Divergences in the Theory of Cyclotron Resonance Lineshape

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사이클로트론 공명 흡수선 이론의 발산에 대하여

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Summary

A new theory of cyclotron resonance lineshape for the system of electrons in interaction with impurities or phonons is introduced. The collision term appearing in the Kubo formalism is dealt with in the relaxation time approximation. The many body conductivity tensor is reduced to two different forms in the single particle formalism. But the two conductivity formulas give the same lineshape function, implying that the two techniques are identical. The thermal relaxation time included in the lineshape function is finite, resulting in the disappearance of any danger of divergence in some higher order terms usually excluded in the formula.

摘　　要

불순물이나 포논과 상호작용하는 전자계에 대한 전기전도도텐서와 사이클로트론 공명 선모양 함수를 도출하였다. 전기전도도텐서를 얻는 과정에서 Kubo공식을 이용하는데 타이론에서 무시한 보정항인 충돌항을 고려하고 이 항을 열적이완시간과 관련시켜 일반적인 형태를 얻었다.

섭동항을 계산하는데 Argyres와 Sigel의 사영연산자방법과 본 논문에서 정의한 사영연산자를 토대로 한

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운동방정식방법을 다루었는데 이들은 서로 동일한 결과로 나타남을 알 수 있었다.

또한 최저차수에서 구한 선모양 함수를 타이론의 결과와 비교했고 고차항에서는 타이론에서 발산하는 것과는 달리 열적이완시간이 포함됨으로써 발산항이 존재하지 않음을 알 수 있다.

1. Introduction

The study of cyclotron resonance lineshape is known to be a powerful tool of investigating electronic structure of solids. The absorption lineshapes in semiconductors are typically broadened by electron-impurity and electron-phonon interactions.

In many different classes of theory [Argyres et al., Badjou et al., Choi et al., Kawabata, Lodder et al., Prasad, Ryu et al., and Suzuki et al.] the kinetic equation approach including the Kubo formalism based on the linear response theory is fundamental to our understanding of effects of interaction on the absorption linewidths and frequency shifts. In dealing with the perturbation, projection techniques and diagram methods are frequently adopted [Argyres et al., Badjou et al., Kawabata, Lodder et al., Prasad, Ryu et al., and Suzuki et al.]. The lineshape functions in this category are usually given in the perturbative expansions in powers of the scattering strength. Argyres et al., however, claim that correct expression for the lineshape function cannot be expanded as such, because some higher order terms diverge for frequencies near the cyclotron frequency.

The purpose of the present work is to reformulate the former theories [Choi et al.] in such a way that the expansion is valid even at the resonance peaks. This paper is organized as follows. In Sec. 2 and 3, we review the Kubo formalism for the electrons in the impurity and/or phonon fields. In Sec. 4, we formulate the theory of cyclotron resonance lineshape by using the two projection techniques [Argyres et al. and Ryu et al.], and show that the expansions do not contain any divergent terms. The last section is devoted to discussion about the formalism. Comparison with some other theories is also made.

2. The Kubo formalism for the Electron Conductivity in the Many-body Formalism

Consider a system of many electrons subject to a time-dependent external electric field of amplitude $\vec{E}$ and angular frequency ω given by

$$\vec{E}(t) = \vec{E} exp(i\omega t). \tag{2.1}$$

Then, the total Hamiltonian of the system is

$$H_T(t) = H + H_1(t), \tag{2.2}$$

$$H_1(t) = -\sum_{i=1}^{\bar{N}} e\vec{r}_i \cdot \vec{E}(t), \tag{2.3}$$

where H denotes the Hamiltonian operator corresponding to the time-independent part, $\bar{N}$ the total number of electrons in the system, and $\vec{r}_i$ the i-th electron position vector.

We suppose that the external field is initially absent and the system is in thermodynamic equilibrium with a temperature T. The initial state can be described in terms of the grand canonnical density operator

$$\rho_0 = \rho(-\infty) = exp(\alpha\mathcal{N} - \beta H)/\Xi \qquad (2.4)$$

with

$$\Xi = Tr\{exp(\alpha\mathcal{N} - \beta H)\}. \qquad (2.5)$$

Here $\alpha \equiv \beta\zeta$ and $\beta \equiv (k_B T)^{-1}$ where k_B and ζ, respectively, are the Boltzmann constant and the chemical potential, and Tr denotes the many-body trace.

