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Mesonic Spectra in the Quarkonium Model

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The mesonic mass spectra of $u\bar{u}, s\bar{s}, c\bar{c}$, and $b\bar{b}$ systems in the 1994 Particle Data are analyzed relativistically using a linear plus Coulomb potential model. The effective quark masses and the corresponding strong coupling constants are determined by the least-squares method and the calculated and the observed masses of the systems are compared. It has been found that either $f_1(1419)$ or $f_1(1512)$ can not be assigned to the $q\bar{q}$ state for low-mass isosinglet assignments. The calculated values of the effective quark masses are 1.54, 1.77, 1.88, and 5.223 GeV, and the strong coupling constants are 0.850, 0.493, 0.474, and 0.332 for the $u\bar{u}$, $s\bar{s}$, $c\bar{c}$, and $b\bar{b}$ respectively.

I. INTRODUCTION

The definition of meson was a particle of mass intermediate between those of the electron and the proton mediating forces in the nuclei, but it is now changed to a strongly interacting particle with an integer spin. Because of the new definition, the number of mesons has increased steadily, and its mass range has been extended up to more than 100 GeV. Since there are several kinds of strongly interacting particles with integer spin, we need to classify them into appropriate categories, if possible. Traditionally the quark-antiquark bound states are quarkonium states, and, of course, these states form the main body of the meson family. Other possibilities composed of quark components are the multi-quark states such as $q^2\bar{q}^2$, q^6, $q^3\bar{q}^3$, and so on. When we include the gluonic components, it is possible to consider hybrid states, such as $q\bar{q}g$, $q\bar{q}gg$, and glueballs composed of two and three gluons gg, ggg. In order to analyze the spectra of these observed mesons, we have to assign each observed state to an appropriate state which can be made available through theoretical considerations.

Theoretically, explicit calculations have been made possible since the applications of potential models to the non-relativistic $c\bar{c}$ systems. The $b\bar{b}$ systems have given more firm ground to check the potential models, and it has been found that the spin dependences become more important for the lighter meson systems. Since so many radial and orbital excitations of quarkonium states exist, almost all the observed mesons can be accounted for by assigning suitable $q\bar{q}$ states with corresponding quantum numbers, except for several states such as glueballs and the exotic states. Hence we will concentrate on the analysis of quarkonium states.

So many isosinglet meson states exist in the 1994 Particle Data [1] that simple counting can lead to the conclusion that states exist which can not be assigned to bound states of a quark and an antiquark. All of these quarkonium states and exotic states can not be analyzed simultaneously to determine the unknown parameters because too many possibilities exist. Instead, we can choose the quarkonium model to find the values of the parameters by the least-squares method.

In this paper, we will consider 480 different assignments for low-mass isosinglet mesons and compare different assignments to determine exotic states which cannot be assigned to any quarkonium states. Then, we calculate the best fitted parameter values for the $u\bar{u}$, $s\bar{s}$, $c\bar{c}$, and $b\bar{b}$ systems by using the least-squares method. The effective quark masses, the strong coupling constants, and the potential parameters can be determined explicitly in this procedure, and it is possible to compare directly the results of assigning radially and orbitally excited states with appropriate quantum numbers to the observed states.

In Section II, we give a brief description of our formalism, and assignments in the quarkonium model are given in Section III. Parameter fit procedures by the least-squares method are shown in Section IV, and the calculated results are presented in Section V. The final section is devoted to discussion.

II. FORMALISMS

When the quark model was proposed [2], only a small number of mesons and baryons were known. Hence, the main problem was to find the compositions of each hadron; radial excitations were not been considered because of the lack of information about the quark-confining potential. Although non-relativistic approximations had been made for various calculations, potential models were not used until the heavy charm quark system J/ψ was observed [3]. For the c and b quarkonium systems, the non-relativistic Schrödinger equation was tried, and various potential models were introduced [4] to predict the energy differences between radially and orbitally excited states. After the first successful estimation of the energy level differences, spin splittings were found to be very important for the quarkonium systems, and several groups have calculated the spin dependences of strongly interacting quarkonium systems.

For heavy quark systems such as $c\bar{c}$ and $b\bar{b}$, the spin-dependent potentials can be safely obtained by calculating the relativistic propagator corrections expanded in the strong coupling constant α_s and inverse quark mass $\frac{1}{m}$ for a suitably chosen Wilson loop. The first-order results are [5]

$$\begin{aligned}V_{SD} = & \frac{1}{m_1 m_2}(\mathbf{s}_1+\mathbf{s}_2)\cdot\mathbf{L}\frac{4}{3}\alpha_s\frac{1}{(r+r_q)^3} \\ & +\frac{1}{2}\left(\frac{\mathbf{s}_1}{m_1^2}+\frac{\mathbf{s}_2}{m_2^2}\right)\cdot\mathbf{L}\left(-\frac{1}{r}\frac{d\epsilon(r)}{dr}+\frac{8}{3}\alpha_s\frac{1}{(r+r_q)^3}\right) \\ & +\frac{1}{3m_1 m_2}(3\mathbf{s}_1\cdot\hat{\mathbf{r}}\mathbf{s}_2\cdot\hat{\mathbf{r}}-\mathbf{s}_1\cdot\mathbf{s}_2)\frac{4}{3}\alpha_s\frac{3}{(r+r_q)^3} \\ & +\frac{2}{3m_1 m_2}\mathbf{s}_1\cdot\mathbf{s}_2\frac{4}{3}\alpha_s 4\pi\frac{1}{r_0^3}e^{-\frac{\pi r^2}{r_0^2}}\end{aligned} \tag{1}$$

where $\mathbf{s}_i$ are the spins and $\mathbf{L}\equiv\mathbf{L}_1=-\mathbf{L}_2$. Since we cannot calculate the form of the spin-independent potential $\epsilon(r)$ from first principles, we have to assume the form of $\epsilon(r)$ [6].

