mark the onset of convective instability. Even though the propagation theory is a rather simple model, the relation of $\tau_m \simeq 4\tau_c$ is consistent with experimental measurements. The present results show that the infinitesimal disturbance sets in at $\tau = \tau_c$ and grows until detected around $\tau \cong 4\tau_c$. This means that secondary motion is very weak during $\tau_c \leq \tau \leq \tau_m$. More refined studies on the η-effect and the nonlinear growth of disturbances are under progress.

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