We also assume that the system is not so far from the equilibrium. Then we may take

$$\rho(t) = \rho_0 + \rho_1(t). \qquad (2.6)$$

and the Liouville equation $i\partial\rho(t)/\partial t = [H_T(t), \rho(t)] + i(\partial\rho(t)/\partial t)_{coll.}$ can be written as [Jones et al.]

$$i\partial\rho_1(t)/\partial t = [H, \rho_1(t)] + [H_1(t), \rho_0] + i(\partial\rho(t)/\partial t)_{coll.} \qquad (2.7)$$

in units in which $\hbar = 1$, where we have used the fact that $[H, \rho_0] = 0$ and neglected the term $[H_1(t), \rho_1(t)]$ in approximation. The last term on the right-hand side represents the interaction with the surroundings during the time which is reached from equilibrium state to nonequilibrium state by the external field. Here we may adopt the "Ansatz" that the collision term appears as a result of the thermal relaxation in the existance of the external field. We then have

$$i(\partial\rho(t)/\partial t)_{coll.} = -i\rho_1(t)/\tau_t, \qquad (2.8)$$

where $\tau_t \approx [k_B(T_e - T)]^{-1}$ is the thermal relaxation time which depends on the difference between the electron temperature T in the equilibrium state and the electron temperature T_e in the nonequilibrium state by the external field since the electron energies between before and after the external field differ under the condition that the background temperatures T are kept.

In order to obtain $\rho_1(t)$, we now define the density operator in the Dirac picture as

$$\rho_{1D}(t) = exp(iHt)\rho_1(t)exp(-iHt). \qquad (2.9)$$

Differentiating Eq. (2.9) and considering Eqs. (2.7) and (2.8), we obtain

$$i\partial\rho_{1D}(t)/\partial t = [H_{1D}(t), \rho_0] - i\rho_{1D}(t)/\tau_t, \qquad (2.10)$$

where

$$H_{1D}(t) = exp(iHt)H_1(t)exp(-iHt). \qquad (2.11)$$

Integrating Eq. (2.10) and taking into account Eq. (2.9) we have

$$\rho_1(t) = (-i)\int_{-\infty}^{t} ds\ exp[-(t-s)/\tau_t] \times exp\{-iH(t-s)\}[H_1(s), \rho_0]exp(iH(t-s)). \qquad (2.12)$$

On the other hand, the average current density can be calculated in terms of the density operator as

$$< \Delta\vec{J} >= Tr[\vec{J}\rho_1(t)], \qquad (2.13)$$

where $\vec{J}$ is the total current operator given by

$$\vec{J} = \sum_{i=1}^{\bar{N}} e(d\vec{r}_i/dt), \qquad (2.14)$$

and $\Delta\vec{J}$ is the total current minus the static value. By using Eqs. (2.3), (2.12) and (2.1) succesively, the k-component of Eq. (2.13)

can be written in the following form :

$$< \Delta J_k >= \sum_l \hat{\Sigma}_{kl}(\bar{\omega}) E_l(t). \qquad (2.15)$$

where l,k=x,y,z and

$$\hat{\Sigma}_{kl}(\bar{\omega}) = FLT(\Sigma_{kl}(t)). \qquad (2.16)$$

Here FLT(f(t)) is the Fourier Laplace transform (FLT) of f(t) defined as

$$FLT(f(t)) \equiv \int_0^\infty f(t) exp(-i\bar{\omega}t) dt, \qquad (2.17)$$

$$\bar{\omega} = \omega - i/\tau_t, \qquad (2.18),$$

and

$$\Sigma_{kl}(t) = (-i) Lim(1/\Omega_0) Tr([\rho_0, \sum_{i=1}^{\bar{N}} e\vec{r}_{il}] J_k(t)), \qquad (2.19)$$

where Ω_0 is the volume of the system, and "Lim" denotes the bulk limit. By considering Eq.(2.14), the time-dependent response function $\Sigma_{kl}(t)$ can be reduced to

$$\Sigma_{kl}(t) = Lim(1/\Omega_0) Tr\{(1/2\pi i) \int_c dz\ exp(\alpha\bar{N} - \beta z) \times (H-z)^{-1}\ J_l\ (H-z)^{-1} J_k(t)/\Xi\}. \qquad (2.20)$$

where c is any simple closed curve enclosing the poles and

$$J_k(t) = exp(iHt) J_k exp(-iHt). \qquad (2.21)$$

Adopting the approximation $H \approx H_0$ in Eq. (2.20), using Cauchy's integral formula, and considering Eq.(2.16) we have

$$\hat{\Sigma}_{kl}(\bar{\omega}) = \sum_{m,n} \frac{F(E_m) - F(E_n)}{E_m - E_n} \times < n \mid J_l \mid m >< < m \mid \hat{J}_k(\bar{\omega}) \mid n >>_B, \qquad (2.22)$$

where $F(E_n)$ is the many-body distribution function for the state $|n\rangle$ with the energy eigenvalue E_n. $\langle A \rangle_B$ denotes the average of A over backgrounds, and

$$\hat{J}_k(\bar{\omega}) = \int_0^\infty exp(-i\bar{\omega}t) J_k(t). \qquad (2.23)$$