As is well-known, various potential models exist with one or two parameters which can be adjusted to fit the observed spectra, and because these potential models have quite similar features for the radial and the orbital excitations of energy levels, we will take explicitly the linear plus Coulomb potential [7]

$$\epsilon(r)=\frac{r}{a^2}-\frac{4}{3}\alpha_s\frac{1}{r}+b \tag{2}$$

with two parameters a and b. The parameter b will be fixed by the observed mass of 1^3S_1 states in order to reduce the uncertainties of the annihilation contribution to the lowest-lying 1^1S_0 states. The choice of a linear plus Coulomb potential is motivated by the QCD asymptotic behaviors [8]. For short distances, the running coupling constant becomes so small that one gluon exchange is expected to be a good approximation, and the Coulomb potential results from this approximation. For large separation between the quark and the antiquark, the intermediate gluon fields are thought to form a linear tube so that the potential increases linearly with distance. It is well known that various phenomenological potential models appropriately accommodate these two features. After the successful applications of the spin-dependent potential models to heavy quarkonium systems, several attempts were made to extend the formalisms to the light quark systems made of up, down, and strange quarks. First attempts used mass formula method to predict remaining states with given radial and orbital excitations where several parameters were determined by observed masses. Later, brute force applications of potential model calculations were made for the light quark meson systems and met with unexpected successes in explaining nearly the whole meson system from π to Υ in one systematic viewpoint [9]. The difference between the heavy quark and the light quark systems is taken to be the relativistic motions of light quarks in contrast to the assumed non-relativistic motions of heavy quarks. The successful applications of the relativised quark model to the whole quarkonium system gave new insights with respect to the strong bound-state problems which have not been explicitly solved for highly relativistic cases.

The reason for the unexpected successes can be explained by introducing effective quark masses in bound-state problems. For a relativistic system of one quark-antiquark pair with masses m_1 and m_2, the equation to be solved is

$$H\Psi=E\Psi \tag{3}$$

where the Hamiltonian H can be written as $H=H_0+\epsilon(r)+V_{SD}$ with

$$H_0=\sqrt{\mathbf{p}_1{}^2+m_1^2}+\sqrt{\mathbf{p}_2{}^2+m_2^2} \tag{4}$$

Because of the square root operators, it is difficult to handle Eq. (3) without expansion, and if we consider the highly relativistic case $\mathbf{p}_i^2\gg m_i^2$, it is better to introduce the momentum-dependent parameter M and to expand as

$$\begin{aligned}\sqrt{\mathbf{p}_i{}^2+m_i^2} &= \sqrt{M^2+m_i^2+\mathbf{p}_i{}^2-M^2} \\ &\cong \frac{M}{2}+\frac{m_i^2}{2M}+\frac{\mathbf{p}_i{}^2}{2M}\end{aligned} \tag{5}$$

where the expansion parameter M includes the momentum expectations in the form [10]

$$M=\sqrt{<\mathbf{p}_i^2>+m_i^2}. \tag{6}$$

The parameter M is called the effective mass in bound-state problems and turns out to be larger than the conventional constituent quark mass by a factor of up to 5 or so for light quark systems. However, in the non-relativistic case, we have

$$\sqrt{m_i^2 + \mathbf{p}_i^2} \cong m_i + \frac{\mathbf{p}_i{}^2}{2m_i}. \qquad (7)$$

Now the two expanded forms in Eqs. (5) and (7) are equivalent if we consider that the spin-independent potential $\epsilon(r)$ contains a constant term as in Eq. (2). We can use the same second-order differential equation to solve Eq. (3) for any system, and the differences between various systems are represented by the magnitudes of the momentum expectations in Eq. (6). The determination of free parameters in the Hamiltonian H can be carried out by comparing the calculated and the observed masses for given combinations of quarks. Although seven parameters appear in H, we can reduce the number of free parameters by several methods. Firstly, we consider only those meson systems composed of equal mass quarks so that we have only one mass parameter M. Secondly, the two parameters r_q and r_0 can be fixed to some values which do not affect the final results [11]. We have tested various values and have chosen for this paper the values $r_q = 10^{-7}$ GeV^{-1} and $r_0 = 1.0$ GeV^{-1}. Finally, the constant b can be determined by the mass of the $1S$ triplet state for each combination of quarks. The remaining three parameters are the effective quark mass M, the potential parameter a, and the strong coupling constant α_s. We vary these parameters to fit the spectra of four different meson systems: $u\bar{u}$ (combination of u and d quarks), $s\bar{s}$, $c\bar{c}$, and $b\bar{b}$. Best values for these parameters have been chosen in such a way that the root-mean-square mass difference

$$\Delta m = \sqrt{\frac{1}{N}\sum_i (E_i^{cal} - E_i^{obs})^2} \qquad (8)$$

becomes a minimum, where E_i^{cal} and E_i^{obs} represent the calculated and the observed masses and N is the number of observed states. We used 11 isotriplet $u\bar{u}$ states, 11 $c\bar{c}$ states, and 12 $b\bar{b}$ states from the 1994 Particle Data; however, in the low-mass isosinglet states, problems of appropriate assignments occur. These assignments in the quarkonium model will be given in the next section.

III. ASSIGNMENTS IN THE QUARKONIUM MODEL

Most of the observed meson states can be explained as $q\bar{q}$ states with given quantum numbers corresponding to appropriate quantum states formed by spin-spin, spin-orbit couplings, and radial and orbital excitations. The assignments of observed states to $q\bar{q}$ states can be carried out without uncertainty for many mesons; however, uncertainties arise when several $q\bar{q}$ states exist with the same quantum numbers. For example, in a given combination of $q\bar{q}$, the quantum numbers of the s-wave triplets are the same as those of the D-wave triplet vector particles, and for different combinations of flavors such as $u\bar{u}$, $s\bar{s}$, $c\bar{c}$, and $b\bar{b}$, the same set of quantum states occurs.

