반도체의 광학적 성질에 관한 다체효과

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Many Body Effects on the Optical Properties of Semiconductors

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Summary

We discuss the influence of finite electron densities and electron-electron interactions on optical properties of N type doped semiconductors.

Introduction

The topics investigated herein depend on the fact that, in sufficiently heavily doped N type semiconductors, the collective excitation frequency of the electron gas can be comparable to or greater than the characteristic vibrational frequencies of the host material. This suggests the possibility that coupled modes may be found between the electron plasmon and the phonons of comparable ferquencies. For example, at the appropriate free carrier density – 10^{17} to 10^{18} electrons cm^{-3} in III-V compounds – the electron plasma frequency is comparable to the longitudinal optical (LO) phonon frequency, and the plasmons and phonons form coupled modes. Such behavior was first predicted by Gurevich et al, by Varga and by Singwi and Tosi (Gurevich, 1962; Varga, 1965; Singwi, 1966). Definitive experimental verification followed shortly thereafter in a variety of polar semiconductor, which investigated Raman scattering spectra resulting from interaction of the incident radiation with mixed phonon-plasmon modes. (Mooridian, 1966; Pinczuk, 1971; Scott, 1971).

In this paper we discuss the role of electron-electron interactions in the LO phonon mediated infrared absorption by doped N type polar semiconductors. We will be concerned with doping strengths such that the electron plasma frequency is comperable to the LO phonon frequency, leading to the possibility that mixed plasmon-phonon modes will participate in the absorption process. Our procedure will be to calculate microscopically the photon life time, $1/\tau(\Omega)$, in the interacting electron gas-phonon system. This can be related directly to the absorption coefficient $\alpha(\Omega)$. Here Ω the incident radiation frequency. To orient the reader, we present a discussion of a simple phenomenology of the absorption process.

Drude dielectric constant and Hamiltonian

Without free carriers, the onset of absorption occurs for an incident photon energy, Ω (we set $\hbar = 1$ everywhere), equal to the band gap energy, Eg. However, as described by Drude early this century, free electrons that undergo inelastic collisions in a characteristic time τ_e provide a mechanism for absorption of radiation less energetic than Eg (Drude, 1900). The explanation is purely classical. The equation of motion of an electron in a parabolic conduction band characterized by an effective mass m^* is

$$m^* \ddot{\vec{X}} + \dot{\vec{X}} / \tau_e = e\vec{E} \ , \qquad (1)$$

where $\vec{E}$ is the electric field at $\vec{X}$. Solution of Eq. (1) yields an expression for the drift velocity

$$\dot{\vec{X}} = e\vec{E}\,\tau_e/m^*\,(1 - i\Omega\tau_e),$$

if $\vec{E}$ has the form $\vec{E}_0 e^{i\Omega t}$. If one relates the conductivity to the drift velocity by $ne\dot{\vec{x}} = \sigma\vec{E}$, one obtains the Drude conductivity formula:

$$\sigma(\Omega) = \frac{ne^2\tau_e}{m^*(1 - i\Omega\tau_e)} \ , \qquad (2)$$

where η is the electron density.

Application of Maxwell's curl equations with $\vec{J} = \sigma\vec{E}$ and $\vec{D} = \epsilon\vec{E}$ leads to

$$\epsilon(\Omega) = \epsilon_\infty - \frac{\omega_p^2}{\Omega(\Omega + i/\tau_e)} \ , \qquad (3)$$

the Drude dielectric constant. Here we have encountered the classical electron plasma frequency defined by

$$\omega_p^2 = 4\pi ne^2 / \epsilon_\infty m^* \tag{4}$$

An expression for the absorption coefficient is easily obtained from Eq. (3). With $k^2 = \Omega^2 \epsilon(\Omega)/c^2$, for $\Omega\tau_e \gg 1$, an approximation valid for much of our discussion, we have

$$K = \frac{\Omega\sqrt{\epsilon}}{c}\ (1+i\omega_p^2 / 2\Omega^3\tau_e) \ .$$

The energy of the radiation field has a spacial dependence proportional to the square of the electric field which varies as $e^{-(\mathrm{Im}\vec{k})\cdot\vec{X}}$. Thus we obtain the Drude expression for the absorption coefficient.

$$\alpha(\Omega) = \frac{4\pi ne^2}{m^*\sqrt{\epsilon}\, c^2 \Omega^2 \tau_e(\Omega)} \tag{5}$$

This relation will enable us to obtain the electron relaxation rate once we have determined the absorption coeffiicient from the microscopic calculation which follows.

we first define our model by writing down the Hamiltonian which includes a description of the basic excitations and their interactions. This Hamiltonian is

$$H = H_L + H_E + H_{EL} + H_{EE} + H_{ER} + H_R \tag{6}$$

where H_L, H_E, and H_R are the Hamiltonians for the phonons, the conduction electrons and the radiation field in the presence of the background dielectric constant ϵ_∞ of the crystal. (By ϵ_∞ we mean the contribution to the dielectric constant from interband transitions. We shall always be concerned with photon frequencies well below the gap here; the method of including interband transitions into the treatment explicitly is discussed in reference (Jensen, 1973)). Then $H_{E\text{-}L}$ is the electron – LO phonon interaction, H_{EE} the electron – electron interaction in the rigid lattice, and H_{ER} the interaction between the electrons and the radiation field.

We have for these terms the explicit forms

$$H_E = \sum_{\vec{k}} \frac{k^2}{2m^*} C^+_{\vec{k}} C_{\vec{k}} \tag{7a}$$

$$H_L = \omega_{LO} \sum_{\vec{k}} (b^+_{\vec{k}} b_{\vec{k}} + \tfrac{1}{2}) \tag{7b}$$

$$H_R = \sum_{\vec{k}\lambda} \omega(\vec{k}) (a^+_{\vec{k}\lambda} a_{\vec{k}\lambda} + \tfrac{1}{2}) \tag{7c}$$

Here m^* is the band structure effective mass, ω_{LO} the LO phonon frequency (we will be concerned with phonon wavelengths sufficiently long that depression in the phonon spectrum is unimportant), and $\omega(\vec{k}) = ck/\sqrt{\epsilon_\infty}$, with c the vacuum velocity of light and λ a polarization index. Finally $C_{\vec{k}}(C^+_{\vec{k}})$, etc. are the annihilation (creation) operations for the electrons, and the other elementary excitations. We have set $\hbar = 1$.

For H_{EL} we have

$$H_{EL} = \int d^3\vec{x}\ \hat{\psi}^+(\vec{x})\, V_{EL}(\vec{x})\, \hat{\psi}(\vec{x}) , \tag{8a}$$

where

$$\hat{\psi}(\vec{x}) = \sum_{\vec{k}} C_{\vec{k}} \frac{e^{i\vec{k}\cdot\vec{x}}}{\sqrt{V}} \tag{8b}$$

is the electron field operator,

$$V_{EL} = \sum_{\vec{q}} i\, \gamma_o(\vec{q}) \hat{\varphi}_{pn}(\vec{q})\, e^{i\vec{q}\cdot\vec{x}} \tag{8c}$$

with $\hat{\varphi}_{pn}(\vec{q}) = b_{\vec{q}} - b^+_{-\vec{q}}$ and $\gamma_o(\vec{q}) = (4\pi e/\epsilon_\infty q)(n_L e^{*2}/2M\omega_{LO}V)^{1/2}$ is the Fröhlich coupling constant. (Hats are used to distinguish the second quantized operators from ordinary c-numbers.) In $\gamma_0(\vec{q})$, n_L is the number of unit cells/unit volume, e^* the Born effective charge, and M is the reduced mass of the unit cell. The electron-electron interaction term is of the form

$$H_{EE} = \frac{1}{2} \sum_{\vec{k},\vec{k}',\vec{q}} V_c(\vec{q})\, C^+_{\vec{k}+\vec{q}}\, C^+_{\vec{k}'-\vec{q}}\, C_{\vec{k}'}\, C_{\vec{k}} \tag{9}$$

with

$$V_c(\vec{q}) = 4\pi e^2/\epsilon_\infty q^2 V$$

Finally, for the electron-radiation interaction we have the $\vec{p}\cdot\vec{A}$ term

$$H_{ER} = -\frac{e}{mc} \int d^3\vec{x}\ \hat{\psi}^{+(b)}(\vec{x})\, \vec{A}(\vec{x})\cdot\vec{p}\, \hat{\psi}^{(b)}(\vec{x}) \tag{10}$$

where $\hat{\psi}^{(b)}(\vec{x})$ is the Bloch function of the electrons in the conduction bands. We now show that Eq.(10) is equivalent to a similar expression, but with the true electron mass replaced with m^* and the Bloch functions replaced by the plane wave states defined by Eq. (8b), provided the electron states are near the conduction band edge. The proof rests on a simple application of $\vec{k}\cdot\vec{p}$ perturbation theory.