3. Reduction to the Single Electron Formalism

By using the Kubo identity, Eq.(2.19) is reduced to

$$\Sigma_{kl}(t) = Lim(1/\Omega_0) \int_0^\beta Tr\{\rho_0 J_l(-i\beta_1) J_k(t)\} d\beta_1, \qquad (3.1)$$

which can be changed, by using the modified Hamiltonian formalism (Fujita), into

$$\Sigma_{kl}(t) = Lim(1/\Omega_0) \lim_{u_l \to 0} \partial/\partial u_l < Tr^e(\rho_0' J_k(t)) >_B, \qquad (3.2)$$

where Tr^e is the many electron trace and

$$\rho_0' \equiv exp(\alpha\bar{N} - \beta H')/Tr^e\{exp(\alpha\bar{N} - \beta H')\}, \qquad (3.3)$$

$$H' = H - \vec{J} \cdot \vec{u} \qquad (3.4)$$

with $\vec{u}$ being a constant vector.

Let us introduce the number operator in terms of the creation and annihilation operators as

$$\hat{n}_{\nu\mu} = a_\mu^+ a_\nu \qquad (3.5)$$

which can be expanded on the following basis :

$$a_\mu^\dagger = \sum_\alpha a_\alpha^\dagger < \alpha \mid \mu >, \tag{3.6}$$

$$a_\nu = \sum_{\alpha'} a_{\alpha'} < \nu \mid \alpha' > . \tag{3.7}$$

Then J_k and H can be expressed in the second quantized formalism as

$$J_k = \sum_\mu \sum_\nu < \mu \mid j_k \mid \nu > \hat{n}_{\nu\mu}. \tag{3.8}$$

and

$$H = H_B + \sum_\mu \sum_\lambda < \lambda \mid h \mid \mu > \hat{n}_{\mu\lambda}. \tag{3.9}$$

Here j_k is the single electron current operator, H_B the background (impurity and/or phonon) Hamiltonian, and h the single electron Hamiltonian which is the unperturbed electron Hamiltonian h_0 plus the scattering potential V :

$$h = h_0 + V. \tag{3.10}$$

In representation in which $H'-\zeta\bar{N}$ is diagonal we may write

$$H' - \zeta N = \sum_\gamma (h'_\gamma - \zeta)\hat{n}_{\gamma\gamma} + H_B, \tag{3.11}$$

$$h'_\gamma \equiv h'_{\gamma\gamma} \tag{3.12}$$

and

$$h' \equiv h - \vec{j}\cdot\vec{u}. \tag{3.13}$$

Then, the main part of Eq. (3.2) becomes

$$Tr^e(\rho'_0 J_k(t)) = tr^e(j_k(t)\bar{n}). \tag{3.14}$$

Here "tr^e" means the single electron trace and

$$j_k(t) = exp[it(h^\times + H_B^\times)]j_k. \tag{3.15}$$

$$\bar{n} = [exp\{\beta(h' - \zeta)\} + 1]^{-1}, \tag{3.16}$$

where $h^\times$ and $H_B^\times$ are the Liouville operators corresponding to h and H_B, respectively. By making the approximation $\bar{n} \approx \bar{n}_0$ in Eq. (3.14) which corresponds to $h \approx h_0$ [Argyres et al., Choi et al., and Kawabata] and considering Eqs. (3.2), (3.14) and (2.15), we obtain

$$< \Delta J_k > = \sum_{i=1}^{N} < \Delta j_k >^{(i)} = N \sum_l \tilde{\sigma}_{kl}(\bar{\omega}) E_l(t) \tag{3.17}$$

for the single electron formalism, and the conductivity tensor $\tilde{\sigma}_{kl}(\bar{\omega})$ is given by

$$\tilde{\sigma}_{kl}(\bar{\omega}) = (-i) \sum_{\alpha,\beta} \frac{f(\epsilon_\beta) - f(\epsilon_\alpha)}{\epsilon_\beta - \epsilon_\alpha}$$
$$< \alpha \mid j_l \mid \beta >< < \beta \mid \hat{R}_k(\bar{\omega}) \mid \alpha >>_B . \tag{3.18}$$