In the 1994 Particle Data, there are 25 isosinglet mesons which are well established and can be taken to be composed of light quarks or gluons. In the quarkonium model, we have to assign these mesons to $u\bar{u}$ or $s\bar{s}$ states, and since the number of states is restricted for given energy ranges the possibility of an exotic state which cannot be assigned to any quarkonium state exists. In order to check-out various possibilities, we need to analyze as many assignments as possible, and these procedures can be reduced significantly by fixing some mesons to appropriate states. We can fix the six states $\eta(547)$, $\eta(1295)$, $\omega(782)$, $\omega(1419)$, $h_1(1170)$, and $\omega_3(1668)$ to the isosinglet $u\bar{u}$ 1^1S_0, 2^1S_0, 1^3S_1, 2^3S_1, 1^1P_1 and 1^3D_3 states, respectively. Furthermore, the five states $\eta'(958)$, $\eta(1420)$, $\phi(1019)$, $\phi_3(1854)$, and $f_4(2044)$ can be fixed to the $s\bar{s}$ 1^1S_0, 2^1S_0, 1^3S_1, 1^3D_3, and 1^3F_4 states [12]. We have found that two states $f_1(1282)$ and $f_2(1275)$ cannot be assigned to any $s\bar{s}$ state; and therefore, these states have to be assigned to the isosinglet $u\bar{u}$ 1^3P_1 and 1^3P_2 states, respectively. The other remaining 12 states have possibilities of being assigned to some $s\bar{s}$ states, and these possibilities are listed in Table 1. There are 5, 4, 2, and 12 possible assignments for the states with quantum numbers $J^{PC} = 1^{--}$, 0^{++}, 1^{++}, and 2^{++}, and we have considered all these 480 cases

Table 1. Possible assignments of isosinglet mesons to $s\bar{s}$ states.

mesons	possible $s\bar{s}$ assignments											
$\omega(1662)$	2^3S_1	2^3S_1	1^3D_1									
$\phi(1680)$		1^3D_1	2^3S_1	2^3S_1	1^3D_1							
$f_0(980)$	1^3P_0			1^3P_0								
$f_0(1300)$		1^3P_0										
$f_0(1598)$			1^3P_0	2^3P_0								
$f_1(1419)$	1^3P_1											
$f_1(1512)$		1^3P_1										
$f_2'(1525)$	1^3P_2	1^3P_2	1^3P_2	1^3P_2	1^3P_2							
$f_J(1709)$						1^3P_2	1^3P_2	1^3P_2	1^3P_2			
$f_2(2011)$	2^3P_2	2^3P_2				2^3P_2				1^3P_2	1^3P_2	1^3P_2
$f_2(2297)$			2^3P_2				2^3P_2			2^3P_2		
$f_2(2339)$	3^3P_2			2^3P_2				2^3P_2			2^3P_2	

to calculate the parameters by the least-squares method. For the 280 cases in which $f_2(2297)$ and $f_2(2339)$ are assigned to the $s\bar{s}$ 2^3P_2 state and $f_2(2011)$ to the $s\bar{s}$ 1^3P_2 state, the Δm are larger than 100 MeV, so we cannot assign these mesons to states above the $s\bar{s}$ states. Of the remaining cases, the number of acceptable assignments with Δm smaller than 100 MeV is 192. In the remaining 5 cases of $J^{PC} = 2^{++}$ assignments, Δm becomes smallest in the case where the 3 states $f_2'(1525)$, $f_2(2011)$, and $f_2(2339)$ are fixed to the isosinglet $s\bar{s}$ 1^3P_2, 2^3P_2, and 3^3P_2 states, respectively. Although it is equally probable to assign $f_0(980)$ or $f_0(1300)$ to the 1^3P_0 $s\bar{s}$ state in the four $J^{PC} = 0^{++}$ assignments, we have chosen $f_0(1300)$, which has larger frequencies than $f_0(980)$, to be assigned to 1^3P_0 state. For the $J^{PC} = 1^{--}$ assignments, both of the states $\omega(1662)$ and $\phi(1680)$ can be assigned to $s\bar{s}$ states, but we have selected $\omega(1662)$ for the 2^3S_1 $s\bar{s}$ state because Δm is smallest in this case. In the long run, we have found that it is equally probable to assign $f_1(1427)$ or $f_1(1512)$ to the 1^3P_1 $s\bar{s}$ state for the acceptable assignments, but we have assigned $f_1(1427)$ to the 1^3P_1 $s\bar{s}$ state for the same reason as above. Hence, we have assigned $\eta'(958)$, $\eta(1420)$, $\phi(1019)$, $\omega(1662)$, $f_2'(1525)$, $f_2(2011)$, $f_2(2339)$, $f_1(1427)$, $f_0(1300)$, $\phi_3(1854)$, and $f_4(2044)$ to the isosinglet $s\bar{s}$ 1^1S_0, 2^1S_0, 1^3S_1, 2^3S_1, 1^3P_2, 2^3P_2, 3^3P_2, 1^3P_1, 1^3P_0, 1^3D_3, and 1^3F_4 states, respectively. In a similar way, we can fix the 11 states $\pi(135)$, $\pi(1300)$, $\rho(770)$, $\rho(1465)$, $b_1(1231)$, $a_2(1318)$, $a_1(1230)$, $a_0(982)$, $\pi_2(1670)$, $\rho_3(1691)$, and $\rho(1700)$ to the isotriplet $u\bar{u}$ 1^1S_0, 2^1S_0, 1^3S_1, 2^3S_1, 1^1P_1, 1^3P_2, 1^3P_1, 1^3P_0, 1^1D_2, 1^3D_3, and 1^3D_1 states, respectively. Furthermore, the 11 states $\eta_c(2979)$, $\eta_c(3590)$, $J/\psi(3097)$, $\psi(3686)$, $\psi(4040)$, $\psi(4415)$, $\chi_{c_2}(3556)$, $\chi_{c_1}(3511)$, $\chi_{c_0}(3415)$, $\psi(3770)$, and $\psi(4159)$ can be fixed to $c\bar{c}$ 1^1S_0, 2^1S_0, 1^3S_1, 2^3S_1, 3^3S_1, 1^3P_2, 1^3P_1, 1^3P_0, 1^3D_1, and 2^3D_1 states. Finally, we can fix the 12 states $\Upsilon(9460)$, $\Upsilon(10023)$, $\Upsilon(10355)$, $\Upsilon(10580)$, $\Upsilon(10865)$, $\chi_{b_2}(9913)$, $\chi_{b_2}(10269)$, $\chi_{b_1}(9892)$, $\chi_{b_1}(10255)$, $\chi_{b_0}(9860)$, $\chi_{b_0}(10232)$, and $\Upsilon(11019)$ to the $b\bar{b}$ 1^3S_1, 2^3S_1, 3^3S_1, 4^3S_1, 5^3S_1, 1^3P_2, 2^3P_2, 1^3P_1, 2^3P_1, 1^3P_0, 2^3P_0, and 4^3D_1 states, respectively.

