On a Lodato Prenearness Space

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Lodato Prenearness 空間에 관하여

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0. Introduction

H. Herrlich [1] has introduced nearness spaces as an axiomatization of the concept of nearness of arbitrary collection of sets. Since that time, these spaces have been used for several different purpose by topologist.

In this paper we will consider a nearness space (X, ξ) which does not satisfy the following condition:

$$\text{If } \mathcal{A} \vee \mathcal{B} \in \xi \text{ then } \mathcal{A} \in \xi \text{ or } \mathcal{B} \in \xi.$$

We call the space (X, ξ) as a Lodato prenearness space.

In particular, the category of Lodato pre-N-spaces and N-maps is denoted by LP-Near and we investigate the basic categorical properties of LP-Near. And also we try to compare LP-Near with different structure.

In the present note, category theory provides the proper tool for constructing some theorems.

I. Categorical Preliminaries

1.1. Definition. A source in a catgeory $\underline{A}$ is a pair $(X,(f_i)_{i\in I})$, where X is an $\underline{A}$ -object and $(f_i:X \to X_i)_{i\in I}$ is a family of $\underline{A}$-morphisms each with domain X. In this case X is called the *domain of the source* and the family $(X_i)_{i\in I}$ is called the *codomain of the source.*

1.2. Definition. Let $\underline{A}$ be a category and $((Y_i,\eta_i)_{i\in I})$ a family of ojbects in $\underline{A}$ indexed by a class I, and let X be a set and $(f_i:X \to Y_i)_{i\in I}$ a source of maps indexed by I.

An $\underline{A}$ -structure ξ on X is called *initial* with respect to $(X, (f_i), (Y_i,\eta_i)_{i\in I})$ if the following conditions are satisfied:

(1) for each $i\in I, f_i:(X,\xi) \to (Y_i,\eta_i)$ is an $\underline{A}$-morphism.

(2) if (Z,ζ) is an $\underline{A}$-object and $g:Z \to X$ is a map such that for each $i\in I$, the map $f_i \cdot g:(Z,\zeta) \to (Y_i,\eta_i)$ is an $\underline{A}$-morphism, then $g:(Z,\zeta) \to (X,\xi)$ is an $\underline{A}$-morphism.

In this case, the source $(f_i:(X,\xi) \to (Y,\eta_i))_{i\in I}$ is also call initial.

Dually we define the final structures.

1.3. Definition. A category $\underline{A}$ is said to be *topological* if for each set X, for any family $((Y_i,\xi_i))_{i\in I}$ of $\underline{A}$-objects and for any family $(f_i:X \to Y_i)_{i\in I}$ of maps, there exists an $\underline{A}$ -structure on X which is initial with respect to $(X, (f_i)_{i\in I}, ((Y_i,\xi_i))_{i\in I})$.

1.4. Definition. Let $\underline{A}$ be a category.

(1) The $\underline{A}$-*fibre* of a set X is the class of all $\underline{A}$-structures on X.

(2) $\underline{A}$ is called *properly fibred* if it satisfies the following conditions:

(i) for each set X, the $\underline{A}$-fibre of X is a set,

(ii) for each one-element set X, the $\underline{A}$-fibre of X has precisely one element,

(iii) if ξ and η are $\underline{A}$-structures on X such that $1_X:(X,\xi) \to (X,\eta)$ and $1_X:(X,\eta) \to (X,\xi)$ are morphisms, then $\xi=\eta$.

1.5. Definition. Let $\underline{C}$ be a category and $\underline{A}$ a subcategory of $\underline{C}$. For any $X \in \underline{C}$, a $\underline{C}$-morphism $f:X \to A$ is called the $\underline{A}$-reflection of X if $A \in \underline{A}$ and for any $A' \in \underline{A}$ and a $\underline{C}$-morphism $g:X \to A'$, there exist a unique $\underline{A}$-morphim $\bar{f}:A \to A'$ with $\bar{f} \cdot f = g$. If every object of $\underline{C}$ has the $\underline{A}$-*reflection*, then $\underline{A}$ is called a *reflective subcategory* of $\underline{C}$.