Here $f(\epsilon_\alpha)$ is the Fermi distribution function for the single electron with the energy ϵ_α in the state $|\alpha\rangle$, and

$$\hat{R}_k(\bar{\omega}) = (\bar{\omega} - h^\times - H_B^\times)^{-1} j_k. \tag{3.19}$$

On the other hand, by applying Kawabata's approach which is based on the Kubo formalism and the Mori method of calculation, we can also obtain a similar form of the conductivity in the single electron formalism as follows :

$$\tilde{\sigma}_M(\bar{\omega}) = \sum_{\alpha,\beta} \frac{f(\epsilon_\beta) - f(\epsilon_\alpha)}{\epsilon_\beta - \epsilon_\alpha}$$
$$\times < < FLT(< \alpha \mid j_l \mid \beta >< \beta \mid j_k(t) \mid \alpha >) >_B . \tag{3.20}$$

Here, if we let $\beta = \alpha + 1$ and $\tau_t \to \infty$, Eq. (3.20) is reduced to that of Kawabata's cyclotron resonance formula. In fact, Eq.

(3.18) can be obtained by calculating Eq. (3.20). Thus the two approaches ultimately give the same result. In the next section we will present two different methods, starting with Eqs. (3.18) and (3.20).

4. Formulation of the Theory

(a) Description of the System

It is well-known that when a static magnetic field is applied along the z-direction of an isotropic semiconductor, characterized by the vector potential $\vec{A}=(0,\ Bx,\ 0)$, the Hamiltonian of the unperturbed electron is given by

$$h_0 = [p_x^2 + (p_y + m\omega_0 x)^2 + p_z^2]/(2m), \qquad (4.1)$$

where $\vec{p}$ and m, respectively, are momentum and the effective mass of the conduction electron and ω_0 is the cyclotron frequency. The eigenvalues ϵ_α and the eigenstates $|\alpha\rangle$ corresponding to h_0 are characterized by the Landau index N and the electron wave vector $\vec{k}$:

$$\epsilon_\alpha \equiv \epsilon_{N,k} = (N + 1/2)\omega_0 + k_z^2/(2m), \qquad (4.2)$$

$$|\alpha\rangle = |N,\vec{k}\rangle$$
$$= A\, exp(iyk_y + izk_z) H_N[(x - X)/r_0]$$
$$\times exp[-(x - X)^2/2r_0^2], \qquad (4.3)$$

where

$$A \equiv (2^N N! r_0 \sqrt{\pi} L_y L_z)^{-1/2}, \qquad (4.4)$$

$$r_0 \equiv (m\omega_0)^{-1/2}, \qquad (4.5)$$

$$X \equiv -k_y/(m\omega_0). \qquad (4.6)$$

Here H_N is the Hermite polynomial and L_y and L_z, respectively, are the normalization lengths in the y- and z-directions.

For the phonon background we have

$$V = \sum_{\vec{q}} (\gamma_{\vec{q}} b_{\vec{q}} + \gamma_{\vec{q}}^{+} b_{\vec{q}}^{+}), \qquad (4.7)$$

$$\gamma_{\vec{q}} = C_{\vec{q}} exp(i\vec{q}\cdot\vec{r}), \qquad (4.8)$$

$$H_B = H_p = \sum_{\vec{q}} \omega_{\vec{q}} b_{\vec{q}}^{+} b_{\vec{q}}, \qquad (4.9)$$

where $b_{\vec{q}}^{+}$ and $b_{\vec{q}}$, respectively, are the creation and annihilation operators of a phonon with monentum $\vec{q}$ and energy $\omega_{\vec{q}}$ and $C_{\vec{q}}$ is the interacton operator.

For the impurity background we have

$$V = \sum_{\vec{q}} \hat{V}_{\vec{q}} exp(i\vec{q}\cdot\vec{r}), \qquad (4.10)$$

$$H_B = H_{imp} = 0, \qquad (4.11)$$

where $\hat{V}_{\vec{q}}$ is the interaction operator in the momentum representation.

(b) The Theory of Lineshape

For the circularly polarized electromagnetic wave of amplitude E and frequency ω, the absorption power is given by [Kawabata]

$$P = (E^2/2) Re[\tilde{\sigma}_{+-}(\bar{\omega})]. \qquad (4.12)$$

where "Re" means "the real part of" and the explicit form of the conductivity tensor $\tilde{\sigma}_{+-}(\bar{\omega})$ for the cyclotron transition can be obained by taking $\beta=\alpha+1$ either in Eq. (3.18) or in Eq. (3.20).