IV. PARAMETER FITTING BY THE LEAST-SQUARES METHOD

In the isotriplet sector, there are 11 fixed states, and we can calculate Δm for this sector. There are now three parameters $2m = m_1 = m_2$, a, and α_s. We can vary these parameters to check for the best fit by minimizing the value of Δm. In fact, we varied the parameters in the regions

$$1.40 \le 2m \le 1.64 \text{ GeV}, \quad 2.2 \le a \le 3.1 \text{ GeV}^{-1}, \quad 0.60 \le \alpha_s \le 0.94, \tag{9}$$

and selected 13, 10, and 15 values respectively for each parameters, so the total number of calculated Δm values

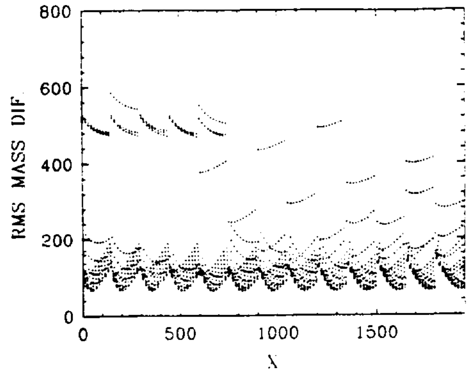

Fig. 1. The root-mean-square mass difference of $u\bar{u}$ meson systems. The total number of calculated Δm values is 1950.

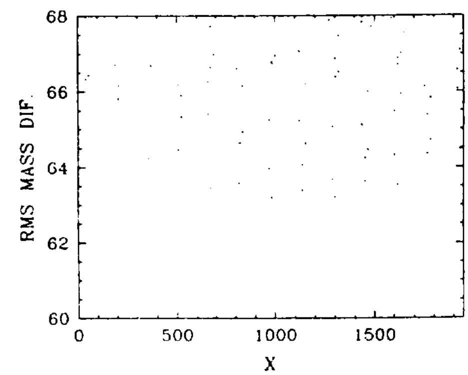

Fig. 2. The root-mean-square mass difference of $u\bar{u}$ meson systems. The y-axis of Fig. 1 is enlarged.

was 1950 (= $13 \times 10 \times 15$). Then, we plotted all of these Δm values along the y-axis in the Fig. 1, but the values of X correspond to the triplets of the parameters. For example, $X = 1$ corresponds to the parameters $2m =$ 1.40 GeV, $a = 2.2$ GeV^{-1}, $\alpha_s = 0.60$, and $X = 2$ to $2m = 1.40$ GeV, $a = 2.2$ GeV^{-1}, $\alpha_s = 0.62$, $\cdots$, and $X = 1950$ to $2m = 1.64$ GeV, $a = 3.1$ GeV^{-1}, $\alpha_s =$ 0.94. The values of Δm are distributed in the region $60 < \Delta m < 800$ GeV, and the minimum value equals 63.20 MeV at $X = 985$ which corresponds to $2m = 1.52$ GeV, $a = 2.70$ GeV^{-1}, and $\alpha_s = 0.84$. However for each value of $2m$, the corresponding minimum values of Δm are distributed in the region $63 < \Delta m < 67$ GeV, and these points are distributed along a curve similar to a quadratic function, as shown in Fig. 2.

For further calculations, we varied these parameters again. The regions of these parameters which include the minimum values of Δm for every $2m$ value of Fig. 1 are

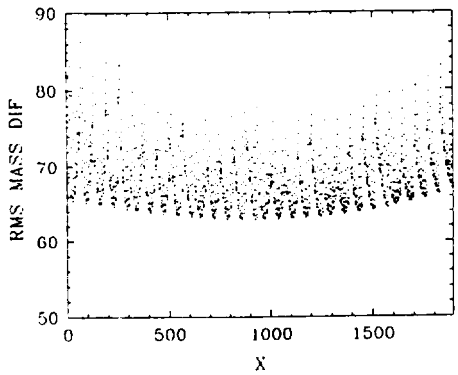

Fig. 3. The root-mean-square mass difference of $u\bar{u}$ meson systems. The total number of calculated Δm values is 1890.

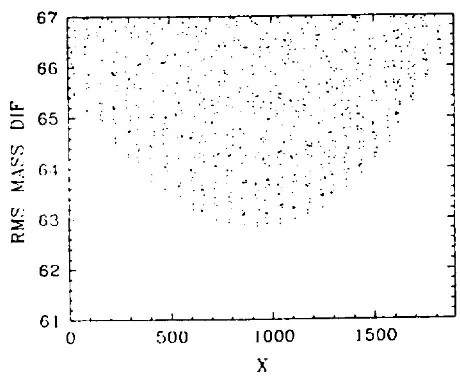

Fig. 4. The root-mean-square mass difference of $u\bar{u}$ meson systems. The y-axis of Fig. 3 is enlarged.

$$
\begin{aligned}
&1.40 \leq 2m \leq 1.69 \text{ GeV}, \\
&2.30 \leq a \leq 3.15 \text{ GeV}^{-1}, \\
&0.77 \leq \alpha_s \leq 0.93,
\end{aligned} \tag{10}
$$

and the selected number of values for each parameter was 30, 7, and 9, respectively, so the total number of calculated Δm values was 1890 (= $30 \times 7 \times 9$). We have also plotted all of these Δm values along the y-axis in Fig. 3, and the y-axis of this figure is enlarged in Fig. 4. From Fig. 4, the minimum value of Δm equals 62.84 MeV at $X = 927$, which correspond to $2m = 1.54$ GeV, $a = 2.70$ GeV^{-1}, and $\alpha_s = 0.85$. We have also shown that the minimum values of Δm are distributed according to a quadratic function as in Fig. 4.

