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# Orbital magnetic response of mesoscopic metallic systems with Kondo impurities

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## Kondo 불순물이 포함된 mesoscopic 금속계의 궤도 자기응답

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## Summary

We propose a novel mechanism effecting both the magnitude and the sign of orbital magnetic response in a mesoscopic metallic system, namely, the electron-electron interaction induced via spin-flip scattering from paramagnetic impurities. We evaluate its contribution to the response due to weak localization effects and compare it with the average response of a canonical ensemble of noninteracting electrons and with the localization correction due to the screened Coulomb interaction. We examine both the singly connected (Landau) and multiply connected (Aharonov-Bohm) sample geometries and explain how various contribution to the total magnetic response can be distinguished by their dependences on the magnetic field and the impurity concentration.

## Introduction

Paramagnetic impurities can play a prominent role in experiments studying magnetic response of small metallic samples. One of aspects is the exchange interaction with the conduction electrons. In a disordered metal, the feedback from quantum interference effects on the orbital degrees of freedom of conduction electrons can have dramatic consequences for the magnetic response. In an earlier work, the weak localization correction to the Kondo susceptibility has been evaluated by Okhawa *et al* (1983). Later, Aronov and Zyuzin (1984) have studied correction to the susceptibility of conduction electrons due to the interaction induced by spin-flip scattering and found that scattering off the same Kondo impurity induces an effective interaction between the conduction electrons.

The interaction-induced localization correction to the orbital magnetic response does not have a

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smallness in the parameter $(k_F l)^{-1}$, where $l$ is the electron mean free path. This result was first obtained for the screened Coulomb interaction (Altshuler *et al.*, 1982) and can be understood as follows. Any localization correction has the smallness of $(k_F l)^{-1}$. However, the diffusion equation for the Cooperon is formally equivalent to the Schrödinger equation with the "mass" $(2D)^{-1}$ which is $(k_F l)^{-1}$ times lighter than the electron mass. Here $D \sim v_F^2 \tau$ is the diffusion constant and $\tau$ is the elastic scattering time. Since the magnetic susceptibility is proportional to the squared Bohr magneton, this factor yields the largeness of $(k_F l)^2$. The overall effect has the largeness of $k_F l$, meaning that it is more pronounced in cleaner samples.

Whereas previous theories apply to macroscopic systems, in this paper we extend the results of Aronov and Zyuzin to the mesoscopic case where the response is defined by the finite-size Cooperon eigenstates. In the mesoscopic case, it is critical that a measurement of orbital response does not typically involve sample contact with electron reservoirs. Consequently, the mechanism whereby the electron phase relaxation would take place in a transport experiment-by means of inelastic collisions in ideal metallic leads (electron reservoirs)-is not ordinarily enacted in a magnetic experiment (Oh *et al.*, 1991; Altshuler *et al.*, 1991). In other words, provided that the electron scattering off the sample boundaries is elastic, sample size is no longer a legitimate scale for the breakdown of phase coherence. To emphasize this circumstance, we shall consider the extreme quantum coherence limit, $L \leq \min\{L_T, L_\psi\}$, where $L$ is the sample size, $L_T \sim \sqrt{D/T}$ is the thermal length, and $L_\psi$ is the phase coherence length. To simplify the notation, we use a system of units $\hbar = k_B = 1$.

## Evaluations of the Kondo effect

In a system with Kondo impurities, the phase-breaking process will be defined predominantly by spin-flip scattering processes, $L_\psi \sim L_s = \sqrt{D\tau_s}$, where $\tau_s$ is the spin-flip scattering time,

$$\frac{1}{\tau_s} = 2\pi\nu_0 \left| \frac{J}{\nu_0} \right|^2 n_s S(S+1), \qquad (1)$$

$\nu_0$ is the density of states at the Fermi surface, and $S$ is the spin of Kondo impurities. Since the interaction strength scales with the concentration of paramagnetic impurities, so will the interaction-induced localization correction. On the other hand, it follows from Eq. (1) that the spin-flip scattering rate will increase with the number of impurities as well leading to the increase of the number of the phase coherence breaking processes. The maximum magnitude of the correction is achieved when these two effects are balanced against each other. The difference with the Coulomb interaction is understood by taking into account the peculiar nature of the Kondo-induced interaction (Aronov and Zyuzin, 1984) shown in Fig. 1, whose strength $\lambda_K$ is given by the third order in the dimensionless exchange con-