In Eq. (10), $\vec{A}(\vec{x})$ is given by

$$\vec{A}(\vec{x}) = \sum_{\vec{q}\lambda} \left\{ \frac{2\pi c^2}{\epsilon_\infty V \Omega(q)} \right\}^{1/2} \hat{e}(\vec{q}\lambda)\, \vec{A}(\vec{q}\lambda)\, e^{i\vec{q}\cdot\vec{x}} \tag{11}$$

where $\hat{e}(\vec{q}\lambda)$ is a polarization vector and $\vec{A}(\vec{q}\lambda) = a_{q\lambda} + a^{+}_{-q\lambda}$. Thus we can rewrite Eq. (10) as

$$H_{ER} = -\frac{e}{c} \sum_{qx} \left\{ \frac{2\pi c^2}{\epsilon_\infty V \Omega(q)} \right\}^{1/2} \hat{e}(\vec{q}\lambda) \vec{A}(\vec{q}\lambda) \cdot \vec{I}(\vec{q}), \quad (12)$$

where

$$I(\vec{q}) = \frac{1}{m} \sum_{\vec{k}\vec{k}'} C^{+}_{\vec{k}'} C_{\vec{k}} \int d^3\vec{x}\, \psi^{+(b)}_{\vec{k}}(\vec{x})\, e^{i\vec{q}\cdot\vec{x}}\, \vec{p}\, \psi^{(b)}_{\vec{k}}(\vec{x}) \quad (13)$$

We expand $\psi^{(b)}_{\vec{k}}(\vec{x})$ about the zone center of the conduction band;

$$|\psi^{(b)}_{\vec{k}}(\vec{x})>_c = e^{i\vec{k}\cdot\vec{x}} \left\{ |0>_c + \frac{1}{m} \sum_{\gamma}{}' |0>_\gamma \frac{<0|\vec{k}\cdot\vec{p}|0>_c}{\varepsilon_{oc} - \varepsilon_{o\gamma}} \right\} \quad (14)$$

Here C and γ label the conduction band and all other bands, respectively. $\varepsilon_{0c}(\varepsilon_{0\gamma})$ is the energy at zero wavevector of the conduction (γ) band, and $|0>_c$ ($|0>_\gamma$) is the electron state at zero wave vector of the conduction band. The prime in the summation in Eq. (14) denotes omission of the conduction band. Rewriting Eq. (13) in terms of Eq. (14), and retaining up to first order $\vec{k}\cdot\vec{p}$ terms we find

$$\begin{aligned}\vec{I}(\vec{q}) = \frac{1}{m} \sum_{\vec{k}\vec{k}'} C^{+}_{\vec{k}'} C_{\vec{k}} \Big\{ &<0|e^{-i\vec{k}'\cdot\vec{x}} e^{i\vec{q}'\cdot\vec{x}}\, \vec{p}\, e^{i\vec{k}\cdot\vec{x}}|0>_c + {}_c<0|e^{-i\vec{k}'\cdot\vec{x}} e^{i\vec{q}\cdot\vec{x}} \vec{p}\, e^{i\vec{k}\cdot\vec{x}}|0>_c \\ &\left[\frac{1}{m} \sum_{\gamma}{}' \frac{|0>_\gamma <0|\vec{k}\cdot\vec{p}|0>_c}{\varepsilon_{co} - \varepsilon_{\gamma o}}\right] \quad (15) \\ &+ \frac{1}{m} \sum_{\gamma}{}' \frac{\gamma<0|\vec{k}\cdot\vec{p}|0>_c^{*}}{\varepsilon_{co} - \varepsilon_{\gamma o}} \gamma<0|e^{-i\vec{k}'\cdot\vec{x}} e^{i\vec{q}\cdot\vec{x}} \vec{p}\, e^{i\vec{k}\cdot\vec{x}}|0>_c \Big\}\end{aligned}$$

Evaluation of the matrix elements that contain exponentials in Eq. (15) leads to

$$\begin{aligned}\vec{I}(q) = \frac{1}{m} \sum_{\vec{k}} C^{+}_{\vec{k}+\vec{q}} C_{\vec{k}} \Big\{ &\vec{k} + \frac{1}{m} \sum_{\gamma}{}' \frac{1}{\varepsilon_{co} - \varepsilon_{\gamma o}} \\ &[{}_c<0|\vec{p}|0>_\gamma \gamma<0|\vec{k}\cdot\vec{p}|0>_c \quad (16) \\ &+ {}_\gamma<0|\vec{k}\cdot\vec{p}|0>_c^{*} {}_\gamma<0|\vec{p}|0>_c]\Big\}.\end{aligned}$$

One further rearrangement of the terms in the square bracket enables us to write

$$\vec{I}(q) = \sum_{\vec{k}} C^{+}_{\vec{k}+\vec{q}} \frac{\vec{k}}{m^*} \quad (17)$$

where we have used the definition of the scalar effective mass(Kittel, 1963)

$$\frac{1}{m^*} = \frac{1}{m} + \frac{2}{m} \sum_{\gamma}{}' \frac{c<0|\vec{p}|0>_\gamma\ \gamma<0|\vec{p}|0>_c}{\varepsilon_{co} - \varepsilon_{\gamma o}} \quad (18)$$

When Eq. (17) is put into Eq. (12) we obtain the same expression that results from replacing m by m* and the Bloch electron by the free electron operator in Eq. (10):

$$H_{ER} = -\frac{e}{m^* c} \int d^3\vec{x}\, \hat{\psi}^{+}(\vec{x})\, \vec{A}(\vec{x}) \cdot \vec{p}\, \hat{\psi}(\vec{x}) \quad (19)$$

Our task is to find the effect of the interaction terms on the lifetime of a photon in the system, and to relate this to the absorption coefficient.

Formalism

In this section we dreive a formal expression for the absorption coefficient by calculating the lifetime of a photon in the interacting system. The calculation is carried out within the framework of many body theory. Specifically, we calculate the imaginary part of the photon self energy which can be related to $\tau_p(\Omega)$ directly. This is anaiogous to the procedure followed by Mills and Burstein (Mills, 1969) in their treatment of Raman scattering in insulators. An alternative approach would be to calculate through diagrammatic analysis the conductivity $\sigma(\Omega)$, and relate this to the absorption coefficient through Eqs. (2) and (5). We begin by defining the imaginary time photon Green's function:

$$P_{\lambda\lambda'}(\vec{Q}, \tau) = -<T_\tau[\varphi(\vec{Q}\lambda, \tau)\varphi^{+}(\vec{Q}\lambda', o)]>, \quad (20)$$

where T_τ is the Wick imaginary time ordering operator, and the angular brackets denote statistical averaging over a complete set of states. We have used

$$\varphi(\vec{Q}\lambda, \tau) = e^{H\tau}(a_{\vec{Q}\lambda} + a^{+}_{-\vec{Q}\lambda}) e^{-H\tau} \quad (21)$$

Straightforward calculation of Eq. (20) when H is taken to include only H_E, H_L and H_R yields the standard result

$$P^{(q)}_{\lambda\lambda'}(\vec{Q}, \tau) = -\delta_{\lambda\lambda'} \left\{ <n(\vec{Q})> (e^{\omega_{\vec{Q}}\tau} + e^{-\omega_{\vec{Q}}\tau}) + e^{-\omega_{\vec{Q}}\tau}\theta(\tau) + e^{\omega_{\vec{Q}}\tau}\theta(-\tau) \right\}. \quad (22)$$

Here $n(\vec{Q})=(e^{\beta\omega_{\vec{Q}}}-1)^{-1}$ is the Bose-Einstein occupation number. The periodicity of $P_{\lambda\lambda'}(\vec{Q},\tau)$ allows us to write

$$P_{\lambda\lambda'}(\vec{Q},\tau)=\beta^{-1}\sum_n e^{-i\omega_n\tau}P_{\lambda\lambda'}(\vec{Q},i\omega_n) \tag{23}$$

where $\omega_n=2\pi n/\beta$. Here $\beta=1/k_BT$ with k_B Boltzmann's constant and T the absolute temperature. Evaluation of the Fourier transform

$$P_{\lambda\lambda'}(\vec{Q},i\omega_n)=\int^{\beta}d\tau e^{i\omega_n\tau}P_{\lambda\lambda'}(\vec{Q},\tau) \tag{24}$$

yields

$$P^{(o)}_{\lambda\lambda'}(\vec{Q},i\omega_n)=-\delta_{\lambda\lambda'}\frac{2\omega_{\vec{Q}}}{\omega^2_{\vec{Q}}+\omega^2_n} \tag{25}$$

It is the generalization of Eq. (25) to include interactions of the photon with the electron gas-phonon system that we are concerned with. We know that the effect of the interactions will be to modify Eq. (25) by causing the addition of a self energy term in the denominator, so that the full photon Green's function can be written

$$P(\vec{Q},i\omega_n)=\frac{-2\omega_{\vec{Q}}}{\omega^2_{\vec{Q}}+\omega^2_n+2\omega_{\vec{Q}}\pi(\vec{Q},i\omega_n)}, \tag{26}$$

where $\pi(\vec{Q},i\omega_n)$ is the photon proper self energy. We have utilized the fact that for crystals with a cubic point group the radiation in the long wave-length limit is purely transverse, so that $P_{\lambda\lambda'}(\vec{Q},i\omega_n)$ is diagonal in the polarization index, and have suppressed λ in the equation for $P(\vec{Q},i\omega_n)$.