Finally, we varied these parameters in the following regions:

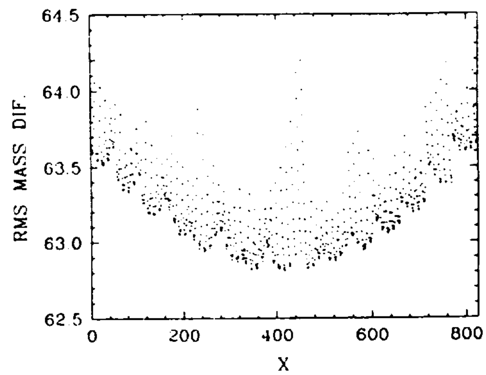

Fig. 5. The root-mean-square mass difference of $u\bar{u}$ meson systems. The total number of calculated Δm values is 825.

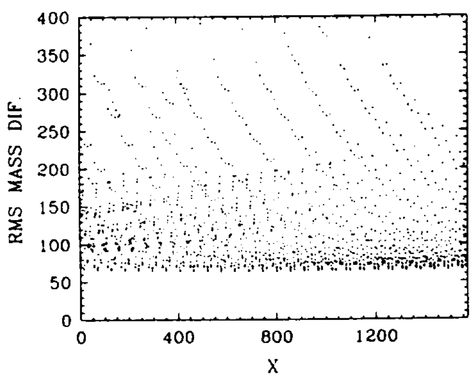

Fig. 6. The root-mean-square mass difference of $s\bar{s}$ meson systems. The total number of calculated Δm values is 1568.

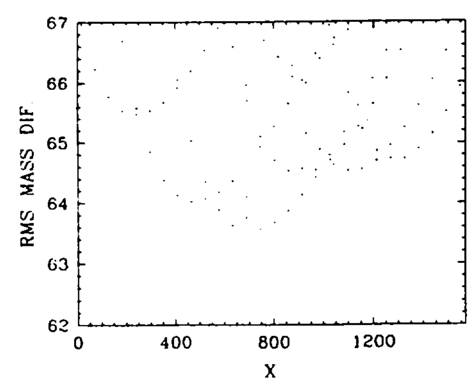

Fig. 7. The root-mean-square mass difference of $s\bar{s}$ meson systems. The y-axis of Fig. 6 is enlarged.

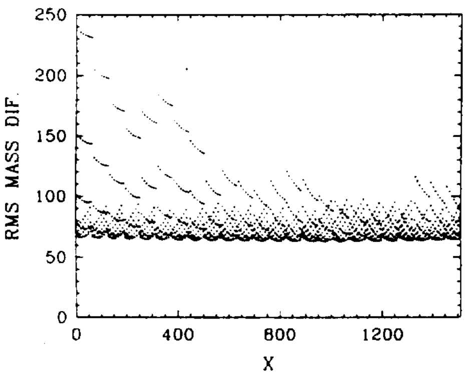

Fig. 8. The root-mean-square mass difference of $s\bar{s}$ meson systems. The total number of calculated Δm values is 1512.

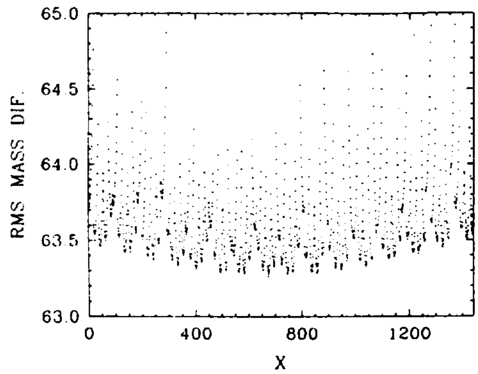

Fig. 10. The root-mean-square mass difference of $s\bar{s}$ meson systems. The total number of calculated Δm values is 1440.

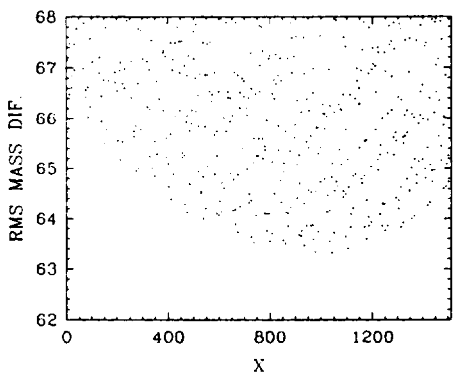

Fig. 9. The root-mean-square mass difference of $s\bar{s}$ meson systems. The y-axis of Fig. 8 is enlarged.

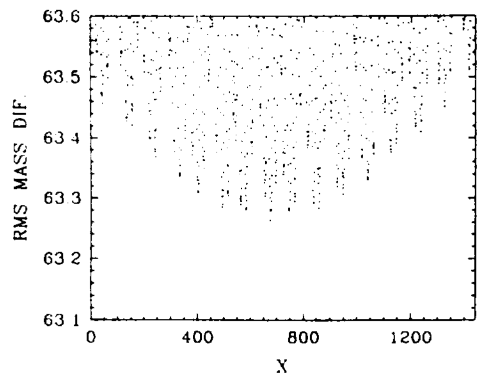

Fig. 11. The root-mean-square mass difference of $s\bar{s}$ meson systems. The y-axis of Fig. 10 is enlarged.

$$1.47 \leq 2m \leq 1.62 \text{ GeV}, \quad 2.54 \leq a \leq 2.92 \text{ GeV}^{-1}, \quad 0.810 \leq \alpha_s \leq 0.888. \tag{11}$$

We selected 16, 5, and 11 values respectively for each parameters, so the total number of calculated Δm values was 825 (= $15 \times 5 \times 11$). We have plotted all of these Δm values in Fig. 5, and the minimum value of Δm equals 62.81 MeV at $X = 416$, which corresponds to $2m = 1.54$ GeV, $a = 2.71$ GeV^{-1}, and $\alpha_s = 0.850$. Therefore, we may say that these are the parameters which produce the best fit in the isotriplet sector.