1.6. Definition. Let P be a class of p-morphisms of $\underline{C}$ and let $\underline{A}$ be a reflective subcategory of $\underline{C}$. If $\underline{A}$-reflection of $\underline{C}$ belongs to P, then $\underline{A}$ is *p-reflective subcategory* of $\underline{C}$.

The following propositions are well-known.

1.7. Proposition. If $\underline{A}$ is a properly fibred topological category and $\underline{B}$ is a full isomorphism closed subcategory of $\underline{A}$, then the following are equivalent:

(1) $\underline{B}$ is bireflective in $\underline{A}$.

(2) $\underline{B}$ is closed under the formation of initial sources.

1.8. Proposition. Let $\underline{A}$ be a full, isomorphism-closed subcategory of properly fibred topological category $\underline{B}$. Then the followings ar equivalent.

(1) $\underline{A}$ is bireflective in $\underline{B}$.

(2) $\underline{A}$ contains all discrete and indiscrete objects of $\underline{B}$ and $\underline{A}$ is closed under the objects of $\underline{B}$ and $\underline{A}$ is closed under the formation of subobjects and products in $\underline{B}$.

II. A Lodato Prenearness Structure and A Semi-Closure Structure

2.1. Notations. Let PX denote the power set of X and let $P^2X=PPX$. For any subset ξ of P^2X we write $\mathcal{A} \bar{\in} \xi$ for $\mathcal{A} \notin \xi$, $Cl_\xi A$ for $\{x \in X: \{\{x\}, A\} \bar{\in} \xi\}$ and $Cl_\xi \mathcal{A}$ for $\{Cl_\xi A: A \in \mathcal{A}\}$. For subsets $\mathcal{A}, \mathcal{L}$ of PX,

$\mathcal{A} < \mathcal{L}$ iff each set $A \in \mathcal{A}$, there is $B \in \mathcal{L}$ with $B \subset A$,

$\mathcal{A} \vee \mathcal{L} = \{A \cup B : A \in \mathcal{A}, B \in \mathcal{L}\}$.

2.2. Definitions. Let X be a set and $\xi \subset PX$. Consider the following axioms:

(N1) if $\mathcal{A} < \mathcal{L}$ and $\mathcal{A} \in \xi$ then $\mathcal{L} \in \xi$.

(N2) if $\cap \mathcal{A} \neq \phi$ then $\mathcal{A} \in \xi$.

(N3) $\phi \neq \xi \neq P^2X$.

(N4) if $(\mathcal{A} \vee \mathcal{L}) \in \xi$ then $\mathcal{A} \in \xi$ or $\mathcal{L} \in \xi$

(N5) if $Cl_\xi \mathcal{A} \in \xi$ then $\mathcal{A} \in \xi$.

ξ satisfying (N1), (N2) and (N3) is called a *prenearness structure* on X. ξ satisfying (N1), (N2), (N3) and (N5) is called a *Lodato prenearness structure* on X. Finally satisfying (NI)–(N5) is called a nearness structure on X. The pair (X,ξ) is called a *(pre-, Lodato pre-) nearness space* -shortly: a *(pre-*, Lodate *pre-)* N-space- iff ξ is a (pre-, Lodato pre-) nearness structure on X.

2.3. Definitions. If (X,ξ) and (Y,η) are pre-N-spaces, then a map $f:X \to Y$ is called a nearness preserving map-shortly: an N-map- $f:(X,\xi) \to (Y,\eta)$ from (X,ξ) to (Y,η) iff $\mathcal{A} \in \xi$ implies $f\mathcal{A} \in \eta$. The category of pre-N-spaces and N-maps is denoted by $\underline{\text{P-Near}}$. Its full subcategory whose objects are Lodato pre-N- spaces is denoted by $\underline{\text{LP-Near}}$. Its full subcategory whose objects are N-spaces is denoted by $\underline{\text{Near}}$.