Model A

For $\beta=\alpha+1$, which is the condition for the cyclotron transition, we have from Eq. (3.18)

$$\tilde{\sigma}_{--}(\bar{\omega}) = (-i\omega_0)^{-1}\sum_{\alpha}[f(\epsilon_\alpha+\omega_0)-f(\epsilon_\alpha)](j_\alpha^-)^* \times <[\hat{R}(\bar{\omega})]_\alpha>_B , \tag{4.13}$$

where

$$j^{\pm} \equiv j_x \pm i j_y , \tag{4.14}$$

$$X_\alpha \equiv <\alpha+1 \mid X \mid \alpha> . \tag{4.15}$$

$$\hat{R}(\bar{\omega}) = (\bar{\omega} - h^\times - H_B^\times)^{-1} j^+ . \tag{4.16}$$

With the use of the projection operators P and P′ defined as

$$PX = j^+(X_\alpha / j_\alpha^+) , \tag{4.17}$$

$$P' = 1 - P , \tag{4.18}$$

by Argyres and Sigel, we have the following relations :

$$(h_0^\times P'X)_\alpha = 0 , \tag{4.19}$$

$$\sum_\beta (P'X)_{\alpha+1,\beta} = \sum_{\beta(\neq\alpha)} X_{\alpha-1,\beta} , \tag{4.20}$$

$$(h_0 j^+)_\alpha = \omega_0 j_\alpha^+ . \tag{4.21}$$

Then from Eq. (4.16), we obtain

$$[\hat{R}(\bar{\omega})]_\alpha = j_\alpha^+[(\bar{\omega}-\omega_0) - (V_{\alpha+1,\alpha+1} - V_{\alpha\alpha}) - i\tilde{B}_\alpha(\bar{\omega})]^{-1} , \tag{4.22}$$

where

$$j_\alpha^+(i\tilde{B}_\alpha(\bar{\omega})) = <\sum_{n=0}^{\infty}[V^\times SG_0(P'V^\times SG_0)^n P'(h^\times + H_B^\times)j^+]_\alpha>_B . \tag{4.23}$$

Here the factor S is given by

$$S = 1 \qquad (\textit{for impurity scatterings}), \tag{4.24a}$$

$$S = \sum_{m=0}^{\infty}(G_0 H_p^\times)^m \quad (\textit{for phonon scatterings}), \tag{4.24b}$$

and

$$G_0 = (\bar{\omega} - h_0^\times)^{-1} . \tag{4.25}$$

(For the details of the calculation technique, see Choi et al.) From Eqs. (4.12), (4.13) and (4.22), we see that lineshape is depicted by the collision factor $\tilde{B}_\alpha(\bar{\omega})$. Thus the present authors named the factor the lineshape function.

Model B

The conductivity tensor can also be obtained from Eq. (3.20). By putting $\beta=\alpha+1$ we have

$$\tilde{\sigma}_{--}(\bar{\omega}) = (\omega_0)^{-1}\sum_\alpha [f(\epsilon_\alpha+\omega_0)-f(\epsilon_\alpha)] < \tilde{Y}_\alpha(\bar{\omega}) >_B . \tag{4.26}$$

Here $\tilde{Y}_\alpha(\bar{\omega})$ is the FLT of $y_\alpha(t)$ given by

$$y_\alpha(t) = (j^-, j^+(t))_\alpha , \tag{4.27}$$

where

$$(A,B)_\alpha \equiv (A^\dagger B)_{\alpha\alpha} = \sum_\beta <\alpha \mid A^\dagger \mid \beta><\beta \mid B \mid \alpha> . \tag{4.28}$$

For the calculation of $\tilde{Y}_\alpha(\bar{\omega})$, we define the projection operators for the state $|\alpha\rangle$ as

$$P_\alpha A = \frac{(j^-, A)_\alpha}{(j^-, j^+)_\alpha} j^+ , \tag{4.29}$$

$$P_\alpha = 1 - P_\alpha . \tag{4.30}$$

This definition is different from Argyres and Sigel's and Kawabata's. Following Mori, we separate $j^+(t)$ into the projective and vertical