In the $s\bar{s}$ sector, 11 states were assigned. Therefore, we could calculate the values of Δm. Firstly, we varied the parameters in the regions

$$1.48 \leq 2m \leq 2.02 \text{ GeV}, \quad 2.2 \leq a \leq 2.5 \text{ GeV}^{-1}, \quad 0.42 \leq \alpha_s \leq 0.68, \tag{12}$$

and the total number of calculated Δm values was 1568 (= $28 \times 4 \times 14$). The minimum value of Δm was 63.56 MeV at $X = 746$, which means $2m = 1.74$ GeV, $a = 2.30$ GeV^{-1}, and $\alpha_s = 0.48$. These data are shown in Fig. 6 and Fig. 7. Secondly, we varied the parameters in the regions

$$1.48 \leq 2m \leq 1.94 \text{ GeV}, \quad 2.24 \leq a \leq 2.42 \text{ GeV}^{-1}, \quad 0.36 \leq \alpha_s \leq 0.60, \tag{13}$$

and the total number of calculated Δm values was 1512 (= $24 \times 7 \times 9$). The minimum value of Δm was 63.32 MeV at $X = 1041$, which means $2m = 1.80$ GeV, $a = 2.34$ GeV^{-1}, and $\alpha_s = 0.50$. These are shown in the Fig. 8 and Fig. 9. For the final procedure of this isosinglet sector, we varied the parameters in the regions

$$1.70 \leq 2m \leq 1.85 \text{ GeV}, \quad 2.29 \leq a \leq 2.36 \text{ GeV}^{-1}, \quad 0.471 \leq \alpha_s \leq 0.516, \tag{14}$$

and the total number of calculated Δm values was 1440

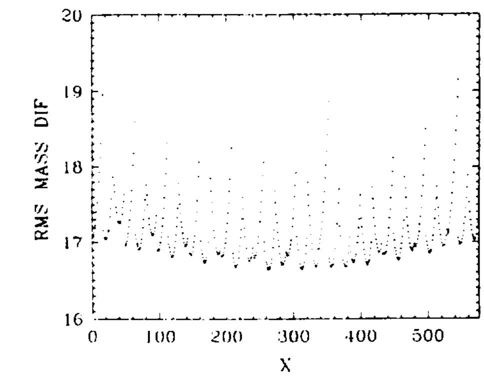

Fig. 12. The root-mean-square mass difference of $c\bar{c}$ meson systems. The total number of calculated Δm values is 576.

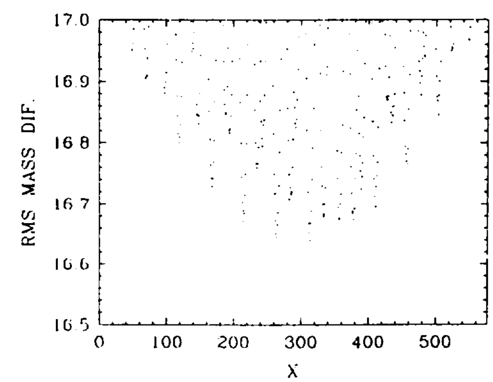

Fig. 13. The root-mean-square mass difference of $c\bar{c}$ meson systems. The y-axis of Fig. 12 is enlarged.

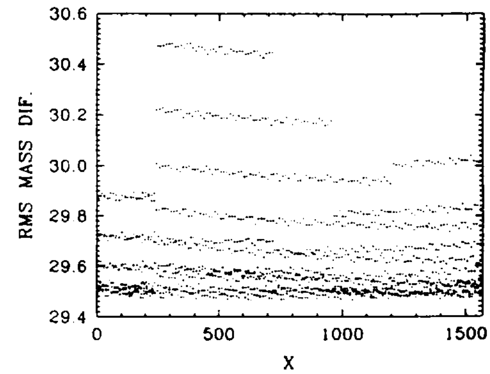

Fig. 14. The root-mean-square mass difference of $b\bar{b}$ meson systems. The total number of calculated Δm values is 1680.

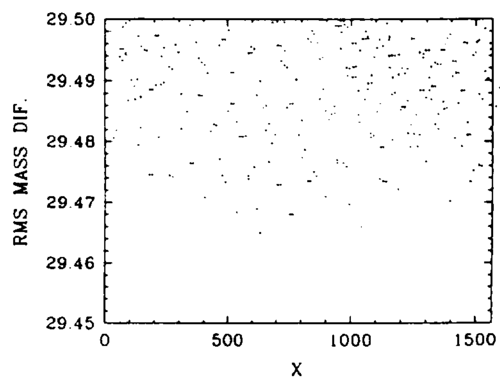

Fig. 15. The root-mean-square mass difference of $b\bar{b}$ meson systems. The y-axis of Fig. 14 is enlarged.

(= 16 × 5 × 18). We have shown all of these values of Δm in Figs. 10 and 11, and the minimum value of Δm for this isosinglet sector is 63.26 MeV and occurs at X = 674, which means $2m$ = 1.77 GeV, a = 2.33 GeV^{-1}, and α_s = 0.493.

In the charmonium sector, we calculated Δm in the same way as before by using 11 states. For the last calculation of this sector, we varied the parameters in the regions

$$\begin{aligned} &1.82 \leq 2m \leq 1.93 \text{ GeV}, \\ &2.49 \leq a \leq 2.52 \text{ GeV}^{-1}, \\ &0.464 \leq \alpha_s \leq 0.494, \end{aligned} \tag{15}$$

and the total number of calculated Δm values was 576 (= 12 × 3 × 16). All of the calculated values of Δm are shown in Figs. 12 and 13, and the minimum value of Δm is 16.64 MeV and have occurrs at X = 313, which corresponds to $2m$ = 1.88 GeV, a = 2.50 GeV^{-1}, α_s = 0.474 .