2.4. Proposition. If X is a set, (Y_i,η_i) is a family in $\underline{\text{LP-Near}}$ indexed by a class I, and $(f_i:X \to Y_i)_{i \in I}$ is a family of maps, then $\beta = \cap\{f_i^{-1}(\eta_i): i \in I\}$ is a Lodato prenearness structure on X, initial with respect to $(X,(f_i)_{i \in I}, (Y_i, \eta_i))_{i \in I}$.

Proof. First of all., let's show that $\beta \in$ LP-Near.

(N1) Suppose $@\in\beta$ and $\mathcal{L}<@$ Then $f(@)\in\eta_i$ for each i, and $f_i(\mathcal{L}) < f_i(@)$ for each i. Thus $f_i(\mathcal{L})\in\eta_i$ for each i, and so $\mathcal{L}\in\beta$.

(N2) Let $\cap @\neq\phi$. Then $\cap f_i(@)\neq\phi$ for each i, which implies $f_i(@)\in\eta_i$ for each i. Thus $@\in\beta$.

(N3) Since $\cap\phi\neq\phi$, $\phi\in\beta$ by (N₂) and $\{\phi\}\not\leq\xi$. Hence $\phi\neq\beta\neq P^2X$.

(N5) Let $Cl_\beta @\in\beta$. Then $f_i(Cl_\beta @)\in\eta_i$ for each i. Since $Cl_\beta f_i(@) < f_i(Cl_\beta @)$ for each i, $Cl_\beta f_i(@)\in\eta_i$ for each i. This implies $f_i(@)\in\eta_i$; $@\in\beta$.

It remains to show that β is initial with to $(X,(f_i)_{i\in I},(Y_i,\eta_i))_{i\in I}$.

Suppose for any $(Z,\zeta)\in$ LP-Near and $g:Z\to X$ is a map such that for each i, the map $f_ig:(Z,\zeta)\to(Y_i,\eta_i)$ is an N-map. Then for any $@\in\zeta$ $f_ig(@)\in\eta_i$ for each i and hence $g@\in\beta$. This completes the proof.

2.5. Theorem. The category LP-Near is a properly fibred topological category.

Proof. It is obvious from proposition 2.4.

2.6. Remark. Final structures in LP-Near can be described in the following way:if Y is a set, (X_i,ξ_i) is a family of Lodato pre-N-spaces, and $(f_i:X_i\to Y)_{i\in I}$ is a family of maps then $\eta=\{\beta\subset PY:\cap\beta\neq\phi\}\cup[\cup\{f_i(\xi_i)i\in I\}]$ is a Lodato prenearness structure on Y, final with respect to $((X_i,\xi_i)_{i\in I},(f_i)_{i\in I},Y)$.

2.7. Proposition. (1) For each set X there is a disrete Lodato prenearness structure β on X, characterized (up to isomorphism) by the fact that $f:(X,\beta)\to(Y,\eta)$ is a morphism for any object $(Y,\eta)\in$ LP-Near and any map $f:X\to Y$.

(2) For each set X there is an indescrete Lodato prenearness structure β on X, characterized (up to isomorphism) by the fact that $f:(Y,\eta)\to(X,\beta)$ is a morphism for any object $(Y,\eta)\in$ LP-Near and any map $f:Y\to X$.

Proof. (1) For any set X, let β be the final structure on X with respect to the empty source. Then it is obvious that β is the discrete structure on X.

(2) Dual of (1).

2.8. Definition. Let X be a set, A function $\alpha:PX\to PX$ is called a *semi - closure structure* [4] on X if it satisfies the following conditions:

(S1) $\alpha(\phi)=\phi$,

(S2) $A\subset\alpha A$ for each $A\in PX$,

(S3) $A\subset B$ implies $\alpha A\subset\alpha B$ for each $A,B\in PX$,

(S4) $\alpha A=\alpha(\alpha A)$, for each $A\in PX$.

