Semiclassical Analysis for $^{16}O+^{208}Pb$ Elastic Scattering at $E_{lab}=216.6$ and 312.6 MeV

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$E_{lab}=216.6$과 312.6 MeV에서 $^{16}O+^{208}Pb$ 탄성산란에 대한 반고전적인 분석

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I. Introduction

The interpretation and description of scattering phenomena in heavy ion reactions has been greatly facilitated by the application of the semiclassical method. The semiclassical methods in the description of heavy ion scattering phenomena are useful approximation techniques when the wavelength associated with the relative motion of the center of mass is very small compared to some typical interaction distance. One of the methods for the analysis of elastic scattering data is based on the use of the conveniently parametrized scattering matrix from which the semiclassical expressions for the scattering amplitude have been derived [Frahn, 1985; McIntyre et al., 1960].

In the recent few years, a number of studies have been made to describe elastic scattering processes between heavy ions in terms of the optical limit to the Glauber model [Charagi et al., 1990; Chauvin et al., 1983; Chauvin et al., 1985; Hegab et al., 1990; Vitturi et al., 1987]. The standard form of the Glauber model [Chauvin et al., 1983] is used for the description of elastic scattering at low and intermediate energies. In particular, it has been shown that some approximations implied in the optical limit of the Glauber theory [Chauvin et al., 1985] could be justified for low-energy heavy ion surface collisions. Vitturi et al. modified the standard form of the optical limit to the Glauber model to account for the Coulomb distortion of the trajectories occurring in the case of heavy ion scattering and applied it successfully to elastic scattering for $E_{lab}=1{,}760$ MeV ^{40}Ar and 390 MeV ^{13}C ions on ^{208}Pb target nuclei. Recently, the semiclassical phase-shift analysis of the elastic

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heavy ion scattering by using the asymtotic Legendre function and modified Glauber model are performed (Cha et al., 1991; Kim, 1991; Kim et al., 1991).

In this paper, we present a semiclassical phase-shift analysis of the elastic scattering data for E_{lab}=216.6 and 312.6 MeV ^{16}O beams on ^{208}Pb target nuclei (Chaudhuri et al., 1986) based on the modified Glauber model. In Sec. II, we present the semiclassical scattering amplitude. Results and conclusions are presented in Sec. III.

II. Semiclassical Scattering Amplitude

The differential cross section for the elastic scattering is given by the following equation :

$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2 \cdots\cdots\cdots\cdots\cdots (1)$$

where the elastic scattering amplitude $f(\theta)$ for spin-zero particle via Coulomb and short-range central forces can be separated into the Rutherford and nuclear parts by writing (Brink, 1985)

$$f(\theta) = f_R(\theta) + \frac{1}{ik}\sum_{\ell=0}^{\infty} (\ell+\tfrac{1}{2})\exp(2i\sigma_\ell)(S_\ell^N - 1)P_\ell(\cos\theta) \cdots\cdots\cdots\cdots\cdots (2)$$

Here, $f_R(\theta)$ is the usual Rutherford scattering amplitude, $\sigma_\ell = \arg\Gamma(\ell+1+i\eta)$ are the Coulomb phase shifts and $\eta = mZ_1Z_2e^2/(\hbar^2 k)$ is the Sommerfeld parameter.

In the modified Glauber model, the nuclear scattering matrix S_ℓ^N is expressed as

$$S_\ell^N = \exp[\frac{2\pi i}{k_{NN}} \Omega_\ell f_{NN}(0)] \cdots\cdots\cdots\cdots\cdots (3)$$

where the scattering amplitude for the nucleon-nucleon scattering at $\theta=0°$, $f_{NN}(0)$, is related to the average nucleon-nucleon total cross section σ_{NN} through

$$f_{NN}(0) = \frac{k_{NN}}{4\pi} \sigma_{NN}(\alpha_{NN}+i) \cdots\cdots\cdots\cdots (4)$$

where α_{NN} is the ratio of the real to the imaginary part of the forward nucleon-nucleon scattering amplitude. To determine the value of σ_{NN} it is convenient to use the expressions given by the Charagi et al. Ω_ℓ is the overlap integral of the nuclear densities along a Rutherford trajectory characterized by a closest approach d such that

$$d = \frac{1}{k}[\eta + [\eta^2 + (\ell+\tfrac{1}{2})^2]^{1/2}] \cdots\cdots\cdots\cdots\cdots (5)$$