Finally, we calculated the Δm values in the bottonium sector. We varied the three parameters in the regions

$$\begin{aligned} &5.171 \leq 2m \leq 5.310 \text{ GeV}, \\ &2.22 \leq a \leq 2.24 \text{ GeV}^{-1}, \\ &0.327 \leq \alpha_s \leq 0.336, \end{aligned} \tag{16}$$

and the total number of calculated Δm values was 1680 (= 140 × 2 × 6). We have shown all of these Δm values in Fig. 14, and, as before, the minimum values of Δm for every $2m$ value are shown clearly in Fig. 15. The minimum value of Δm is 29.47 MeV and occurrs at X = 623, which corresponds to $2m$ = 5.223 GeV, a = 2.23 GeV^{-1}, and α_s = 0.332. If we vary the parameters a and α_s more densely than those shown here, the distribution of these local minimum values is thought to be similar to plot of quadratic function, as before.

Table 2. Calculated and observed masses of mesons in units of MeV. The numbers in parentheses are the values of the experimental masses.

	$u\bar{u}$			$s\bar{s}$		$c\bar{c}$		$b\bar{b}$	
	Exp.(I=1)	Cal.	Exp.(I=0)	Exp.	Cal.	Exp.	Cal.	Exp.	Cal.
1^1S_0	$\pi(135)$	220	$\eta(547)$	$\eta'(958)$	873	$\eta_c(2979)$	2975		9431
2^1S_0	$\pi(1300)$	1349	$\eta(1295)$	$\eta(1420)$	1587	$\eta_c(3590)$	3620		10000
3^1S_0		1819			2046		4032		10351
1^3S_1	$\rho(770)$	770	$\omega(782)$	$\phi(1019)$	1019	$J/\psi(3097)$	3097	$\Upsilon(9460)$	9460
2^3S_1	$\rho(1465)$	1483	$\omega(1419)$	$\omega(1662)$	1651	$\psi(3686)$	3673	$\Upsilon(10023)$	10009
3^3S_1		1907			2093	$\psi(4040)$	4070	$\Upsilon(10355)$	10358
4^3S_1		2248			2463	$\psi(4415)$	4403	$\Upsilon(10580)$	10643
5^3S_1		2546			2794		4700	$\Upsilon(10865)$	10895
1^1P_1	$b_1(1231)$	1339	$h_1(1170)$		1478		3522		9900
2^1P_1		1784			1941		3937		10260
1^3P_2	$a_2(1318)$	1397	$f_2(1275)$	$f_2'(1525)$	1501	$\chi_{c_2}(3556)$	3541	$\chi_{b_2}(9913)$	9909
2^3P_2		1829	$f_J(1709)$	$f_2(2011)$	1963		3955	$\chi_{b_2}(10269)$	10268
3^3P_2		2176	$f_2(2297)$	$f_2(2339)$	2346		4299		10561
1^3P_1	$a_1(1230)$	1235	$f_1(1282)$	$f_1(1427)$	1454	$\chi_{c_1}(3511)$	3503	$\chi_{b_1}(9892)$	9892
2^3P_1		1702			1917		3919	$\chi_{b_1}(10255)$	10253
1^3P_0	$a_0(982)$	954	$f_0(980)$	$f_0(1300)$	1303	$\chi_{c_0}(3415)$	3416	$\chi_{b_0}(9860)$	9864
2^3P_0		1609	$f_0(1581)$		1809		3857	$\chi_{b_0}(10232)$	10232
1^1D_2	$\phi_2(1670)$	1656			1783		3799		10146
1^3D_3	$\rho_3(1691)$	1677	$\omega_3(1668)$	$\phi_3(1854)$	1785		3799		10148
1^3D_1	$\rho(1700)$	1580	$\phi(1680)$		1768	$\psi(3770)$	3784		10140
2^3D_1		1943			2162	$\psi(4159)$	4139		10443
3^3D_1		2257			2509		4451		10706
4^3D_1		2541			2825		4735	$\Upsilon(11019)$	10945
1^3F_4		1901		$f_4(2044)$	2024		4016		10341

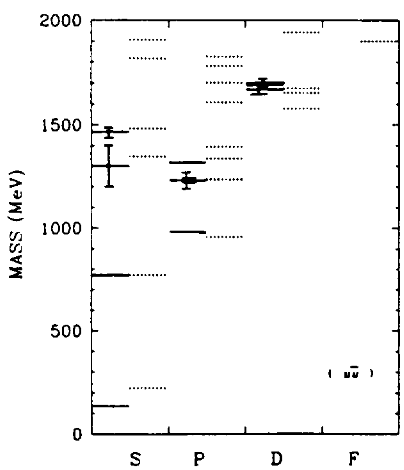

Fig. 16. Positions of the S-, P-, and D-wave $u\bar{u}$ levels. Solid lines represent experimental spectra, and dashed lines represent calculated spectra.

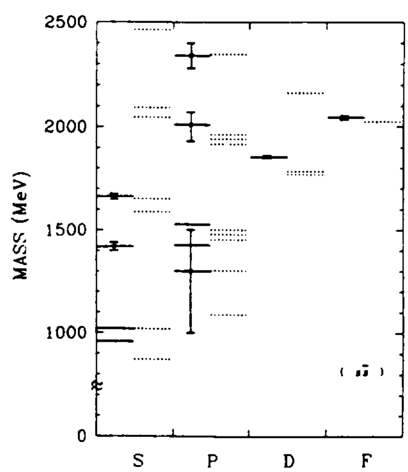

Fig. 17. Positions of the S-, P-, and D-wave $s\bar{s}$ levels. Solid lines represent experimental spectra, and dashed lines represent calculated spectra.

V. CALCULATED RESULTS

The calculated, along with the experimental, spectra are shown in Table 2 and plotted in Figs. 16, 17, 18, and 19. The determined values of the parameters are also displayed in Table 3, where the minimum values of Δm are given. In Table 2, 24 isosinglet mesons are assigned to $u\bar{u}$ and $s\bar{s}$ states, and the remaining meson is $f_1(1512)$. This meson cannot be assigned to the 2^3P_1 $s\bar{s}$ state which is predicted to have a mass of 1915 MeV.