The pair (X,α) is called a *semi-closure space.* For a convinience, we shall agree to use α as $\{A\subset X:\alpha A=A\}$.

2.9. Definition. If (X,α) and (Y,α') are semi-closure spaces, then a map $f:(X,\alpha)\to(Y,\alpha')$ from (X,α) to (Y,α') is called *s-continuous* iff for each $A\in\alpha'$, $f^{-1}(A)\in\alpha$.

Note that the identity map is s-continuous, and the composition of two s-continuous maps is also s-continuous. The category of semi-closure spaces and s-continuous maps is denoted by SCL. The semi-closure structure on X is a generalization of the more familiar Kuratowski closure operator on X.

2.10. Remark. Let $(X,\alpha)\in$ SCL, then $X,\phi\in\alpha$, and for every $A_i\in\alpha$ $(i\in I)$, $\bigcup_{i\in I}A_i\in\alpha$. But the intersection of two elements of α is not an element of α.Therefore α is not a topology on X.

2.11. Proposition. Let $(X,\alpha)\in$ SCL. Define $A\in\tau_\alpha$ iff for each $B\in\alpha$, $A\cap B\in\alpha$. Then τ_α is a topology on X.

Proof. It is obvious ϕ, $X\in\tau_\alpha$ since $\phi\cap B=\phi$, $X\cap B=B$ for each $B\in\alpha$. Let $A_1, A_2\in\tau_\alpha$. Then $A_1\cap B\in\tau_\alpha$ and $A_2\cap B\in\tau_\alpha$ for each $B\in\alpha$. But $A_1\cap A_2\cap B\subset\alpha(A_1\cap A_2\cap B)\subset\alpha(A_1\cap B)\cap\alpha(A_1\cap B)=(A_1\cap B)\cap(A_2\cap B)$ for each $B\in\alpha$. Then $A_1\cap A_2\cap B=\alpha(A_1\cap A_2\cap B)$ for each $B\in\alpha$, and hence $A_1\cap A_2\in\tau_\alpha$. Let $A_i\in\tau_\alpha$ for any $i\in I$. Then $\alpha((\cup A_i)\cap B)=\alpha(\cup(B\cap A_i))=\cup\alpha(B\cap A_i)$ for each $i\in I$ and $B\in\alpha$. So $\bigcup_{i\in I}A_i\in\tau_\alpha$. This completes the proof.

III. The Main Theorem

3.1. Therem. *Thecategory* LP-Near *is bireflective in* P-Near.

Proof. Suppose $(f_i:(X,\beta) \to (Y_i,\beta_i))_{i\in I}$ is an initial source in P-Near and for each $i\in I$, $(Y_i,\beta_i)\in$ LP-Near.

Let's show that β satisfies (N5). Suppose $Cl_\beta @\in\beta$.

Then $Cl_\beta @\in f_i^{-1}(\beta_i)$ for each i, and so $f_i(Cl_\beta @)\in\beta_i$ for each i.

Since $Cl_\beta(f_i@) < f_i(Cl_\beta @)$, $Cl_\beta(f_i(@))\in\beta_i$ for each i. But β_i is a Lodato prenearness structure on Y_i for each i, $f_i(@)\in\beta_i$ for each i. Thus $@\in f_i^{-1}(\beta_i)$ for each i, and (X,β) is an object of LP-Near.

This completes the proof because of proposition 1.8.

3.2. Theorem. *The category* Near *is bireflective in* LP-Near.

Proof. For any $(X,\beta)\in$ LP-Near, we define $\xi=\xi(\beta)$ as follows:

$@\in\overline{\xi}$ iff there exist $@_1, @_2, ..., @_n$ in $\overline{\beta}$ with $@_1 V @_2 V @_3 .., @_n < @$

We claim that ξ is a nearness structure on X.