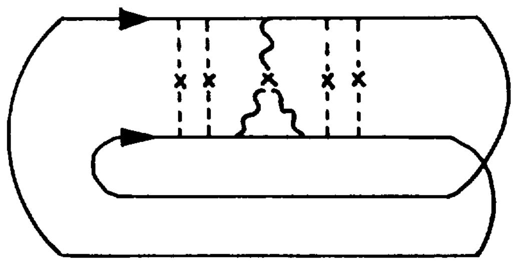

Fig. 1. Cooperon correction to Gibbs free energy with Kondo impurities.
Solid line denotes one-electron Green's function, dashed lines are the elastic scattering from impurities, and wavy line, the Kondo interaction.

stant $J$ ($J$>0 for antiferromagnetic coupling). $\lambda_K \propto J^2 n_s$ being $n_s$ the concentration of Kondo impurities. In the Aharonov-Bohm case, the analytical calculation of the orbital response due to the Kondo-induced interaction is based on the following expression for the correction to the Gibbs free energy which can be described by the inverse Cooperon (Oh *et al.*, 1991):

$$\delta F = -8\lambda_s^{-1} \sum_{wk>0}^{1/\tau} \sum_{n=-\infty}^{\infty} \frac{w_k}{w_k + E_c(2\pi)^2(n+2\phi/\phi_2)^2}, \qquad (2)$$

where $w_k = 2\pi kT$ is the Matsubara frequency and $k$ is the integer, and $L$ is the circumference of the ring. Taking summation on $n$ (harmonics) first and then on Matsubara frequency, we obtain, upon differentiation on external magnetic field $H$, the following expression for the Kondo-induced magnetic moment of the ring:

$$M = 4\mu_B J(2mL^2\tau_s^{-1}) \frac{\sin(4\pi\phi/\phi_0)}{\cosh(\sqrt{2\pi}L/L_T) - \cos(4\pi\phi/\phi_0)}, \qquad (3)$$

where $\mu_B = e/2mc$ is the Bohr magneton. Eq. (3) is valid for two-dimensional samples and for three-dimensional slabs, whose thickness satisfies the constraint a $\ll L_s, L_T$. The response of a disk (of equal circumstance $L$) is equivalent in the same fashion. However, the form of the inverse Cooperon is slightly different with a change:

$$\omega_k + \tau_H^{-1}, \tau_H^{-1} = 2\pi^2 E_c(2\phi/\phi_0)^2, \qquad (4)$$

leading to the following expression for the Kondo-induced magnetic moment of the disk:

$$M = 4J\tau_s^{-1}\left(\frac{\partial \tau_H^{-1}}{\partial H}\right) T \int_0^\infty dt \frac{t\exp(-t/\tau_H)}{\exp(2\pi Tt)-1} \qquad (5)$$

From the correction of the Gibbs free energy due to localization correction due to kondo impurities (LCKI), we find the average correction to the magnetic moment in the linear response regime

$$M \sim \mu_B J(k_F l) \begin{cases} (T\tau_s)^{-1}(\phi/\phi_0), & L \le L_T \le L_s \\ (\phi/\phi_0), & L \le L_s \le L_T \end{cases}. \qquad (6)$$

In terms of competition between the strength of the Kondo-induced interaction and the spin-flip scattering rate as a function of impurity concentration, our result points to the former tipping the balance in its favor. However, our anaytical calculation does not extend beyond the case of $L \le L_s$. At any rate, the condition $T \sim \tau_s^{-1}$ signals a spin-glass transition and it is not very meaningful to consider significantly lower temperatures. Below the transition, It is conceivable that the interaction is induced by processes involving frozen spins. This will be addressed in a future work. In the remainder of this paper we shall assume that $L \le L_T \le L_s$ or $\tau_s^{-1} \le T \le E_c = D/L^2$. Above $\phi_c$, the Kondo-induced correction decays as $M \sim \mu_B J(k_F l)(E_c\tau_s)^{-1}(\phi_0/\phi)$. Combining this with Eq. (6) and noticing that $\phi_c \sim (T/E_c)^{\frac{1}{2}}\phi_0$, we obtain:

$$M \sim \mu_B J(k_F l)(LL_T/L_s^2) \begin{cases} \phi/\phi_c, & \phi \le \phi_c \\ \phi_c/\phi, & \phi \ge \phi_c \end{cases}. \qquad (7)$$