components with respect to the j^- as

$$j^+(t) = P_\alpha j^+(t) + \bar{P}_\alpha j^+(t)$$

$$= Z_\alpha(t) j^+ + \int_0^t Z_\alpha(s) f_1(t-s) ds, \quad (4.31)$$

where

$$Z_\alpha(t) \equiv \frac{y_\alpha(t)}{(j^-, j^-)_\alpha}, \quad (4.32)$$

$$f_1(t) \equiv exp[i\bar{P}_\alpha(h^\times + H_B^\times)t] f_1, \quad (4.33)$$

$$f_1 = i\bar{P}_\alpha(h^\times + H_B^\times) j^+. \quad (4.34)$$

In order to obtain $y_\alpha(t)$ or $Z_\alpha(t)$, we differentiate Eq. (4.32) as

$$\frac{d}{dt} Z_\alpha(t) = i\omega_\alpha Z_\alpha(t) - \int_0^t \Gamma_\alpha(t-s) Z_\alpha(s) ds, \quad (4.35)$$

where

$$\Gamma_\alpha(t) = \frac{(j^-, (h^\times + H_B^\times) f_1(t))_\alpha}{(j^-, j^+)_\alpha}, \quad (4.36)$$

$$\omega_\alpha = \frac{(j^-, (h^\times + H_B^\times) j^+)_\alpha}{(j^-, j^+)_\alpha} = \omega_0 + V_{\alpha+1,\alpha+1} - V_{\alpha\alpha}. \quad (4.37)$$

Then comparison of Eqs. (4.27), (4.32) and the FLT of Eq. (4.35) leads to

$$< \hat{Y}_\alpha(\bar{\omega}) >_B = \langle \frac{(j^-, j^+)_\alpha}{i(\bar{\omega} - \omega_\alpha) + \tilde{\Gamma}_\alpha(\bar{\omega})} \rangle_B, \quad (4.38)$$

where $\tilde{\Gamma}_\alpha(\bar{\omega})$ is the FLT of $\Gamma_\alpha(t)$ defined by Eq. (4.36), which can be calculated further via Eqs. (4.33), (4.34) and (4.36) as

$$(j^-, j^+)_\alpha \Gamma_\alpha(t) = (j^-, V^\times exp[i\bar{P}_\alpha(h^\times + H_B^\times)t] \bar{P}_\alpha(h^\times + H_B^\times) j^+)_\alpha, \quad (4.39)$$

where we have used $(\bar{P}_\alpha X)_\alpha = 0$. (For the detailed procedure of calculation, see Ryu et al.) By taking into account the relation $P_\alpha h_0^\times G_0 \bar{P}_\alpha X = 0$, the FLT of Eq. (4.39) leads to

$$j_\alpha^+(i\tilde{\Gamma}_\alpha(\bar{\omega})) = < \sum_{n=0}^{\infty} [V^\times S G_0 \{\bar{P}_\alpha V^\times S G_0\}^n \bar{P}_\alpha \times (h^\times + H_B^\times) j^+]_\alpha >_B, \quad (4.40)$$

where S and G_0 are defined as Eqs. (4.24a), (4.24b) and (4.25).

Comparing Eqs. (4.38) and (4.40), respectively, with Eqs. (4.22) and (4.23) we see that

$$(j_\alpha^+)^*[\tilde{R}(\bar{\omega})]_\alpha = i\tilde{Y}_\alpha(\bar{\omega}), \quad (4.41)$$

$$\tilde{B}_\alpha(\bar{\omega}) = \tilde{\Gamma}_\alpha(\bar{\omega}). \quad (4.42)$$

In other words, we have arrived at the same formula, starting with different expressions and using different techniques. We see that Eqs. (4.12), (4.13), (4.26), (4.22), (4.38), (4.23), and (4.40) build the flame of the theory. It is to be noted that the lineshape function $\tilde{B}_\alpha(\bar{\omega})$ [or $\tilde{\Gamma}_\alpha(\bar{\omega})$] has been expanded with respecct to $V^\times$ which corresponds to the scattering potential.

5. Disscussion and Conclusion

(a). The Lowest Order Approximation

For n=0 in Eq. (4.23) or Eq. (4.40) we obtain for the impurity scatterings

$$i\tilde{B}_\alpha(\bar{\omega}) = < \sum_{\beta(\neq\alpha+1)} \frac{V_{\alpha+1,\beta}(V_{\beta\alpha+1} - V_{\beta-1\alpha} j_{\beta-1}^+ / j_\alpha^+)}{\bar{\omega} - \epsilon_\beta + \epsilon_\alpha}$$

$$+\sum_{\beta(\neq\alpha)}\frac{(V_{\alpha\beta}-V_{\alpha+1,\beta+1}j_\beta^+/j_\alpha^+)V_{\beta\alpha}}{\bar{\omega}-\epsilon_{\alpha+1}+\epsilon_\beta}>_{imp}, \quad (5.1)$$

where we have used Eq. (4.20) and

$$(G_0X)_{\beta\alpha}=(\bar{\omega}-\epsilon_\beta+\epsilon_\alpha)^{-1}X_{\beta\alpha}. \quad (5.2)$$

Eq. (5.1) is given in the second order of the scattering potential and is enough for sufficiently weak scatterings. Eq. (5.1) is similar to some others' expression [Kawabata and Lodder et al.], the main difference being that ω has been replaced by $\bar{\omega}\equiv\omega-i/\tau_t$.