Assuming a Gaussian distribution of the nuclear density (Chauvin et al., 1983; Vitturi et al., 1987)

$$\rho_i(r) = \rho_i(0)\exp(-r^2/a_i^2) \quad (i=P,T) \cdots\cdots\cdots (6)$$

for both target and projctile, Ω_ℓ is given by

$$\Omega_\ell = \rho_P(0)\rho_T(0)\pi^2 \frac{a_P^3 a_T^3}{a_P^2+a_T^2} \exp(\frac{-d^2}{a_P^2+a_T^2}) \cdots (7)$$

The semiclassical approximation assumes that contributions to the cross section come mainly from the large angular momenta. We consider the Legendre polynominals, taken as a special case of the associated Legendre functions. The asymptotic form of the Legendre function can be written as (Amado et al., 1985)

$$P_\ell^m(\cos\theta) \simeq (\ell+\tfrac{1}{2})^m (\frac{\theta}{\sin\theta})^{1/2}$$

$$J_{-m}[(\ell+\tfrac{1}{2})\theta] \cdots\cdots (8)$$

which is valid for all m and all angles except when $(\pi-\theta)\leq\ell^{-1}$. Using the Eq. (8) with m=0 and taking into account Eq. (2), the nuclear scattering amplitude can be rewritten as

$$f_N(\theta) \simeq \frac{1}{ik}\left(\frac{\theta}{\sin\theta}\right)^{1/2}\sum_{\ell=0}^{\infty}(\ell+\tfrac{1}{2})\exp(2i\sigma_\ell)$$
$$(S_\ell^N-1)J_0[(\ell+\tfrac{1}{2})\theta]. \cdots\cdots (9)$$

Ⅲ. Results and Conclusions

The elastic cross sections are obtained from the scattering amplitude Eqs. (2) and (9). The Gaussian nuclear density parameters for ^{16}O are calculated from $a=R_{RMS}/\sqrt{1.5}$, $\rho(0)=A/(a\sqrt{\pi})^3$ and RMS radius R_{RMS}^i can be taken from Charagi et al.. And we have used the ^{208}Pb Gaussian nuclear density adjusted by Karol to reproduce the surface tail of density.

The calculated results for the $^{16}O+^{208}Pb$ elastic scattering angular distributions at 216.6 and 312.6 MeV incident energies are presented in Fig.1. The fits are satisfactory and the corresponding parameters used in the calculation are listed in Table 1. The solid and dashed lines which represent the calculated elastic scattering cross sections obtained by using Eq. (2) and (9), respectively, show nearly identical fits. In table 1, $d_{1/2}$ is the strong absorption distance, for which $T(d)=1/2$, and σ_R the total reaction cross section. In both cases, numerical results agree well with the observed data [Chaudhuri et al., 1986].

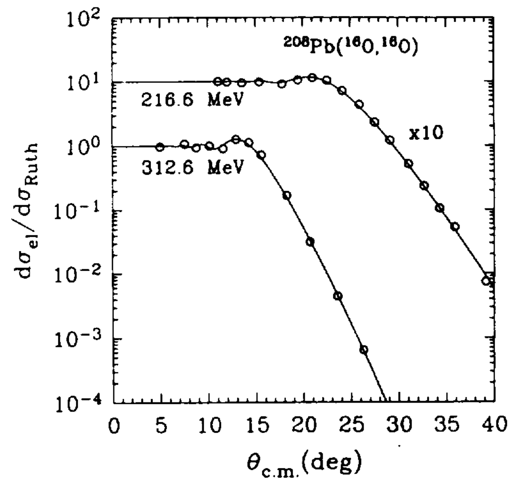

Fig. 1. Elastic scattering angular distributions for $^{16}O+^{208}Pb$ systems at E_{lab}=216.6 and 312.6 MeV. The open circles denote the observed data taken from Chaudhuri et al.. The solid and dashed lines are the calculated results from Eqs. (2) and (9), respectively. (Note that the two lines show nearly identical fits).

Transparency coefficients T_ℓ are plotted as a function of angular momentum ℓ in Fig.2 (a). In this figure we can see that lower partial waves are totally absorbed and the T_ℓ's increase very rapidly in a narrow localized angular momentum zone. The real

Table 1. Nucleon-nucleon collision data and total reaction cross sections for $^{16}O+^{208}Pb$ system at E_{lab}=216.6 and 312.6 MeV.