The orbital contribution of conduction electrons must be distinguished in experiment from the Curie response of paramagnetic impurities. To compare the two effects, we notice that in terms of the susceptibility per unit volume Eq. (6) can be rewritten as

$$\chi \sim J|\chi_L|(l/a)(T\tau_s)^{-1}, \qquad (8)$$

where $\chi_L = -(1/3)\mu_B^2\nu_0$ is the Landau susceptibility. In the 2D-case, $l/a$ in Eq. (8) is replaced by $k_F l$. It is convenitent to use the same parameters as in Eq. (9) in the expression for the Curie susceptibility,

$$\chi_{curie} \sim |\chi_L| J^{-2} (T\tau_s)^{-1}. \qquad (9)$$

Even if the susceptibilities in Eqs. (8) and (9) are comparable in magnitude and the exchange coupling is attractive, $J>0$, it is still possible to distinguish the two effects if one takes into account that the Kondo-induced contribution is highly non-linear, as is seen from Eqs. (6) and (7) and the periodic oscillations of the Aharonov-Bohm response. This circumstance is ever more pertinent near a spin-glass transition where the cusp of spin susceptibility can be quite pronounced. It should be stressed that the periodic oscillations of the Aharonov-Bohm response are eventually cut-off by the magnetic field penetrating the annulus of the ring and freezing spin degrees of freedom. This happens when $\mu_B H \sim T$. Thus, the observation of oscillations associated with the Kondo-induced interaction is possible when $L \leq L_T(k_F l)^{-1/2}$; in sufficiently clean systems this is not in contradiction with the condition $L \leq L_T$

## Discussion

We start the comparison between Kondo and other effects by considering meaning of an isolated sample, the typical situation of magnetic measurements. The absence of phase and energy relaxation at the sample boundaries establishes (Serota *et al.*, 1987; Altshuler *et al.*, 1991; Oh *et al.*, 1991) the relevance of the magnitude field $H_c$ such that $H_c L L_T \sim \phi_0 = hc/e$, which corresponds to the flux through the sample, $H_c L^2$, of order $\phi_c \sim \phi_0 (L/L_T) \leq \phi_0$. For instance, the average orbital magnetic response of a canonical ensemble of noninteracting electrons grows lineary below and peaks at $\phi_c$. Above $\phi_c$, it falls off as $\phi^{-1}$. The localization correction due to the screened Coulomb interaction, on the other hand, peaks at $\sim \phi_0$ (Oh *et al.*, 1991). However, in either case the Aharonov-Bohm response of a narrow ring is periodically repeated with the period $\phi_0/2$. In what follows, we show that LCKI peaks at $\phi_c$. The implications of Eq. (6) are as follows. Suppose that we fix the concentration of paramagnetic impurities and consider the effect of temperature variations. As the temperature is lowered, the maximum, $M_{max} \sim \mu_B J(k_F l)(L L_T / L_s^2)$, grows in magnitude just as it shifts toward smaller fluxes until $T \sim \tau_s^{-1}$. At this point, it saturates at $M_{max} \sim \mu_B J(k_F l)(L/L_s)$. It is interesting that the latter is actually bigger, $M_{max} \sim \mu_B J(k_F l)$, when $L_s \sim L$ (meaning that $\phi_c \sim \phi_0$). Therefore, the magnitude of the maximum of the Kondo-induced interaction correction does not favor quantum coherence of the sample. It should be contrasted with the average response of a system of non-interacting electrons (Oh *et al.*, 1991), in which case the maximum is larger for a larger phase coherence length. It should be noticed that the dependence on the flux in Eq. (7) is same as for the average response of non-interacting electrons because of the similarity of the analytical formalism. However, in the latter case, as well as for the response fluctuations (Oh *et al.*, 1991), the underlying physics is based on the repulsive level statistics (Altshuler and Shklovskii, 1986) which tends to avoid level crossings. Mathematically, it is manifested by the dependence of the orbital response on the average interlevel spacing $\Delta$, which, in turn, depends on the sample geometry. For instance, $\Delta$ is larger in a ring than in disk of equal cidcumstance : its relation to the dimensionless sample conductance $g$ is clear from $\Delta \sim E_c/g$. The role of $\Delta$ in the Kondo mechanism considered here is reflected only by the condition $T, \tau_s^{-1} > \Delta$

which signifies the breakdown of the diffusion approximation.

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〈국문초록〉

# Kondo 불순물이 포함된 mesoscopic 금속계의 궤도 자기응답


mesoscopic 금속계의 궤도 자성응답의 크기와 부호에 영향을 미치는 새로운 메카니즘, 즉 상자성 불순물과의 spin-flip 산란에 의해 유도되는 전자-전자간 상호작용을 논하고자 한다. 그 작용이 약한 국소화 효과의 자기응답에 미치는 기여를 계산하였으며, 독립전자로 구성된 정준 앙상블의 평균응답 및 차폐된 Coulomb 상호작용에 의한 국소화의 자기응답과도 비교하였다. 단일연결(Landau)과 다중연결(Aharnov-Bohm)의 기하학적 모형에 적용하였으며, 총 자기응답에 대한 각 기여가 자기장과 불순물의 농도에 따라 어떻게 주어지는 가를 설명하였다.