(N1) Let $\mathcal{L}<@$ and $@\in\xi$. Assume that $\mathcal{L}\in\overline{\xi}$. Then there exist $@_1, @_2, ..., @_n$ in β with $@_1 V @_2 V, . . . V @_n < \mathcal{L} < @$ Hence $@\in\overline{\xi}$ which is a contradiction. Thus $\mathcal{L}\in\xi$

(N2) Suppose that $\cap @\neq\phi$ and $@\in\overline{\xi}$. Then $@_1 V @_2 V ... V @_n < @$, for some $@_1, @_2, ..., @_n$ in $\overline{\beta}$. But $@_1 V ... V @_n < @ < \cap @$ and we have $\cap @\in\overline{\xi}$. This implies $\cap @=\phi$. This is a contradiction.

(N3) By (N2), $\xi\neq\phi$. Since $\overline{\beta}\neq\phi$, we may choose $@\in\overline{\xi}$ with $@<@$. Then $@=\overline{\xi}$ and so $\xi\neq P^2X$.

(N4) If $@\in\overline{\xi}$ and $\mathcal{L}\in\overline{\xi}$, obviously $@ V \mathcal{L}\in\overline{\xi}$.

(N5) Let $@\in\overline{\xi}$. Then $@_1 V ... V @_n < @$ for some $@_1, ..., @_n$ in $\overline{\beta}$. Note that for any $B_i\in @_i$, $1\leqslant i\leqslant n$, we have $\overset{n}{\underset{i=1}{V}}\{\{x\}, B_i\} < \{\{x\}, \overset{n}{\underset{i=1}{\cup}} B_i\}$ and so $Cl_\beta(\cup B_i)\subset\overset{n}{\underset{i=1}{\cup}} Cl_\beta B_i$. Now $\overset{n}{\underset{i=1}{V}}\{Cl_\beta B_i : B_i\in @_i\} < \{Cl_\xi A : A\in @\}$

But $\{Cl_\beta B_i : B_i\in @_i\}\in\overline{\xi}$, $1\leqslant i\leqslant n$, and so $Cl_\xi @\in\overline{\xi}$. Therefore $(X,\xi)\in$ Near, where $\xi=\xi(\beta)$.

Let 1_X: $(X,\beta)\to(X,\xi)$ be the identity map. If $@\in\overline{\xi}$ then there exists $@_1, ..., @_n$ in $\overline{\beta}$ with $@_1 V ... V @_n < @$. Thus $@\in\xi$ and $1_X @=@\in\overline{\beta}$ Hence 1_X is an N-map. Take any object $(Y,\eta)\in$ Near and take any N-map $f:(X,\beta)\to(Y,\eta)$. It remains to show that $f:(X,\xi)\to(Y,\eta)$ is an N-map. Take $@\in\overline{\eta}$. Then $f^{-1}(@)\in\overline{\beta}$ and $f^{-1}(@)<f^{-1}(@)$. So that $f^{-1}(@)\in\overline{\xi}$.

3.3. Proposition. LP-Near *is a subcategory of* P-Near *containing all discret and all indiscrete spaces and being closed under the formation of subobjects, products in* P-Naer.

Proof. It is obvious from proposition. 1.8.

3.4. Definition. A pre-N-space (X,ξ) is called *regular* iff $@(<_\xi)\in\xi$ implies $@\in\xi$, where $@(<_\xi)=\{B\subset X$: there exist $A\in @$ such that $\{A, X-B\}\in\overline{\xi}\}$.

3.5. Proposition. *Every regular pre-N-space is a Lodato pre-N-space.*

Proof. Let (X,ξ) be a regular pre-N-space. We must show that ξ satisfies (N5). Suppose $@\subset PX$ with $\{Cl_\xi A : A\in @\}\in\xi$. Assume $@\in\overline{\xi}$. Then $@(<_\xi)\in\overline{\xi}$ because of being regular. Take any $B\in @(<_\xi)$, there exist $A\in @$ such that $\{A, X-B\}\in\overline{\xi}$ If $x\in X-B$ then $\{A,\{x\}\}\in\overline{\xi}$, which implies $x\notin Cl_\xi B$ and also $Cl_\xi A\subset B$. So we have $@(<_\xi)<\{Cl_\xi A : A\in @\}$. Thus $@(<_\xi)\in\xi$. This is a contradiction.