The Dyson equation which can be solved to obtain Eq. (26) may be represented as shown in Figure (1), where the crosshatched bubble is the photon self energy.

The analytic continuation of $P(\vec{Q},i\omega_n)$ to $P(\vec{Q},\Omega+i\mu)$, where μ is a real infinitesimal, contains the information about the photon lifetime which we require. Before carrying out a detailed analysis of the photon self energy, we outline how we obtain from Eq. (26) the photon lifetime and the absorption coefficient once $\pi(\vec{Q},i\omega_n)$ is known.

Using the relation

$$\omega_{\vec{Q}}=CQ/\sqrt{\epsilon_\infty}$$

we have

$$P(\vec{Q},\Omega+i\mu)=\frac{-2CQ\sqrt{\epsilon_\infty}}{C^2Q^2-\Omega^2\{\epsilon_\infty-2i\omega_Q\epsilon_\infty\pi_R(\vec{Q},\Omega)/\Omega^2\}+2i\epsilon_\infty\omega_{\vec{Q}}\pi_I(\vec{Q},\Omega)} \tag{27}$$

Here we have written

$$\pi(\vec{Q},\Omega+i\mu)=\pi_R(\vec{Q},\Omega)+i\pi_I(\vec{Q},\Omega). \tag{28}$$

We rewrite Eq. (27) as

$$P(\vec{Q},\Omega)=\frac{-2CQ\sqrt{\epsilon_\infty}}{C^2Q^2-\Omega^2\epsilon_R(\Omega)+2i\epsilon_\infty\omega_{\vec{Q}}\pi_I(\vec{Q},\Omega)}, \tag{29}$$

where

$$\epsilon_R(\Omega)=\epsilon_\infty[1-2\omega_{\vec{Q}}\pi_R(Q,\Omega)/\Omega^2] \tag{30}$$

is the real part of the dielectric constant. The pole of $P(\vec{Q},\Omega)$ is shifted off the real frequency axis by the imaginary part of the self energy. With the real time photon propagator, $P(\vec{Q},t)$ related to $P(\vec{Q},\Omega)$ by

$$P(\vec{Q},t)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}d\omega e^{i\Omega t}P(\vec{Q},\Omega), \tag{31}$$

we see that $\vec{p}(\vec{Q},t)$ (and thus the vector potential and electric field) decays like $e^{-t/2\tau_p}$, where we have written the complex frequency for which the denominator vanishes as

$$\Omega=\Omega_R+i/2\tau_p \tag{32}$$

Of course the energy density decays as e^{-t/τ_p}. By setting the denominator of Eq. (29) equal to zero, and expressing the complex freqency as in Eq. (32), we have

$$C^2Q^2-(\Omega_R+i/2\tau_p)^2\epsilon_R(\Omega_R+i/2\tau_p)+2i\epsilon_\infty\omega_{\vec{Q}}\pi_I(\vec{Q},\Omega_R)=0. \tag{33}$$

Assuming that the pole is not too far from the real axis, the imaginary part of Eq. (33) can be rewritten

$$\frac{1}{\tau_p(\Omega)}=\frac{2\epsilon_\infty\omega_{\vec{Q}}\pi_I(\vec{Q},\Omega)}{\Omega[\epsilon_R(\Omega)+\frac{\Omega}{2}\frac{\partial\epsilon_R(\Omega)}{\partial\Omega}]}. \tag{34}$$

We related $1/\tau_p(\Omega)$ to the abosrption coefficient $\alpha(\Omega)$ by $1/\alpha(\Omega)\tau_p(\Omega)=V_g(\Omega)$, the group velocity of the photon at frequency Ω. Noting that

$$V_g(\Omega)=C\sqrt{\epsilon_R}\left[\epsilon_R+\frac{1}{2}\Omega\frac{\partial E_R}{\partial\Omega}\right]^{-1} \tag{35}$$

we have, finally,

$$\alpha(\Omega)=\frac{2}{C\sqrt{\epsilon_R}}\frac{\epsilon_\infty\omega_{\vec{Q}}}{\Omega}\pi_I(\vec{Q},\Omega) \tag{36}$$

We not proceed to evaluate $\pi_I(\vec{Q},\Omega)$ by diagrammatic methods. A very helpful guide in this analysis is the diagram-

matic representation of the self energy when there are no electron-electron interactions present. To lowest nonvanishing order in electron-phonon interaction, the self energy can be representeded in Figure (2). The dashed lines represent the bare LO phonon propagator given by the zero order approimation to

$$D(i\omega_n) = \int_0^\beta d\tau e^{i\omega_n \tau} < T_\tau [\hat{\varphi}_{pn}(\tau)\hat{\varphi}_{pn}(o)] > , \qquad (37)$$

where $\varphi_{pn}(\tau) = e^{H\tau}\varphi_{pn}e^{-H\tau}$. A straightforward calculation shows that

$$D_o(i\omega_n) = \frac{-2\omega_{Lo}}{\omega_{Lo}^2 + \omega_n^2} \qquad (38)$$

The bubbles represent the particle-hole propagator in the free electron gas. It is given by

$$\chi_o(\vec{Q}, i\Omega_m) = \frac{2}{\beta}\sum_{\vec{k}, i\omega_n} G(\vec{k}, i\omega_n) G(\vec{k}+\vec{Q}, i\omega_n + i\Omega_m), \qquad (39)$$

where $G(\vec{k}, i\omega_n)$ is the free electron propagator which satisfies

$$G(\vec{k}, i\omega_n) = [i\omega_n - (\mathcal{E}(\vec{k}) - \mu)]^{-1} \qquad (40)$$

Here μ is the chemical potential of the electron gas. The factor of two in Eq. (39) accounts for the electron spin. $\mathcal{E}(\vec{k})$ is electron energy.

Since our concern is for incident radiation of vanishingly small wave vector $\vec{Q}$, the first and last terms in Figure (2) contribute no imaginary part to the self energy and are of no interest to us. This is a consequence of the small $\vec{Q}$ limit of $\chi_o(\vec{Q}, \Omega)$:

$$\lim_{Q\to o} \chi_o(\vec{Q}, \Omega) = \frac{n}{m^*}\frac{Q^2}{\Omega^2} , \qquad (41)$$

which is purely real.

Mills has shown that the remaining three terms of Figure (2) lead to the second order perturbation theory results of Jensen. In this treatment one has the photon absorbed by excitation of a conduction electron, mediate by sacttering of the electron from an LO phonon, as shown in Figure (3).

Using standard diagrammatic procedures, we find that the contributions to the photon self energy from terms a, b and c in Figure (2) are given by

$$\pi^{(a)}(\vec{Q}, i\Omega_m) = \frac{1}{\beta^2}\sum_{\vec{k}\,\vec{q}}\sum_{\substack{i\omega_n \\ i\omega_m}} m_{\vec{Q}}(\vec{k})\, m_{\vec{Q}}(\vec{k})^* \gamma_o^2(q)$$

$$\times D(i\omega_m) G(\vec{k}, i\omega_n) G^2(\vec{k}, i\omega_n + i\Omega_m)$$

$$\times G(\vec{k}+\vec{q}, i\omega_n + i\omega_m + i\Omega_m) , \qquad (42a)$$

$$\pi^{(b)}(\vec{Q}, i\Omega_m) = \frac{1}{\beta^2}\sum_{\vec{k}\,\vec{q}}\sum_{\substack{i\omega_n \\ i\omega_m}} m_Q(\vec{k})\, m_Q(\vec{k})^* \gamma_o^2(q)$$

$$\times D(i\omega_m) G^2(\vec{k}, i\omega_n) G(\vec{k}+\vec{q}, i\omega_n + i\omega_m)$$

$$\times G(\vec{k}, i\omega_n + i\Omega_m) \qquad (42b)$$

and

$$\pi^{(c)}(\vec{Q}, i\Omega_m) = \frac{1}{\beta^2}\sum_{\vec{k}\,\vec{q}}\sum_{\substack{i\omega_n \\ i\omega_m}} m_{\vec{Q}}(\vec{k})\, m_{\vec{Q}}(\vec{k}+\vec{q})^* \gamma_o^2(q)$$

$$\times D(i\omega_m) G(\vec{k}, i\omega_n) G(\vec{k}, i\omega_n + i\Omega_m)$$

$$\times G(\vec{k}+\vec{q}, i\omega_n + i\Omega_m + i\omega_m)$$

$$\times G(\vec{k}+\vec{q}, i\omega_n + i\omega_m) \qquad (42c)$$

In these expressions, $\beta = 1/k_B T$, $i\omega_n = 2\pi i(n+1/2)\, k_B T$ is the fermion frequency of the finite termperature many body formalism, and $i\omega_m = 2\pi i m\, k_B T$ is the boson frequency.