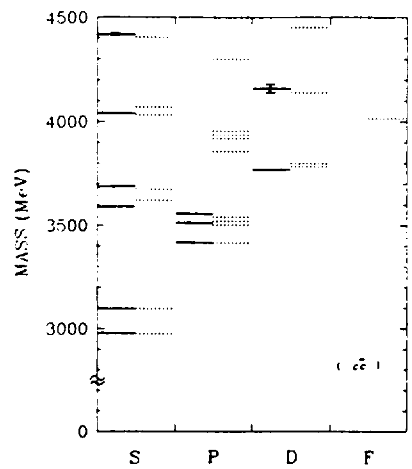

Fig. 18. Positions of the S-, P-, and D-wave $c\bar{c}$ levels. Solid lines represent experimental spectra, and dashed lines represent calculated spectra.

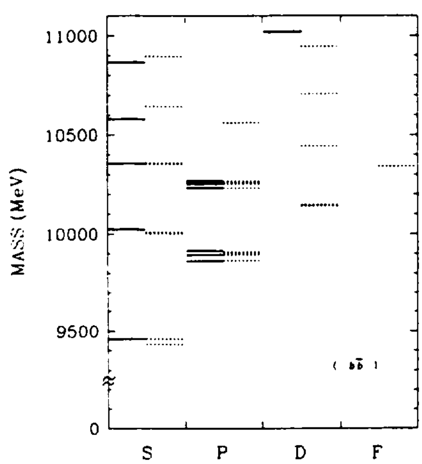

Fig. 19. Positions of the S-, P-, and D-wave $b\bar{b}$ levels. Solid lines represent experimental spectra, and dashed lines represent calculated spectra.

However, it can be assigned to the 1^3P_1 $s\bar{s}$ state as in Table 1, and we have found that it is equally probable to assign $f_1(1419)$ or $f_1(1512)$ to the 1^3P_1 $s\bar{s}$ state. Another possibility is to assign one of these two state to the 2^3P_1 $u\bar{u}$ state which is predicted to be at 1702 MeV, but the mass differences between the predicted and observed states are large enough to include this possibility. Then, we can conclude that either $f_1(1419)$ or $f_1(1512)$ cannot be assigned to any $q\bar{q}$ state.

Now we turn to the determined parameters listed in Table 3. We can see that the strong coupling constant

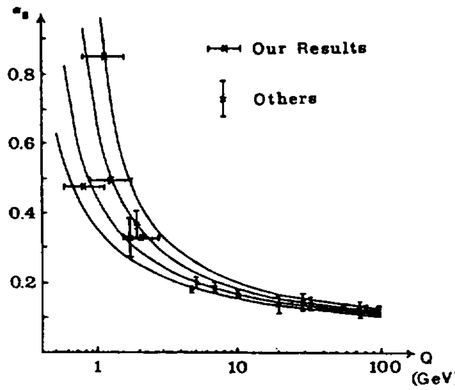

Fig. 20. The values of α_s with horizontal error bars determined by Eq. (21) are compared with other experimental data points. Solid curves correspond to $\Lambda_{\overline{MS}}$ = 350, 250, 150, and 100 MeV from top to bottom.

Table 3. Determined parameters for each quarkonium system.

	M (GeV)	α_s	a (GeV^{-1})	Δm (MeV)
$u\bar{u}$	1.54	0.850	2.71	62.81
$s\bar{s}$	1.77	0.493	2.33	63.26
$c\bar{c}$	1.88	0.474	2.50	16.64
$b\bar{b}$	5.223	0.332	2.23	29.47

varies according to the related momentum included in the effective quark mass M [13]. The ratio between the momentum expectation value and the effective quark mass is

$$f = \frac{\sqrt{<\mathbf{p}^2>}}{\sqrt{<\mathbf{p}^2> + m^2}}, \tag{17}$$

which can be calculated when the quark masses are given. For example, if we take the quark masses in unit of GeV as [14]

$$m_u = 0.33, \quad m_s = 0.45, \quad m_c = 1.5, \quad m_b = 4.5, \tag{18}$$

the f factor becomes

$$f_u = 0.976, \quad f_s = 0.967, \quad f_c = 0.603, \quad f_b = 0.508 \tag{19}$$

for each quark system. By considering this factor, we may choose the relevant momentum scale μ for α_s as

$$\mu = fM. \tag{20}$$

However, it is well-known that there exist scale ambiguities associated with the determination of the renormalization scale for α_s. These ambiguities can be used to estimate the theoretical errors in α_s [15], and, in general, the best fitted energy scale turns out to be considerably smaller than the typical energy scale in scattering problems [13]. In bound-state problems, it is not so easy to fix the appropriate scale boundaries, and one lower limit can be set at half the typical scale, as in some scattering examples [16]. For this choice, we have

$$\frac{1}{2} fM \leq \mu \leq fM, \tag{21}$$

and the error bars can be drawn as in Fig. 20 in which other experimental data are also shown. We can see that the new results from spectroscopic analysis turn out to be complementary with respect to those from high-energy analyses.

VI. DISCUSSION

We have calculated the strong coupling constants and the effective quark masses for four different meson systems, $u\bar{u}$, $s\bar{s}$, $c\bar{c}$, and $b\bar{b}$, by considering first-order spin-dependent forces with the linear plus Coulomb potential model to fit the observed mass spectra. These parameters were determined by the least-squares method, and the calculated and observed masses were compared. For low-mass isosinglets, we considered 480 possible assignments and found 192 acceptable cases from which we can conclude that either $f_1(1419)$ or $f_1(1512)$ should be an exotic state not assignable to any $q\bar{q}$ state. From the determined effective quark masses, we deduced the relevant momentum scales for α_s, and the results were consistent with other experimental data. We also showed in some figures that minimum values of Δm exist for each quarkonium system, but it remains to be confirmed by using a genetic algorithm that these minimum values of Δm are really local minima.

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