3.6. Definition. A semi-closure space (X,α) is called *symmetric* iff $x\in\alpha\{y\}$ implies $y\in\alpha\{x\}$ for each pair (x,y) of elements of X. The category of symmetric semi-closure spaces and s-continuous maps is denoted by S-SCL.

3.7. Proposition. *Let (X, β) be a Lodato* *pre-N-space. Then the map Cl_β:PX → PX is a symmetric semi-closure structure on X.*

Proof. We shall show that Cl_β satisfies (S1)-(S4).

(S1) By (N3), $\{\phi, \{x\}\} \notin \beta$, This implies $Cl_\beta \phi = \phi$.

(S2) Suppose $x \in A$ for each $A \in PX$. Then $\cap \{\{x\}, A\} \neq \phi$, which implies $\{\{x\}, A\} \in \beta$. Thus $x \in Cl_\beta A$, and so $A \subset Cl_\beta A$ for each $A \in PX$.

(S3) Let $A \subset B$ for each $A, B \in PX$ and let $x \in Cl_\beta A$. Then $\{\{x\}, A\} \in \beta$ and $\{B, \{x\}\} < \{\{x\}, A\}$. And also $\{B, \{x\}\} \in \beta$. Hence $Cl_\beta A \subset Cl_\beta B$.

(S4) For each $A \in PX$, $Cl_\beta A \subset Cl_\beta(Cl_\beta A)$. To show $Cl_\beta(Cl_\beta A) \subset Cl_\beta A$, pick any $x \in Cl_\beta (Cl_\beta A)$. Then $\{\{x\}. Cl_\beta A\} \in \beta$. Since $\{Cl_\beta\{x\}, Cl_\beta A\} < \{\{x\}, Cl_\beta A\}$, we have $\{Cl_\beta\{x\}, Cl_\beta A\} \in \beta$ and also $\{\{x\}, A\} \in \beta$ because of β satisfying (N5).

It remains to show that Cl_β is symmetric. If $x \in Cl_\beta\{y\}$, then $\{\{x\}, \{y\}\} \in \beta$. But $\{\{x\}, \{y\}\} < \{\{y\}, \{x\}\}$, which implies $y \in Cl_\beta\{x\}$. Hence Cl_β is symmetric.

3.8. Proposition. *If* (X, α)∈S-SCL, *then there exists* (X, β) ∈ LP-Near *such that* $\alpha = Cl_\beta$.

Proof. Let's define β as follows: $@ \in \beta$ iff $\cap \{\alpha A : A \in @\} \neq \phi$. Let's show that (X, β)∈ LP-Near.

(N1) Let $\mathcal{L} < @$. If $@ \in \beta$ then $\cap\{\alpha A : A \in @\} \neq \phi$, which implies $\cap \{\alpha A : A \in @\} \subset \cap \{\alpha B : B \in \mathcal{L}\}$ and $\cap\{\alpha B : B \in \mathcal{L}\} \neq \phi$. i.e. $\mathcal{L} \in \beta$.

(N2) Suppose $\cap @ \neq \phi$. Then $\cap \{A : A \in @\} \neq \phi$. Since $\cap\{A : A \in @\} \subset \cap \{\alpha A : A \in @\}$, $@ \in \beta$.

(N3) Since $\cap\{\phi\} = \phi$, $\{\phi\} \in \beta$. This implies $\beta \neq P^2 X$. Since $\alpha\phi = \phi$ and $\cap\phi \neq \phi$, $\beta \neq \phi$.