Also

$$m_{\vec{Q}}(\vec{k}) = -e\left(\frac{2\pi}{V\omega_{\vec{Q}}\epsilon_\infty}\right)^{1/2} \hat{e}(\hat{Q}\lambda)\cdot\vec{V}_{\vec{k}} \qquad (43)$$

is the matrix element of the $\vec{p}\cdot\vec{A}$ term, and $\sqrt{\vec{k}} = \vec{k}/m^*$ is the group velocity of the Bloch electron of wave vector.

Our task is to generalize the approximation to the self energy in such a way that includes the electron-electron interaction while reducing to Eq. (42) when H_{EE} is set to zero. The first step is to define an effective electron–electron interaction which includes both the Coulomb term H_{EE}, and the Fröhlich interaction term H_{EE}. In fact we can represent just such an interaction diagrammatically as shown in Figure (4). This propagator, $\Gamma(\vec{q}, i\omega_m)$, is just the RPA effective electron-electron interaction which includes the phonon propagator as well as the Coulomb interactions.

A natural generalization of the photon self energy

$$\pi(\vec{Q}, i\Omega_m) = \pi^{(a)}(\vec{Q}, i\Omega_m) + \pi^{(b)}(\vec{Q}, i\Omega_m) + \pi^{(c)}(\vec{Q}, i\Omega m) \qquad (44)$$

given by Eqs. (42) and (44) is obtained by replacing the product of the phonon propagator $D(i\omega_m)$ and $\gamma_o^2(\vec{q})$ everywhere by $\Gamma(\vec{q}, i\omega_m)$. As will be demonstrated below, however, retaining only the three terms of Eq. (44) is an unsatisfactory approximation. Thp reason is that upon letting the Fröhlich coupling strength, $\gamma_o(\vec{q})$, approach

zero, leaving just an interacting electron gas, $\pi_I(\vec{Q}, \Omega)$ remains non zero. But as is well known, an electron gas characterized by a parabolic dispersion relation between electron energy and wave vector is unable to absorb radiation (Hopfied, 1965). Thus we require our approximation for $\pi(\vec{Q}, i\Omega_m)$ to satisfy the constaint $\lim_{\gamma_o \to o} \pi_I(\vec{Q}, \Omega) \to 0$.

We have guessed the form of the additional terms in $\pi(\vec{Q}, i\Omega_m)$ necessary to build into the self energy this momentum conserving constraint. The diagrams corresponding to these terms are the last two of Figure (5), which is a representation of our approximation to the photon self energy. Our guess was motivated by the repeated apprearance of the two terms, (d) and (e), with the first three terms, (a), (b) and (c) in the literature. Indeed, as we shall see below, the approximation for the photon self energy given in Figure (5) satisfies the momentum conserving constraint when $\gamma_o(\vec{q}) \to o$, and is the approximation we shall calculate below.

Before presenting the calculation for the photon self energy we comment on the structure of the effective electron-electron interaction. The expression corresponding to Figure (4) is given by

$$\Gamma(\vec{q}, i\omega_m) = \frac{V_c(\vec{q}) + \gamma_o^2(\vec{q})\, D_o(i\omega_m)}{1 - (V_c(\vec{q}) + \gamma_o^2(\vec{q})\, D_o(i\omega_m))\chi_o(\vec{q}, i\omega_m)} \tag{45}$$

The dielectric constant, $\epsilon(\vec{q}, \omega)$, which characterizes the modification of the bare Coulomb potential by the interactions of the electron gas-phonon system, can be defined by

$$\Gamma(\vec{q}, \omega) = \frac{V_{coul}(\vec{q})}{\epsilon(\vec{q}, \omega)}, \tag{46}$$

where

$$V_{coul}(\vec{q}) = \frac{4\pi e^2}{q^2}. \tag{47}$$

Using Eq. (45) in Eq. (46), we find, after some algebra,

$$\epsilon(\vec{q}, \omega) = \epsilon_\infty - \frac{4\pi e^2}{q^2}\chi_o(\vec{q}, \omega) + \frac{4\pi n_c e^{*2}}{M(\omega_{To}^2 - \omega^2)} \tag{48}$$

Here ω_{T0} is the T0 optical phonon frequency at $\vec{q} \to O$. We note from Eq. (46) that $\epsilon(\vec{q}, \omega)$ vanishes at the same frequencies for which there are poles of $\Gamma(\vec{q}, \omega)$; these are the normal mode frequencies of the interacting system, so $\Gamma(\vec{q}, \omega)$ can be regarded as a propagator that describes the collective excitations of the system. When γ_o is set to zero, the pole of $\Gamma(\vec{q}, \omega)$ occurs at the wave vector dependent plasma frequency, while the Landau damping of collective electron excitations is accounted for in the region of $(\vec{q}, \omega)$ space for which the imaginary part of $\chi_o(\vec{q}, \omega)$ is non zero.

We now turn to the calculation of $\pi(\vec{Q}, i\Omega m)$ given by

$$\pi(\vec{Q}, i\Omega m) = \pi^{(a)}(\vec{Q}, i\Omega m) + \pi^{(b)}(\vec{Q}, i\Omega m) + \pi^{(c)}(\vec{Q}, i\Omega m) + \pi^{(d)}(\vec{Q}, i\Omega m) + \pi^{(e)}(\vec{Q}, i\Omega m), \tag{49}$$

where each term on the right is represented by a diagram is Figure(5).

We begin with the diagrams in Figure (5) corresponding to $\pi^{(a)}$ and $\pi^{(b)}$. These have the form (for small $\vec{Q}$, with a factor of two for spin)

$$\pi^{(a)}(\vec{Q}, i\Omega m) = \frac{2}{\beta^2}\sum_{\vec{k}\,\vec{q}}\sum_{i\omega_n\, i\omega_m} m_{\vec{Q}}(\vec{k}, m_{\vec{Q}}(\vec{k})^* P(\vec{q}, i\omega_m) \times G(\vec{k}, i\omega_n) G^2(\vec{k}, i\omega_n + i\Omega m) \times G(\vec{k}+\vec{q}, i\omega_n + i\omega_m + i\Omega m) \tag{50a}$$

$$\pi^{(b)}(\vec{Q}, i\Omega_{ml}) = \frac{2}{\beta^2}\sum_{\vec{k}\,\vec{q}}\sum_{i\omega_n\, i\omega_m} m_{\vec{Q}}(\vec{k}, m_{\vec{Q}}(k)^* \Gamma(\vec{q}, i\omega_n) \times G^2(\vec{k}, i\omega_n) G(\vec{k}, i\omega_n + i\Omega m) \times G(\vec{k}+\vec{q}, i\omega_n + i\omega_m) \tag{50b}$$

We rearrange $\pi^{(a)}(\vec{Q}, i\Omega_{ml})$. Using the form of $G(\vec{K}, Z)$ given by Eq. (40) we have the identity

$$G(\vec{k}, i\omega_n) G(\vec{k}, i\omega_n + i\Omega m) = \frac{1}{i\Omega m}[G(\vec{k}, i\omega_n) - G(\vec{k}, i\omega_n + i\Omega m] \tag{51}$$

This may be used to split $\pi^{(a)}(\vec{Q}, i\Omega m)$ into two distinct parts $\pi^{(a)}(\vec{Q}, i\Omega m)$ and $\pi^{(a2)}(\vec{Q}, i\Omega m)$:

$$\pi^{(a1)}(\vec{Q}, i\Omega m) = \frac{2}{\beta^2}\sum_{\vec{k}\,\vec{q}}\sum_{i\omega_n\, i\omega_m} \frac{m_{\vec{Q}}(\vec{k}) m_{\vec{Q}}(\vec{k})^* \Gamma(\vec{q}, i\omega_m)}{i\Omega m} \times G(\vec{k}, i\omega_n) G(\vec{k}, i\omega_n + i\Omega m) \times G(\vec{k}+\vec{q}, i\omega_n + i\omega_m + i\omega_m) \tag{52a}$$

and

$$\pi^{(a2)}(\vec{Q}, i\Omega m) = -\frac{2}{\beta}\sum_{\vec{k}\,\vec{q}}\sum_{i\omega_n\, i\omega_m} \frac{m_{\vec{Q}}(\vec{k}) m_{\vec{Q}}(\vec{k})^* \Gamma(\vec{q}, i\omega_m)}{i\Omega m} \times G(\vec{k}+\vec{q}, i\omega_n + i\Omega m + i\omega_m) \times G^2(\vec{k}, i\omega_n + i\Omega m) \tag{52b}$$

We will presently see that $\pi^{(a_2)}$ is exactly cancelled by a piece of $\pi^{(b)}$.

