Analysis of ^{16}O + ^{208}Pb Elastic Scattering with Eikonal Approximation

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Abstract

Using the phase shift analysis based on the eikonal approximation, the analysis for elastic scattering data of ^{16}O + ^{208}Pb system at E_{lab}=793 and 1503 MeV are performed. The experimental data and the calculated results are found to agree each other comparatively.

I. Introduction

Much interest is being focused in recent years on heavy-ion elastic scattering. Elastic scattering gives us information on the heavy-ion optical potential and forms a basis for a further description of heavy-ion reactions using either the DWBA or the semiclassical approach to these process. To extract the nuclear parameters from elastic scattering, one can use an exact phase shift analysis, which at high energies is tedious and difficult, or one can use various approximation schemes to compute the phase shifts.

In this study, we will use the eikonal phase shift to analyse the heavy-ion elastic scattering. The phase shift in the eikonal approximation is obtained from the integral equation by further approximating the WKB results[1,2,3]. The physical assumption of the eikonal approximation is that the energy of a projectile is sufficiently high that its classical trajectory is deflected little from a straight line. Then, the integral in the equation for the phase shifts may be performed along an almost straight-line path.

A systematic analysis[4] of heavy-ion reaction data in terms of an eikonal approach has been presented and illustrated by a wide range of examples in the projectile energy ($E/A_p \simeq 30$ - 350 MeV) and the mass numbers of colliding nuclei ($4 \simeq A_p \simeq 40$, $12 \simeq A_T \simeq 208$). Recently, the analysis[5] of elastic scattering data for ^{12}C (E_{lab} = 420 MeV) + ^{40}Ca, ^{90}Zr and ^{16}O(E_{lab} = 1503 MeV) + ^{40}Ca, ^{90}Zr is investigated in terms of eikonal approximation and compared with the modified Glauber model[6,7].

This paper reproduces a phase shift analysis of the elastic scattering data[8,9] for E_{lab} = 793 and 1503 MeV ^{16}O beams on ^{208}Pb target nuclei based on the eikonal approximation. In Sec. Ⅱ, we present the relevant theoretical details. Finally, results and conclusions are presented in Sec. Ⅲ

Ⅱ. Theoretical Details

The elastic scattering amplitude for spin-zero particles via Coulomb and short-range central forces is given by

$$f(\theta) = \frac{1}{ik}\sum_{l=0}^{\infty}(l+\frac{1}{2})(S_l - 1)P_l(cos\theta). \tag{1}$$

Here, k is the wave number and the scattering matrix S_l is related to the phase shift for the l-th partial wave. Since the Coulomb interaction between heavy-ions is strong, it is convenient to separate the Coulomb contribution by writing $S_l = exp(2i\sigma_l)S_l^N = exp(2i\sigma_l + 2i\delta_l)$ where $\sigma_l = arg\Gamma(l+1+i\eta)$ are the Coulomb phase shifts, δ_l the nuclear phase shifts and $\eta = mZ_1Z_2e^2/(\hbar^2 k)$ the Sommerfeld parameter. Then, the scattering amplitude $f(\theta)$ can be separated into the Rutherford and nuclear parts by writing[3,10]

$$f(\theta) = f_R(\theta) + f_N(\theta) \tag{2}$$

where the Rutherford scattering amplitude $f_R(\theta)$ is given by

$$f_R(\theta) = -\frac{\eta}{2k sin^2\frac{\theta}{2}} exp[2i\sigma_0 - i\eta ln(sin^2\frac{\theta}{2})] \tag{3}$$

and the nuclear scattering amplitude is expressed as

$$f_N(\theta) = \frac{1}{ik}\sum_{l=0}^{\infty}(l+\frac{1}{2}) exp(2i\sigma_l)(exp(2i\delta_l)-1)P_l(cos\theta). \tag{4}$$

A first-order WKB expression for the nuclear elastic phase shift δ_l can be written as[1, 2, 3, 11]

$$\delta_l = \int_{r_t}^{\infty} k_l(r)dr - \int_{r_c}^{\infty} k_c(r)dr \tag{5}$$

where r_t and r_c are the turning points corresponding to the local wave numbers $k_l(r)$ and $k_c(r)$ given by

$$k_l(r) = k[1-(\frac{2\eta}{kr}+\frac{(l+\frac{1}{2})^2}{k^2r^2}+\frac{V_n(r)}{E})]^{1/2}$$

$$k_c(r) = k[1-(\frac{2\eta}{kr}+\frac{(l+\frac{1}{2})^2}{k^2r^2})]^{1/2}. \tag{6}$$