E_{lab} (MeV)	σ_{NN} (mb)	α_{NN}	$\rho_P(0)$ (fm^{-3})	a_P (fm)	$\rho_T(0)$ (fm^{-3})	a_T (fm)	σ_R (mb)	$d_{1/2}$ (fm)
216.6	485	0.44	0.265	2.213	2.59	3.45	3218	12.59
312.6	322	0.91	0.265	2.213	2.59	3.45	3605	12.32

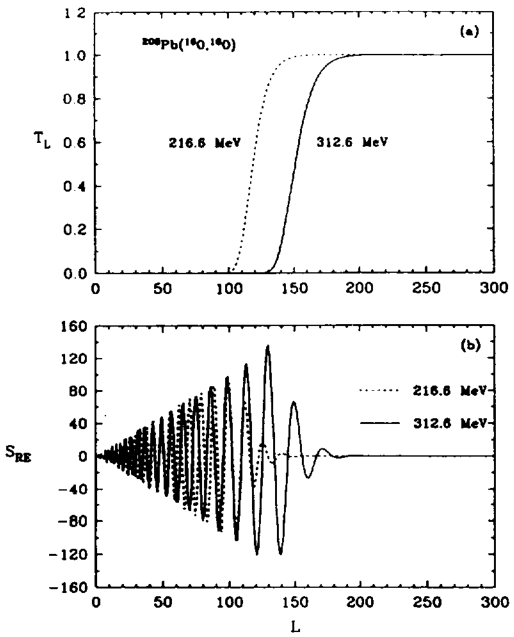

Fig. 2. (a) Transparency coefficients T_ℓ, (b) the real parts (S_{RE}) of the term, $i(\ell+\frac{1}{2})\exp(2i\sigma_\ell)(S_\ell^N-1)$, for the $^{16}O+^{208}Pb$ system plotted versus the orbital angular momentum ℓ.

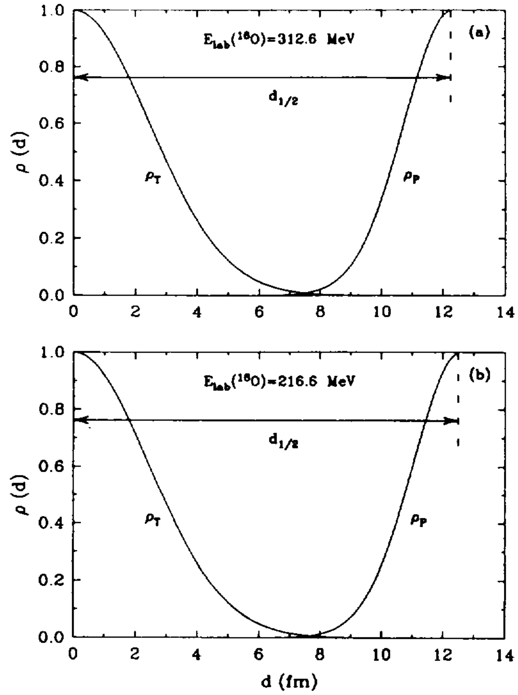

Fig. 3. Overlap of the projectile and target densities. The distance between the two nuclei is set equal to the strong absorption distance $d_{1/2}$ given in Table 1.

parts (S_{RE}) of the term, $i(\ell+\frac{1}{2})\exp(2i\sigma_\ell)(S_\ell^N-1)$, in Fig.2(b) represent the partial wave contributions to the cross section in terms of orbital angular momentum ℓ.

Figure 3 show the overlaps of the projectile and target densities when the distance between the two particles is set equal to the strong absorption distance $d_{1/2}$ given in Table 1. In both cases, the magnitudes of the overlap density are too small to consider their effects.

In conclusion, reasonable fits of elastic scattering angular distributions for $E_{lab}=216.6$ and 312.6 MeV ^{16}O beams on ^{208}Pb target nuclei were obtained using the semiclassical method based on the modified Glauber model. These good agreement suggests that differential cross sections for $^{16}O+^{208}Pb$ system at $E_{lab}=216.6$ and 312.6 MeV are governed by nucleon-nucleon cross sections. It has also turned out that the two numerical elastic scattering cross sections obtained by using the exact Legendre functions and their asymtotic form in calculating the nuclear scattering amplitude agree well with the observed data [Chaudhuri et al., 1987] and show almost the same structures.

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〈摘要〉

E_{lab}=216.6과 312.6 MeV에서 $^{16}O+^{208}Pb$ 탄성산란에 대한 반고전적인 분석

실험실계에서의 입사핵 (^{16}O) 에너지가 216.6과 312.6 MeV이고 표적핵 ^{208}Pb인 탄성산란에 대한 실험데이타를 수정된 Glauber 모델을 사용하여 분석하였다. 계산된 결과들은 실험데이타와 좋은 일치를 보여주었다.