The summations over internal frequencies are next converted in the usual fashion into contour integrations. After performing the sum over $i\omega_n$ in $\pi^{(a_1)}$,

$$\pi^{(a_1)}(\vec{Q}, i\Omega m) = \frac{2}{\beta} \sum_{\vec{k}\vec{q}} \frac{m_{\vec{Q}}(\vec{k}) m_{\vec{Q}}(\vec{k})^*}{(i\Omega m)^2} [f(\vec{k}) - f(\vec{k}+\vec{q})] \times \sum_{i\omega_m} \Gamma(\vec{q}, i\omega_m) \left[\frac{-1}{i\omega_m + \epsilon(\vec{k}) - \epsilon(\vec{k}+\vec{q})} + \frac{1}{i\Omega m + i\omega_m + \epsilon(\vec{k}) - \epsilon(\vec{k}+\vec{q})} \right] \tag{53}$$

We rearrange $\pi^{(b)}(\vec{Q}, i\Omega m)$ in a similar fashion. This gives

$$\pi^{(b)}(\vec{Q}, i\Omega m) = \pi^{(b_1)}(\vec{Q}, i\Omega m) + \pi^{(b_2)}(\vec{Q}, i\Omega m), \tag{54}$$

where

$$\pi^{(b_1)}(\vec{Q}, i\Omega_m) = -\frac{2}{\beta^2} \sum_{\vec{k}\vec{q}} \sum_{\substack{i\omega_n \\ i\omega_m}} \frac{m_{\vec{Q}}(\vec{k}) m_{\vec{Q}}(\vec{k})^*}{i\Omega_m} \Gamma(\vec{q}, i\omega_m) \times G(\vec{k}, i\omega_n + i\Omega_m) G(\vec{k}, i\omega_n) \times G(\vec{k}+\vec{q}, i\omega_n + i\omega_m) \tag{55a}$$

and

$$\pi^{(b_2)}(\vec{Q}, i\Omega_m) = \frac{2}{\beta^2} \sum_{\vec{k}\vec{q}} \sum_{\substack{i\omega_n \\ i\omega_m}} \frac{m_{\vec{Q}}(\vec{k})\, m_{\vec{Q}}(\vec{k})^*}{i\Omega_m} \Gamma(\vec{q}, i\omega_n) \times G^2(\vec{k}, i\omega_n) G(\vec{k}+\vec{q}, i\omega_n + i\omega_m). \tag{55b}$$

We now shift ω_n by Ω_m in $\pi^{(b_2)}$ and obtain the identity $\pi^{(b_2)} = -\pi^{(a_2)}$. Thus we have that the sum of these two self energy terms reduces to $\pi^{(a)+\pi(b)} = \pi^{(a_1)+\pi(b_1)}$. Performing the sum over $i\omega_n$ in $\pi^{(b_1)}$ just as we did in Eq. (53) yields after letting $i\omega_m \to i\omega_m + i\Omega_m$ in the result,

$$\pi^{(a)}(\vec{Q}, i\Omega_m) + \pi^{(b)}(\vec{Q}, i\Omega_m) = \frac{2}{\beta(i\Omega_m)^2} \sum_{\vec{k}\vec{q}} m_{\vec{Q}}(\vec{k}) m_{\vec{Q}}(\vec{k})^* \times [f(\vec{k}) - f(\vec{k}+\vec{q})] \times \sum_{i\omega_m} \left[\frac{1}{i\Omega_m + i\omega_m + \epsilon(\vec{k}) - \epsilon(\vec{k}+\vec{q})} - \frac{1}{i\omega_m + \epsilon(\vec{k}) - \epsilon(\vec{k}+\vec{q})} \right] \times [\Gamma(\vec{q}, i\omega_m) - \Gamma(\vec{q}, i\omega_m + i\Omega_m)] \tag{56}$$

where $f(\vec{k})$ is the Fermi Dirac number density $f(\vec{k}) = (e^{\beta(\epsilon(\vec{k})-\mu)}+1)^{-1}$.

To combine the result in Eq. (56) with the contributions from diagrams (d) and (e) in Figure (5), it is convenient to split the right hand side of Eq. (56) into two parts using the explicit form of $\Gamma(\vec{q}, i\omega_m)$. One has

$$\pi^{(a)} + \pi^{(b)} = [\pi^{(a)+\pi(b)}]_I + [\pi^{(a)+\pi(b)}]_{II}, \tag{57}$$

where

$$[\pi^{(a)+\pi(b)}]_I = \frac{-2}{(i\Omega_m)^2 \beta} \sum_{\vec{k}\vec{q}} \sum_{i\omega_m} m_{\vec{Q}}(\vec{k}) m_{\vec{Q}}(\vec{k})^* \times [f(\vec{k}) - f(\vec{k}+\vec{q})] \times \left[\frac{1}{i\omega_m + \epsilon(\vec{k}) - \epsilon(\vec{k}+\vec{q})} - \frac{1}{i\omega_m + i\Omega_m + \epsilon(\vec{k}) + \epsilon(\vec{k}+\vec{q})} \right] \times \frac{\tilde{V}(\vec{q}, i\omega_m) - \tilde{V}(\vec{q}, i\omega_m + i\Omega_m)}{[1 - \tilde{V}(\vec{q}, i\omega_m)\chi_o(\vec{q}, i\omega_m)]\,[1 - \tilde{V}(\vec{q}, i\omega_m + i\Omega_m)\chi_o(\vec{q}, i\omega_m + i\Omega_m)]} \tag{58a}$$

and

$$[\pi^{(a)+\pi(b)}]_{II} = \frac{-2}{(i\Omega_m)^2 \beta} \sum_{\substack{\vec{k}\vec{q} \\ \vec{k}}} \sum_{i\omega_m} m_{\vec{Q}}(\vec{k}) m_{\vec{Q}}(\vec{k})^* \times [f(\vec{k}) - f(\vec{k}+\vec{q})] \times [f(\vec{k}) - f(\vec{k}+\vec{q})] \times \left[\frac{1}{i\Omega_m + i\omega_m + \epsilon(\vec{k}) - \epsilon(\vec{k}+\vec{q})} - \frac{1}{i\omega_m + \epsilon(\vec{k}) - \epsilon(\vec{k}+\vec{q})} \right] \times \left[\frac{1}{i\Omega_m + i\omega_m + \epsilon(\vec{k}) - \epsilon(\vec{k}+\vec{q})} - \frac{1}{i\omega_m + \epsilon(\vec{k}) - \epsilon(\vec{k}+\vec{q})} \right] \times \frac{\tilde{V}(\vec{q}, i\omega_m)\tilde{V}(\vec{q}, i\omega_m + i\Omega_m)}{[1 - \tilde{V}(\vec{q}, i\omega_m)\chi_o(\vec{q}, i\omega_n)]\,[1 - \tilde{V}(\vec{q}, i\omega_m + i\Omega_m)\chi_o(\vec{q}, i\omega_m + i\Omega_m)]}, \tag{58b}$$

where $\tilde{V}(\vec{q}, i\omega_m) = V_c(\vec{q}) + \gamma_o^2(\vec{q}) D_o(i\omega_m)$.