In the above equation , $V_n(r)$ is the nuclear potential of Woods-Saxon type given by

$$V_n(r) = -\frac{V_0}{1+exp((r-R_v)/a_v)} - i\frac{W_0}{1+exp((r-R_w)/a_w)}. \tag{7}$$

In the high energy limit, we can consider nuclear potential $V_n(r)$ as a perturbation. Thus, the turning point r_t may be taken to be coincident with $r_c = \{\eta + [\eta^2 + (l+\frac{1}{2})^2]^{1/2}\}/k$ and

$$k_l(r) - k_c(r) = k_c(r)[1 - \frac{2\mu V_n(r)}{\hbar^2 k_c^2(r)}]^{1/2} - k_c(r) \simeq -\frac{\mu V_n(r)}{\hbar^2 k_c(r)}. \tag{8}$$

If we substitute Eq. (8) into Eq. (5) and rearrange the terms, we find that the phase shift in terms of r_c instead of l, is given by

$$\delta_l(r_c) \simeq -\frac{\mu}{\hbar^2 k}\int_{r_c}^{\infty}\frac{rV_n(r)}{\sqrt{r^2-r_c^2}}dr. \tag{9}$$

Furthermore, we have adopted a cylindrical coordinate system and decompose the vector $\vec{r}$ as $\vec{r} = \vec{r_c} + z\hat{n}$ where the z component of $\vec{r}$ lies along $\hat{n}$ and $\vec{r_c}$ is perpendicular to $\hat{n}$. We may, therefore, write Eq. (9) as

$$\delta(r_c) = -\frac{\mu}{\hbar^2 k}\int_0^{\infty} V_n(\sqrt{r_c^2+z^2})dz. \tag{10}$$

This formula is the same form as the eikonal approximated expression[12,13] with an impact parameter b. Instead of b, we use the distance of closest approach, r_c. By taking $V_n(r)$ as the optical Woods-Saxon potential fitted to elastic scattering data, we use the approximated nuclear phase shift, Eq. (10), in the general expression for the elastic scattering amplitude.

The semiclassical approximation assumes that contributions to the cross section come mainly from the large angular momenta. The asymptotic form of the Legendre function can be written as[14]

$$P_l^m(cos\theta) \simeq (l+\frac{1}{2})^m(\frac{\theta}{sin\theta})^{1/2}J_{-m}[(l+\frac{1}{2})\theta], \tag{11}$$

which is valid for all m and all angles except when $(\pi-\theta)\leq l^{-1}$. Using the Eq. (11) with $m=0$ and taking into account Eq. (4), the nuclear scattering amplitude can be rewritten as.

$$f_N(\theta) \simeq \frac{1}{ik}(\frac{\theta}{sin\theta})^{1/2}\sum_{l=0}^{\infty}(l+\frac{1}{2})exp(2i\sigma_l)(exp(2i\delta_l)-1)J_0[(l+\frac{1}{2})\theta]. \tag{12}$$

This formula is similar to Eq. (5) of Ref. [14]. We use a summation by parts instead of an integral over the impact parameter b.

Ⅲ. Results and Conclusions

We have applied the eikonal approximation to elastic scatterings of ^{16}O + ^{208}Pb at E_{lab} = 793 and 1503MeV. The elastic scattering cross sections are obtained from the Rutherford scattering amplitude of Eq. (3) and the nuclear scattering amplitude of Eqs. (4) and (12) by using the partial