For the phonon scatterings, we have

$$j_\alpha^+(i\dot{B}_\alpha(\bar{\omega}))=<(V^xSG_0P'(h^x+H_p^x)j^+)_\alpha>_p \quad (5.3)$$

in the lowest order approximation, where

$$G_{\alpha\beta}\equiv SG_0=\sum_{m=0}^{\infty}(G_0H_p^x)^mG_0=(\bar{\omega}-h_0^x-H_p^x)^{-1}. \quad (5.4)$$

We define the phonon state as [Ryu et al.]

$$|q>\equiv|n_{\vec{q}1},n_{\vec{q}2},n_{\vec{q}3},\cdots,n_{\vec{q}i},\cdots>, \quad (5.5)$$

and take into account the following matrix element :

$$<q|<\beta|G_{\alpha\beta}X|\alpha>|q'>=\frac{<q|<\beta|X|\alpha>|q'>}{\bar{\omega}-\epsilon_\beta+\epsilon_\alpha-<q|H_p|q>+<q'|H_p|q'>}. \quad (5.6)$$

It should be noted that the matrix element is given with respect to both the electron states ($|\alpha\rangle,|\beta\rangle$) and the phonon states ($|q\rangle,|q'\rangle$). Then, with the help of Eqs. (4.20), (5.5) and (5.6) the lineshape function in Eq. (5.3) leads to

$$\begin{aligned}i\dot{B}_\alpha(\bar{\omega})=&\sum_{\vec{q}}(1+n_{\vec{q}})\\&\times\Big(\sum_{\beta(\neq\alpha-1)}\frac{(\gamma_{\vec{q}})_{\alpha+1,\beta}[(\gamma_{\vec{q}}^+)_{\beta,\alpha+1}-(\gamma_{\vec{q}}^+)_{\beta-1,\alpha}j_{\beta-1}^+/j_\alpha^+]}{\bar{\omega}-\epsilon_\beta+\epsilon_\alpha-\omega_{\vec{q}}}\\&+\sum_{\beta(\neq\alpha)}\frac{[(\gamma_{\vec{q}})_{\alpha\beta}-(\gamma_{\vec{q}})_{\alpha+1,\beta+1}j_\beta^+/j_\alpha^+](\gamma_{\vec{q}}^+)_{\beta\alpha}}{\bar{\omega}-\epsilon_{\alpha+1}+\epsilon_\beta+\omega_{\vec{q}}}\Big)\\&+\sum_{\vec{q}}n_{\vec{q}}\\&\times\Big(\sum_{\beta(\neq\alpha+1)}\frac{(\gamma_{\vec{q}}^+)_{\alpha+1,\beta}[(\gamma_{\vec{q}})_{\beta,\alpha-1}-(\gamma_{\vec{q}})_{\beta-1,\alpha}j_{\beta-1}^+/j_\alpha^+]}{\bar{\omega}-\epsilon_\beta+\epsilon_\alpha+\omega_{\vec{q}}}\\&+\sum_{\beta(\neq\alpha)}\frac{[(\gamma_{\vec{q}}^+)_{\alpha\beta}-(\gamma_{\vec{q}}^+)_{\alpha+1,\beta+1}j_\beta^+/j_\alpha^+](\gamma_{\vec{q}})_{\beta\alpha}}{\bar{\omega}-\epsilon_{\alpha+1}+\epsilon_\beta-\omega_{\vec{q}}}\Big)\end{aligned} \quad (5.7)$$

where $n_{\vec{q}}$ is the phonon distribution function. Eq. (5.7) is similar to Choi and Chung's formula, and Lodder and Fujita's function. The critical difference is that ω has been replaced by $\bar{\omega}$.