We turn now to $\pi^{(c)}$. Using manipulations similar to those above we obtain,

$$\pi^{(c)}(\vec{Q}, i\Omega_m) = \frac{1}{i\Omega_m\beta}\sum_{\vec{k}\vec{q}}\sum_{i\omega_m}\frac{m_{\vec{Q}}(\vec{k})m_{\vec{Q}}(\vec{k}+\vec{q})}{i\omega_m+\epsilon(\vec{k})-\epsilon(\vec{k}+\vec{q})}$$

$$\times\frac{[f(\vec{k})-f(\vec{k}+\vec{q})]\,[\Gamma(\vec{q}, i\omega_m)-\Gamma(\vec{q}, i\omega_m+i\Omega_m)]}{i\omega_m+i\Omega_m+\epsilon(\vec{k})-\epsilon(\vec{k}+\vec{q})} \quad (59)$$

Again using the explicit expression for $\Gamma(\vec{q}, i\omega_m)$ we rearrange $\pi^{(c)}$:

$$\pi^{(c)}(\vec{Q}, i\Omega_m) = \pi^{(c1)}(\vec{Q}, i\Omega_m)+\pi^{(c2)}(\vec{Q}, i\Omega_m), \quad (60)$$

where

$$\pi^{(c_1)}(\vec{Q}, i\Omega_m) = \frac{2}{(i\Omega_m)^2\beta}\sum_{\vec{k}\vec{p}}\sum_{i\omega_m} m_{\vec{Q}}(\vec{k})m_{\vec{Q}}(\vec{k}+\vec{q})$$

$$\times [f(\vec{k})-f(\vec{k}+\vec{q})]$$

$$\times \left[\frac{1}{i\omega_m+\epsilon(\vec{k})-\epsilon(\vec{k}+\vec{q})} - \frac{1}{i\omega_m+i\Omega_m+\epsilon(\vec{k})+\epsilon(\vec{k}+\vec{q})}\right] \quad (61a)$$

$$\times \frac{(\tilde{V}(\vec{q}, i\omega_m) - \tilde{V}(\vec{q}, i\omega_m+i\Omega_m))}{[1-\tilde{V}(\vec{q}, i\omega_m)\chi_o(\vec{q}, i\omega_m)]\,[1-\tilde{V}(\vec{q}, i\omega_m+i\Omega_m)\chi_o(\vec{q}, i\omega_m+i\Omega_m)]},$$

and

$$\pi^{(c_2)}(\vec{Q}, i\Omega_m) = \frac{2}{(i\Omega_m)^2\beta}\sum_{\vec{k}\vec{k}'\vec{q}}\sum_{i\omega_m} m_{\vec{Q}}(\vec{k})m_{\vec{Q}}(\vec{k}+\vec{q})^*$$

$$\times [f(\vec{k})-f(\vec{k}+\vec{q})]\,[f(\vec{k}')-f(\vec{k}'+\vec{q})]$$

$$\times \Gamma(\vec{q}, i\omega_m)\Gamma(\vec{q}, i\omega_m+i\Omega_m) \quad (61b)$$

$$\times \left[\frac{1}{i\omega_m+i\Omega_m+\epsilon(\vec{k})-\epsilon(\vec{k}+\vec{q})} - \frac{1}{i\omega_m+\epsilon(\vec{k})-\epsilon(\vec{k}+\vec{q})}\right]$$

$$\times \left[\frac{1}{i\omega_m+i\Omega_m+\epsilon(\vec{k}')-\epsilon(\vec{k}'+\vec{q})} - \frac{1}{i\omega_m+\epsilon(\vec{k}')-\epsilon(\vec{k}'+\vec{q})}\right].$$

We note at this point that $\pi^{(c1)}(\pi^{(c2)})$ is identical to $[\pi^{(a)}+\pi^{(b)}]_I \times [\pi^{(a)}+\pi^{(b)}]_{II}$, with the replacement of a single electron-photon matrix element $m_{\vec{Q}}(\vec{k}+\vec{q})^*$ by $m_{\vec{Q}}(\vec{k})^*$.

The remaining terms $\pi^{(d)}$ and $\pi^{(e)}$ are evaluated just as the earlier ones. Performing the sums over the fermion internal frequencies yields results that may be written

$$\pi^{(d)}(\vec{Q}, i\Omega_m) = \frac{2}{\beta(i\Omega_m)^2}\sum_{\vec{k}\vec{k}'\vec{q}}\sum_{i\omega_m} m_{\vec{Q}}(\vec{k})m_{\vec{Q}}(\vec{k}')^*$$

$$\times [f(\vec{k}')-f(\vec{k}'+\vec{q})]$$

$$\times [f(\vec{k})-f(\vec{k}'+\vec{q})]\,\Gamma(\vec{q}, i\omega_m)\Gamma(\vec{q}, i\omega_m+i\Omega_m) \quad (62a)$$

$$\times \left[\frac{1}{i\Omega_m+i\omega_m+\epsilon(\vec{k})-\epsilon(\vec{k}+\vec{q})} - \frac{1}{i\omega_m+\epsilon(\vec{k})-\epsilon(\vec{k}+\vec{q})}\right]$$

$$\times \left[\frac{1}{i\Omega_m+i\omega_m+\epsilon(\vec{k}')-\epsilon(\vec{k}'+\vec{q})} - \frac{1}{i\omega_m+\epsilon(\vec{k}')-\epsilon(\vec{k}'+\vec{q})}\right]$$

and

$$\pi^{(b)}(\vec{Q}, i\Omega_m) = \frac{-2}{\beta(i\Omega_m)^2}\sum_{\vec{k}\vec{k}'\vec{q}}\sum_{i\omega_m} m_{\vec{Q}}(\vec{k})\,m_{\vec{Q}}(\vec{k}'+\vec{q})^*$$

$$\times [f(\vec{k})-f(\vec{k}+\vec{q})]\,[f(\vec{k}')-f(\vec{k}'+\vec{q})]$$

$$\times \Gamma(\vec{q}, i\omega_m)\Gamma(\vec{q}, i\omega_m+i\Omega_m) \quad (62b)$$

$$\times \left[\frac{1}{i\Omega_m+i\omega_m+\epsilon(\vec{k}')-\epsilon(\vec{k}'+\vec{q})} - \frac{1}{i\omega_m+\epsilon(\vec{k}')-\epsilon(\vec{k}'+\vec{q})}\right]$$

$$\times \left[\frac{1}{i\Omega_m+i\omega_m+\epsilon(\vec{k}')-\epsilon(\vec{k}'+\vec{q})} - \frac{1}{i\omega_m+\epsilon(\vec{k}')-\epsilon(\vec{k}'+\vec{q})}\right]$$

Putting all the results together yields $\pi^{(T)} = \pi_1^{(T)}+\pi_2^{(T)}$, where

$$\pi_1^{(T)}(\vec{Q}, i\Omega_m) = \frac{2}{\beta(i\Omega_m)^2}\sum_{\vec{k}\vec{k}'\vec{q}}\sum_{i\omega_m} m_{\vec{Q}}(\vec{k})$$

$$\times [m_{\vec{Q}}(\vec{k}+\vec{q})-m_{\vec{Q}}(\vec{k})+m_{\vec{Q}}(\vec{k}')-m_{\vec{Q}}(\vec{k}'+\vec{q})]^*$$

$$\times [f(\vec{k})-f(\vec{k}+\vec{q})]\,[f(\vec{k}')-f(\vec{k}'+\vec{q})]$$

$$\times \Gamma(q, i\omega_m)\Gamma(\vec{q}, i\omega_m+i\Omega_m) \quad (63a)$$

$$\times \left[\frac{1}{i\omega_m+i\Omega_m+\epsilon(\vec{k}')-\epsilon(\vec{k}'+\vec{q})} - \frac{1}{i\omega_m+\epsilon(\vec{k})-\epsilon(\vec{k}+\vec{q})}\right]$$

$$\times \left[\frac{1}{i\omega_m+i\Omega_m+\epsilon(\vec{k}')-\epsilon(\vec{k}'+\vec{q})} - \frac{1}{i\omega_m+\epsilon(\vec{k}')-\epsilon(\vec{k}'+\vec{q})}\right],$$

and

$$\pi_2^{(T)}(\vec{Q}, i\Omega_m) = \frac{2}{\beta(i\Omega_m)^2}\sum_{\vec{k}\vec{q}}\sum_{i\omega_m} m_{\vec{Q}}(\vec{k})$$

$$\times [m_{\vec{Q}}(\vec{k}+\vec{q})-m_{\vec{Q}}(\vec{k})]^* \times [f(\vec{k})-f(\vec{k}+\vec{q})]$$

$$\times \left[\frac{1}{i\omega_m+i\Omega_m+\epsilon(\vec{k})-\epsilon(\vec{k}+\vec{q})} - \frac{1}{i\omega_m+\epsilon(\vec{k})-\epsilon(\vec{k}+\vec{q})}\right] \quad (63b)$$

$$\times \frac{\tilde{V}(\vec{q}, i\omega_m) - \tilde{V}(\vec{q}, i\omega_m)}{[1-\tilde{V}(\vec{q}, i\omega_m)\chi_o(\vec{q}, i\omega_m)]}$$

$$\times \frac{1}{[1-\tilde{V}(\vec{q}, i\omega_m+i\Omega_m)\chi_o(\vec{q}, i\omega_m+i\Omega_m)]}$$