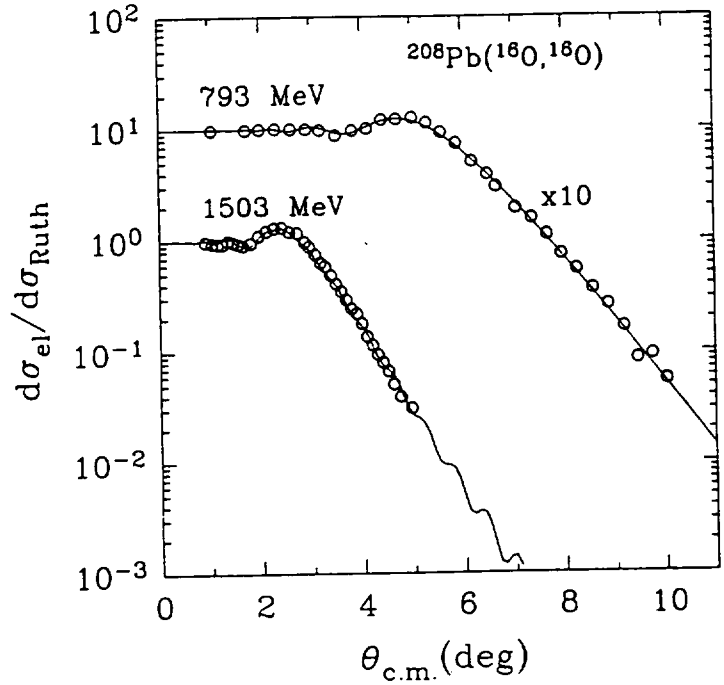

Figure 1. Elastic scattering angular distributions for $^{16}O+^{208}Pb$ system at E_{lab} = 793 and 1503 MeV. The open circles denote the observed data taken from Ref. [8, 9]. The solid and broken curves are the calculated cross sections from Eqs. (4) and (12), respectively, for nuclear component of the scattering amplitude. (Note that the two lines show nearly identical fits)

wave sum in Eq. (1) . The numerical results of elastic scattering angular distributions for $^{16}O+^{208}Pb$ at E_{lab} = 793 and 1503 MeV are presented in figure 1. The fits are satisfactory and the corresponding parameters of Woods-Saxon potential are given in table 1. In figure 1, the full and bro-

Table 1. Parameters of the fitted Woods-Saxon optical potential for ^{16}O beams at E_{lab} = 793 and 1503 MeV

Energy (MeV)	V_0 (MeV)	r_v (fm)	a_v (fm)	W_0 (MeV)	r_w (fm)	a_w (fm)
793	50.7	1.074	0.760	40.7	1.104	0.777
1503	66.4	1.067	0.733	93.0	1.062	0.520

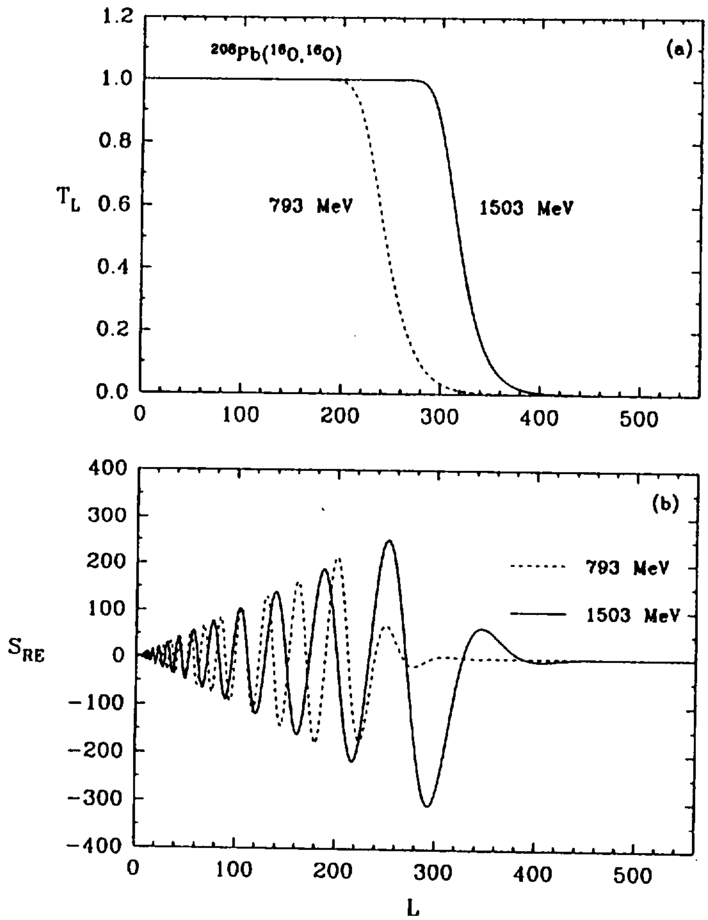

Figure 2. (a) Transmission functions, (b) the real part (S_{RE}) of the term, $\frac{1}{i}(l+\frac{1}{2})exp(2i\sigma_l)(S_l^N-1)$, for the $^{16}O+^{208}Pb$ system plotted versus the orbital angular momentum l.

ken curves represent the calculated cross sections obtained by using Eqs. (4) and (12), respectively, for the nuclear component of the scattering amplitude. The two numerical results agree well with the observed data.