To verify (N5), we first will prove $\alpha = Cl_\beta$. If $x \in \alpha A$ for each $A \in PX$, than $\alpha\{x\} \cap \alpha A \neq \phi \Rightarrow \{\{x\}, A\} \in \beta$, $x \in Cl_\beta A$. If $y \in Cl_\beta A$ for each $A \in PX$, then $\alpha\{y\} \cap \alpha A \neq \phi$ Take any $x \in \alpha\{y\} \cap \alpha A$. Then $x \in \alpha\{y\}$, which is implies $y \in \alpha\{x\}$. Thus $y \in \alpha\{x\} \subset \alpha A$. i.e. $y \in \alpha A$.

(N5) Suppose $\{Cl_\beta A : A \in @\} \in \beta$. Then $\cap\{\alpha(Cl_\beta A) : A \in @\} \neq \phi$; $\cap \{\alpha\alpha A ; A \in @\} \neq \phi$; $\{\alpha A : A \in @\} \neq \phi$; $@ \in \beta$.

3.9. Definition. A Lodato pre-N-space (X,β) is called a *C_β-Lodato pre-N-space* if it satisfies the following condition:

If $@ \in \beta$ then $\cap\{Cl_\beta A : A \in @\} \neq \phi$.

We denote the category of C_β-Lodato pre-N-spaces and N-maps by CLP-Near.

3.10. Theorem. S-SCL *and* CLP-Near *are isomorphic as categories.*

Proof. Define a functor F:S-SCL → CLP-Near as follows: for any (X, α)∈ S-SCL, F(X,α)=(X, β_α), where $\beta_\alpha = \{@ \subset PX : \cap\{\alpha A : A \in @\} \neq \phi\}$ and for any f:(X,α) → (Y,α') in S-SCL, Ff=f: (X, β_α) → (Y, $\beta_{\alpha'}$). Then obviously (X, β_α)∈ CLP-Near, and since for any $@ \in \beta_\alpha$ $\cap \{\alpha A : A \in @\} \neq \phi$ and f is s-continuous, hence $\cap \{\alpha' f(A) : A \in @\} \neq \phi$. i.e. $f(@) \in \beta_{\alpha'}$.

Therefore Ff=f:(X, β_α) → (Y, $\beta_{\alpha'}$) is an N-map.

Define a functor G: CLP-Near → S-SCL as follows: for any (X, β) ∈ CLP-Near, G(X,β)=(X, Cl_β) and for any f:(X, β) → (X, β') in CLP-Near, Gf=f: (X, Cl_β) → (X, $Cl_{\beta'}$).

It is obvious (X, Cl_β) ∈ S-SCL and Gf=f:(X,Cl_β) → (X, $Cl_{\beta'}$) is an N-map.

It remains to show that GF=id, GF=id. One can easily prove that $\alpha = \alpha_{\beta_\alpha}$ and $\beta = \beta_{\alpha_\beta}$, hence GF(X, α)=(X, α) and GF(f)=f, FG(X, β)=(X, β) and FG(f)=f. This completes the proof.

3.11. Remark. Theorem 3.11. is analogous that the category *T-Near* of topological N-spaces and N-maps and the category of R_0-Top of R_0-topological spaces and continuous maps are isomorphic.[1]

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國 文 抄 錄

本 論文에서는, nearness構造로 부터 Lodato prenearness構造를 定義하여 이 Lodato prenearness空間과 N-寫像들의 category LP-Near에 대해 硏究 조사하였다. 그 결과로써,

(1) LP-Near는 prenearness空間과 N-寫像들의 category P-Near의 bireflective subcategory가 되고, 또한

(2) nearness空間과 N-寫像들의 category Near도 LP-Near의 bireflective subcategory가 됨을 보였으며,

끝으로, category LP-Near의 subeategory인 CLP-Near를 소개하여, 이 CLP-Near가 symmetric semi-closure空間과 S-연속함수들의 category S-SCL과는 서로 同値임을 증명하였다.