But since $m_{\vec{Q}}(\vec{k}) \propto \hat{e}\cdot\vec{V}_{\vec{k}}$, $m_{\vec{Q}}(\vec{k}+\vec{q}) - m_{\vec{Q}}(\vec{k}) + m_{\vec{Q}}(\vec{k}') - m_{\vec{Q}}(\vec{k}'+\vec{q}) = 0$ for electrons in a parabolic band, and $\pi_1^{(T)}$ vanishes identically. We thus have that $\pi^{(T)} = \pi_2^{(T)}$. One may cast $\pi_2^{(T)}$ into the form

$$\pi^{(T)}(\vec{Q}, i\Omega_m) = \frac{1}{(i\Omega_m)^2\beta} \sum_{\vec{q}, i\omega_m} m^2_{\vec{Q}}(\vec{q})\gamma_0^2(q)$$

$$\times \frac{[D_0(i\omega_m)-D_0(i\omega_m+i\Omega_n)]}{[1-\tilde{V}(\vec{q}, i\omega_m)\chi_0(\vec{q}, i\omega_m)]} \qquad (64)$$

$$\times \frac{[\chi_0(\vec{q}, i\omega_m+i\Omega_m)-\chi_0(\vec{q}, i\omega_m)]}{[1-\tilde{V}(\vec{q}, i\omega_m+i\Omega_m)\chi_0(\vec{q}, i\omega_m+i\Omega_m)]} .$$

It is evident that in the absence of electron phonon coupling, we find $\pi^{(T)}(i\Omega_m)$ vanishes in the limit as $\vec{Q} \to o$, for electrons in a parabolic band. This is insured by the factor of $\gamma_0^2(\vec{q})$ that has been isolated by the manipulations here.

The last step is to perform the sum over, which we calculate in the standard fashion. We define

$$U(\vec{q}, i\Omega_m) = \frac{1}{\beta} \sum_{i\omega_m} \frac{[D_0(i\omega_m)-D_0(i\omega_m+i\Omega_m)]}{[1-\tilde{V}(\vec{q}, i\omega_m)\chi_0(\vec{q}, i\omega_m)]}$$

$$\times \frac{[\chi_0(\vec{q}, i\omega_m+i\Omega_m) - \chi_0(\vec{q}, i\omega_m)]}{[1-\tilde{V}(\vec{q}, i\omega_m+i\Omega_m)\chi_0(\vec{q}, i\omega_m+i\Omega_m)]} \qquad (65)$$

$$= \frac{1}{\beta}\sum_{i\omega_m} \tilde{U}(q, z, i\Omega_m)\,|_{\,z = i\omega_m}$$

Since both $\chi_0(\vec{q}, Z)$ and $D_0(z)$, the analytic continuations of the function defined by Eqs. (38) and (39), respectively, are analytic everywhere away from the branch lines along the real z axis, we write

$$\bar{U}(q, z, i\Omega_m) = \frac{1}{2\pi i}\int_C \frac{\tilde{U}(q, w, i\Omega_m)}{\omega - z}\, d\omega \qquad (66)$$

where C is the contour shown in Figure (6).

Since $\tilde{U}(\vec{q}, z, i\Omega_m)$ falls off sufficiently fast as $|z| \to \infty$, we have, upon performing the sum over $i\omega_m$,

$$U(\vec{q}, i\Omega_m) = \frac{1}{2\pi i}\int_{-\infty}^{+\infty} d\omega n(\omega)\Big\{\tilde{U}(\vec{q}, \omega+i\epsilon, i\Omega_m) - \tilde{U}(\vec{q}, \omega-i\epsilon, i\Omega_m) + \tilde{U}(\vec{q}, \omega-i\Omega_m+i\epsilon, i\Omega_m) - \tilde{U}(\vec{q}, \omega+i\Omega_m-i\epsilon, i\Omega_m)\Big\} , \qquad (67)$$

where $n(\omega)$ is the Bose-Einstein number density, $n(\omega) = (e^{\beta\omega} - 1)^{-1}$, and ϵ is a positive infinitesimal quantity.

As outlined in the discussion preceding Eq. (27), we are interested in the analytic continuation of $U(\vec{q}, i\Omega_m)$ to $U(\vec{q}, \Omega+i\eta)$. Then with $\Lambda(\vec{q}, z) = (1 - \tilde{V}(\vec{q}, z)\chi_0(\vec{q}, z))^{-1}$, Eq.(67) becomes

$$\begin{aligned} U(\vec{q}, \Omega+i\eta) = \int_{-\infty}^{+\infty} d\omega n(\omega)\Big\{ & \Lambda(\vec{q}, \omega+\Omega^+)\,[\,(\chi_0(\vec{q}, \omega+\Omega^+) \\ & - \chi_0(\vec{q}, \omega^+))\times(D_0(\omega+\Omega^+) - D_0(\omega^+))\,\Lambda(q, \omega^+) \\ & - (\chi_0(\vec{q}, \omega+\Omega^+ - \chi_0(\vec{q}, \omega^-))\times(D_0(\omega+\Omega^+) \\ & - D_0(\omega^-))\,\Lambda(\vec{q}, \omega^-)] + \Lambda(\vec{q}, \omega-\Omega^-)\,[(\chi_0(q, \omega-\Omega^-) \\ & - \chi_0(\vec{q}, \omega^+))\times(D_0(\omega-\Omega^-) - D_0(\omega^+))\,\Lambda(\vec{q}, \omega^+) \\ & - (\chi_0(\vec{q}, \omega-\Omega^-) - \chi_0(\vec{q}, \omega^-))\,(D_0(\omega-\Omega^-) \\ & - D_0(\omega^-))\,\Lambda(\vec{q}, \omega^-))\Big\} , \end{aligned} \qquad (68)$$

where the + (–) sign indicates $+i\eta$ ($-i\eta$), with η a positive infinitesimal frequency.

In the first half of the terms in Eq. (68) we change the integration variable to $\omega-\Omega/2$, while in the remaining terms we change to $\omega+\Omega/2$. Then, using the relations

$$\chi_0(\vec{q}, -\omega\pm i\eta) = F_1^0(\vec{q}, \omega) \mp F_2^0(\vec{q}, \omega) = \chi_0(\vec{q}, \omega\mp i\eta) , \qquad (69a)$$

$$D_0(-\omega\pm i\eta) = D_0(\omega\mp\eta) \qquad (69b)$$

and

$$\Lambda(\vec{q}, -\omega\pm\eta) = \Lambda(\vec{q}, \omega\mp\eta) , \qquad (69c)$$

which follow from the eveness(oddness) with respect to ω of $F_1^0(q, \omega)$ ($F_2^0(q, \omega)$, Eq. (38), and the definition of $\Lambda(\vec{q}, z)$, respectively, we find, after some straightforward manipulations,

$$\begin{aligned} \mathrm{Im}\,U(\vec{q}, \Omega) = \frac{1}{\pi}\int_0^{\infty} d\omega(n(\omega-\Omega/2) - n(\omega+\Omega/2)) \\ \times\Big\{\mathrm{R_e}\,[\Lambda(\vec{q}, \omega+\Omega/2^+)\Lambda(\vec{q}, \omega-\Omega/2^-) \\ \times(\chi_0(\vec{q}, \omega+\Omega/2^+) - \chi_0(\vec{q}, \omega-\Omega/2^-)) \\ \times(D_0(\omega+\Omega/2^+) - D_0(\omega-\Omega/2^-))] \\ - \mathrm{R_e}\,[\Lambda(\vec{q}, \omega-\Omega/2^-)\Lambda(\vec{q}, \omega-\Omega/2^+) \\ \times(\chi_0(\vec{q}, \omega-\Omega/2^+)\chi_0(\vec{q}, \omega-\Omega/2^-)) \\ \times(D_0(\omega-\Omega/2^+) - D_0(\omega-\Omega/2^-))]\Big\} \end{aligned} \qquad (70)$$

Further algebraic rearrangement enables us to write a final form for the imaginary part of $\pi^{(T)}(\vec{Q}\ \Omega+i\mu)$, the analytic continuation of given in Eq. (64):

$$I_m \pi^{(T)}(\vec{Q}, \Omega+i\mu) = \frac{1}{2\pi\Omega^2} \sum_{\vec{q}} m_Q{}^2(\vec{q})\gamma_0^2(\vec{q})\sinh(\beta\Omega/2)$$

$$\times \int_0^\infty \frac{dx}{\sinh(\beta x_+/_2)\sinh(\beta x_-/_2)} \times \sum_{\sigma=\pm1} [I_m\{\Lambda(\vec{q}, x_\sigma + i\mu) D_0(x_\sigma + i\mu)\} \times I_m\{\Lambda(\vec{q}, x_{-\sigma}+i\mu)\chi_0(\vec{q}, x_{-\sigma} + i\mu)\} - I_m\{\Lambda(\vec{q}, x_{-\sigma}+i\mu)\chi_0(\vec{q}, x_{-\sigma}+i\mu) D_0(x_{-\sigma} + i\mu)\} \times I_m\{\Lambda(\vec{q}, x_{\sigma}+i\mu)\}] , \quad (71)$$

where we have used

$$X_\sigma = x + \sigma\,\Omega/2 \quad (72)$$

with $\sigma=\pm1$.