Transmission functions, $T_l = 1-|S_l^N|^2$, are plotted as a function of angular momentum l in Fig. 2(a). In this figure, we can see that lower partial waves are totally absorbed and T_l decrease rapidly in a narrow localized angular momentum zone. The real parts (S_{RE}) of the terms, $\frac{1}{i}(l+\frac{1}{2})exp(2i\sigma_l)(S_l^N-1)$, in Fig. 2(b) represent the partial wave contributions to the cross sections in terms of orbital angular momentum l.

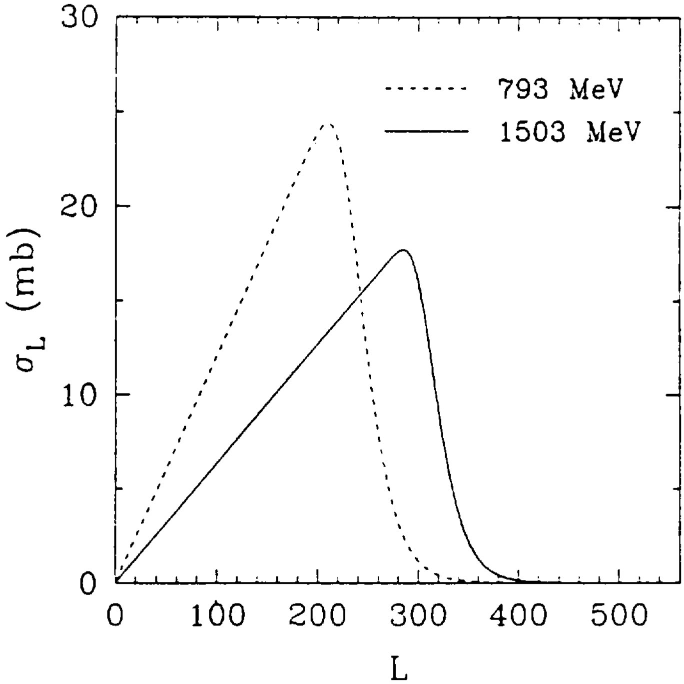

Figure 3. Partial wave reaction cross section, $\sigma_l = \frac{\pi}{k^2}(2l+1)(1-|S_l^N|^2)$, for the $^{16}O+^{208}Pb$ system as a function of orbital angular momentum l.

The partial wave contributions, $\sigma_l = \frac{\pi}{k^2}(2l+1)(1-|S_l^N|^2)$, to the total reaction cross section as a function of l are also presented in Fig. 3. This

Table 2. Total reaction cross sections(σ_R obtained from the transmission function and $\sigma_{R,1/2} = \pi r_{1/2}^2$ from the strong absorption radius) and strong absorption radius($r_{1/2}$) obtained in the eikonal aproximation. Also given in parenthesis is the critical angular momentum $l_{1/2}$

Energy (MeV)	σ_R (mb)	$\sigma_{R,1/2}$ (mb)	$r_{1/2}$ (fm)
793	3710	4045	11.347(244)
1503	3290	3435	10.456(318)

figure shows that the regions of higher partial waves almost never contribute to the total reaction cross section. In table 2, $r_{1/2}$ is the strong absorption radius and σ_R the total reaction cross section. The strong absorption radius, for which $T_l = \frac{1}{2}$, provides a good estimate of the reaction cross section $\sigma_{R,1/2} = \pi r^2_{1/2}$.

In conclusion, we have shown that it is possible to give a satisfactory account of elastic scattering of $^{16}O+^{208}Pb$ system at E_{lab} = 793 and 1503 MeV within the phase shift analysis based on the eikonal appoximation. We can see that the strong absorption radius gives a good measure of the reaction cross section in terms of $\sigma_{R,1/2} = \pi r^2_{1/2}$. It has also turned out that the two calculated results from the approximated scattering amplitude and the exact one show nearly identical fits and are in overall good agreement with the experimental data.

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Eikonal 근사를 이용한 ^{16}O + ^{208}Pb 탄성산란 분석

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Eikonal 근사에 기초를 둔 위상이동량 분석을 이용하여 E_{lab}=793과 1503 MeV인 ^{16}O + ^{208}Pb계의 탄성산란 데이타를 분석하였다. 계산결과들은 실험데이타와 비교적 잘 일치를 하였다