(b). Validity of the Expansion

We like to show that Eq. (4.23) or Eq. (4.40) does not include any terms which diverge. In other words, the lineshape function $\dot{B}_\alpha(\bar{\omega})$, or $\dot{\Gamma}_\alpha(\bar{\omega})$, is finite even at $\omega=\omega_0$. For proof of the validity of the expansion, it suffices to show that each term in Eq. (4.23) or Eq. (4.40) does not diverge at $\omega=\omega_0$. The part for n=0 in Eq. (4.23) or Eq. (4.40) does not contain any terms which are in danger of divergence. For n=1, we obtain

$$\begin{aligned}[j_\alpha^+i\dot{B}_\alpha(\bar{\omega})]_{n=1}=&<\sum_{\beta(\neq\alpha+1)}(\bar{\omega}-\epsilon_\beta+\epsilon_\alpha)^{-1}\\&\times\Big[\sum_{\gamma(\neq\alpha+1)}(\bar{\omega}-\epsilon_\gamma+\epsilon_\alpha)^{-1}V_{\alpha+1,\beta}V_{\beta\gamma}\Omega_{\gamma\alpha}\\&-\sum_{\gamma(\neq\alpha)}(\bar{\omega}-\epsilon_\beta+\epsilon_\gamma)^{-1}V_{\alpha+1,\beta}(P_\alpha\Omega)_{\beta\gamma}V_{\gamma\alpha}\Big]\\&+\sum_{\beta(\neq\alpha)}(\bar{\omega}-\epsilon_{\alpha+1}+\epsilon_\beta)^{-1}\\&\times\Big[\sum_{\gamma(\neq\alpha)}(\bar{\omega}-\epsilon_{\alpha+1}+\epsilon_\gamma)^{-1}\Omega_{\alpha+1,\gamma}V_{\gamma\beta}V_{\beta\alpha}\end{aligned}$$

$$- \sum_{\gamma(\neq\alpha+1)} (\bar{\omega} - \epsilon_\gamma + \epsilon_\beta)^{-1} V_{\alpha+1,\gamma} (\bar{P}_\alpha \Omega)_{\gamma\beta} V_{\beta\alpha}] >_B , \tag{5.8}$$

where $\Omega \equiv (h^\times + H_B^\times)j^+$. The terms which are considered to be in danger of divergence come from the parts containing $(\bar{P}_\alpha \Omega)_{\beta\gamma}$ and $(\bar{P}_\alpha \Omega)_{\gamma\beta}$ in Eq. (5.8). The first is rewritten as

$$1st = < \sum_{\beta(\neq\alpha+1)} (\bar{\omega} - \epsilon_\beta + \epsilon_\alpha)^{-1} (\bar{\omega} - \epsilon_\beta + \epsilon_{\beta-1})^{-1}$$

$$\times V_{\alpha+1,\beta} V_{\beta-1,\alpha} (\omega_0 + V_{\alpha+1,\alpha+1} - V_{\alpha\alpha}) j^+_{\beta-1} >_B . \tag{5.9}$$

The only term which is in danger of divergence is for $\beta=\alpha$ and $\omega=\omega_0$. In that case we have

$$1st = < \frac{V_{\alpha+1,\alpha} V_{\alpha-1,\alpha} (\omega_0 + V_{\alpha+1,\alpha+1} - V_{\alpha,\alpha}) j^+_{\alpha-1}}{(-i/\tau_t)(\omega - i/\tau_t)} >_B . \tag{5.10}$$

We now see that "1st" is finite at $\omega=\omega_0$ since τ_t is finite. The proof for all other terms can be performed in similar ways. Therefore, the thermal relaxation time τ_t acts as a convergence factor for higher order terms.

(c). Conclusions

We have derived the conductivity tensor and the cyclotron resonance lineshape function for the system of electrons in interaction with impurities and phonons. For the derivation of conductivity tensor, the Kubo formalism has been utilized. The collision term usually neglected in some theories have been included here, leading to the appearance of the thermal relaxation time, which comes from the scatterings in the existence of the electromagnetic field.

The perturbation has been dealt with by the two techniques. The difference is that the Fourier-Laplace transforming is performed first in one while last in the other. The projection operators defined by Argyres and Sigel as Eqs. (4.17) and (4.18) look different from those defined by the present authors as Eqs. (4.29) and (4.30). But the two types are identical with each other in cyclotron transition problems, yielding the same result.

The lineshape functions obtained are similar in form to some other authors'. The critical difference, however, is that the thermal relaxation time τ_t is included. Unless τ_t approaches the infinity, any danger of divergence in some higher order terms of the lineshape functions disappears.

There are several important issues under continuing study, including electron-electron interactions, indirect transitions, and non-parabolic band formalisms. Furthermore, the inclusion of exchange effects will be required [Badjou et al.] if the density of electrons is not so small. All these works are left for future studies.

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