Finally, using Eq. (71) in Eq. (36) for the absorptoin coefficient, and averaging over the direction of $\vec{q}$, we obtain

$$\alpha(\Omega) = \frac{2e^2\sinh(\beta\Omega/2)}{3c\sqrt{\epsilon_R}\ m^{*2}\Omega^3} \int_0^\infty \frac{dx}{\sinh(\beta x_+/_2)\sinh(\beta x_-/_2)} \times \sum_{\vec{q}} q^2\gamma^2(q) \sum_{\sigma=\pm1} \times [I_m\{\Lambda(\vec{q}, x_\sigma+i\mu) D_0(x_{\sigma+i\mu})\}$$

$$\times I_m\{\Lambda(\vec{q}, x_{-\sigma}+i\mu)\chi_0(q, x_{-\sigma}+i\mu)\}$$

$$- I_m\ \Lambda(\vec{q}, x_{-\sigma}+i\mu)\chi_0(\vec{q}, x_{-\sigma}+i\mu) D(x_{-\sigma}+i\mu)]$$

$$\times I_m\{\Lambda(\vec{q}, x_\sigma+i\mu)\}] . \quad (73)$$

This expression for the absorption coefficient is our principal result. Further calculation must proceed numerically due to the form of $\chi_0(\vec{q}, \omega)$ and $\Lambda(\vec{q}, \omega)$. Before caryrying out an analysis of Eq. (73), however, we consider two limiting cases in which our expression for the absorption coefficient reduces to forms appropriate to simpler models.

Concluding remarks

In this paper we saw that, contrary to the light scattering studies, the absorption spectrum in the Drude tail is nearly unaffected by coupling of the electron plasma with the lattice LO phonon. The leason was clear: The excitations needed to take an electron from its virtual excited state are those with characteristic wave vector considerably longer than the Thomas-Fermi wave vector. In this small wavelength region the electron gas is unable to form a well defined collective oscillating mode that can couple with the LO phonon, and the excitations are just the nearly bare phonon and the single particle electron states.

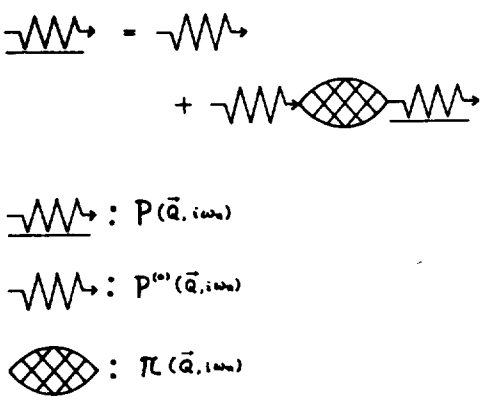

Figure 1. Diagrammatic representation of the Dyson equation satisfied by the photon Green's function. The single and double lines represent the zero order and exact Gree's functions, respectively. The self energy part is represented by the crosshatched bubble.

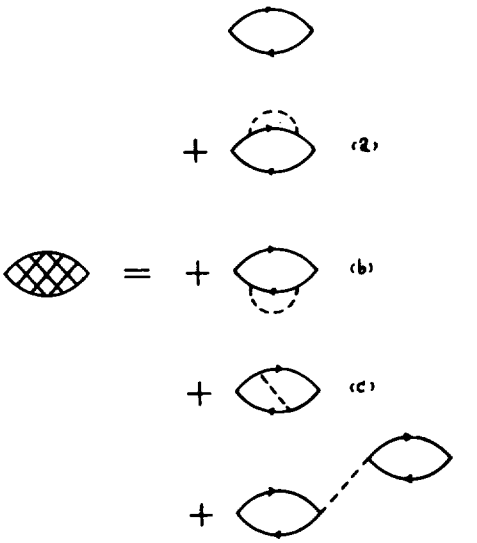

Figure 2. Lowest nonvanishing self energy terms when the electron-electron interaction is taken to vanish. The dashed line represents a phonon propagator.

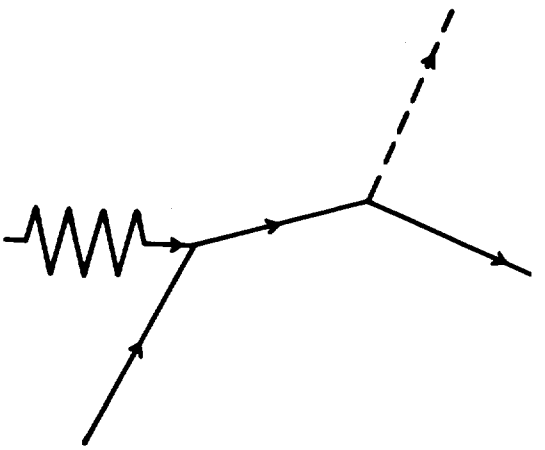

Figure 3. An illustration of a second order process which provides the LO phonon mediated contribution to. The electron is coupled to the LO phonon by the Fröhlich interaction in polar materials.

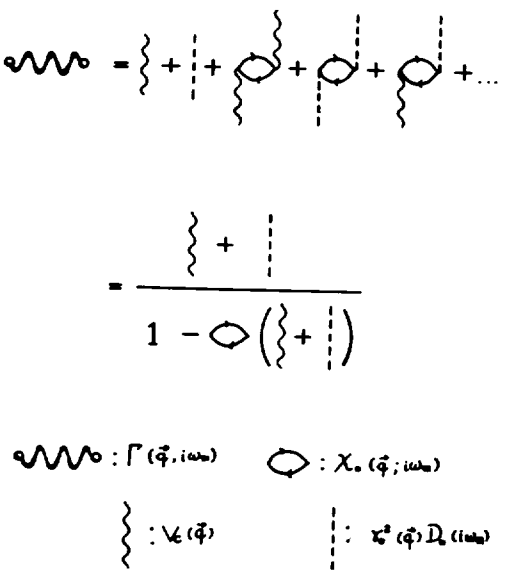

Figure 4. A generalization of the effective electron interaction obtained within the RPA is seen to describe the interaction of the electron plasma with the phonon lattice.

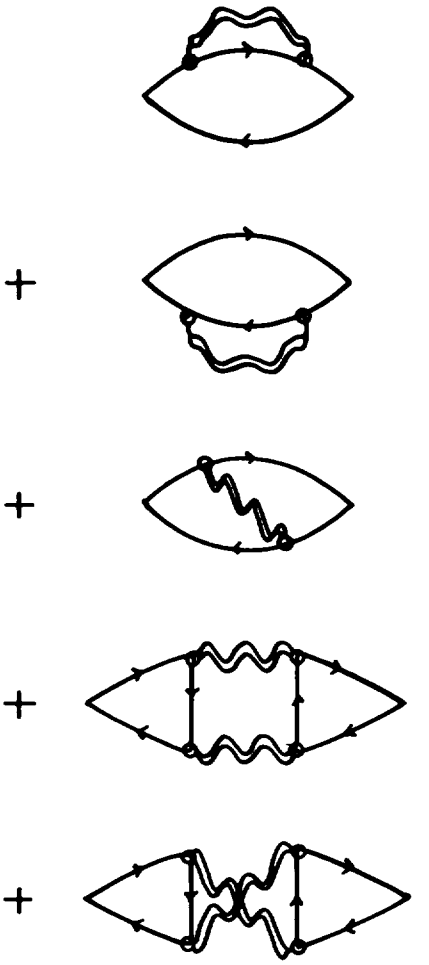

Figure 5. Approximation to the photon self energy used in calculating the absorption coefficient. This is a mementum conserving approximation that vanishes when the strength of the electron phonon interaction goes to zero.

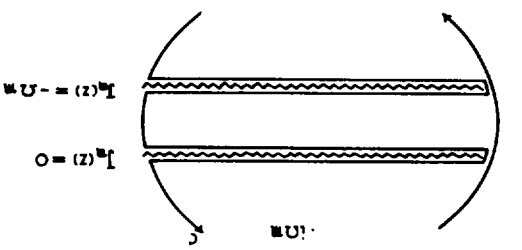

Figure 6. Contour of integration in Eq. (66).

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