Effect of Interfacial Drag Force on the Numerical Stability of the Two-Fluid Model

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이상유체에 있어서 경계면 Drag 힘이 수치적 안정성에 미치는 영향에 대한 고찰

이 재 영

Summary

In order to enhance the stability limit of the I. C. E. (Implicit courant Eulerian) Method, the Stability enhancing Two-step method (SETS) implemented in TRAC-PFI adds the stabilization step to the basic step of the I. C. E. method. The matrix size of the SETS method is smaller than that of ordinary fully-implicit methods. However, the momentum stabilization steps enlarge the matrix size of the SETS method as dimension increases.

In order to reduce the matrix size of the SETS method in multi-dimensional problems and to study the effect of interfacial drag force on stability, the stability analysis of SETS method without momentum stabilization steps (SETS-WM) is done here. One dimensional stability criterion is obtained by von Neumann stability analysis. It is found that the interfacial drag force enlarges the stability limit considerably. When SETS-WM is numerically tested, stability problems have not been encounted with several numerical simulations with time step sizes restricted by the stability limit derived here.

1. Introduction

Lots of computer codes for the thermal hydraulic safety analysis of nuclear power plants have been developed. In these codes the time step size is determined on the basis of numerical stability and accuracy.

In order to analyze the large break loss of coolant accident, the I. C. E. (Implicit

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Courant Eulerian) method was developed by taking the sonic term implicitly and the convective terms explicity. Therefore, the I. C. E. method has the material convective limit which is large enough for such an accident to ensure accuracy.

However, the small break loss of coolant accident having been emphasized after the T. M. I. accident indicates that the material convective limit of the I. C. E. method is too short to analyze such a mild and long accident. Therefore, the following efforts have been dene to enhance numerical stability :

1. Fully-implicit method : by treating every term implicitly, the unconditionallystable numerical scheme can be obtained with the large matrix size. Computation burden from the large matrix size can be mitigated by use of the large time step size. ATHENA [1] is one of computer codes using this scheme.

2. SETS(Stability-emhancing Two-Step) method : In order to make the smaller matrix size than the fully implicit method and to use the efficient numerical algorithm of the I. C. E. method, the stabilization steps of convective terms are added to the basic step of the I. C. E. method. TRAC-PF1[2] and RELAP5-MOD2[3] uses this scheme. As dimension increases, the size of matrix increases due to momentum stabilization steps. If the momentum stabilization steps are removed from the SETS method, the production of the small matrix size is possible.

Therefore, numerical stability analysis is performed to the SETS method without momentum stabilization steps (SETS-WM). The effect of the interfacial drag force on stability is emphasized here.

2. Numerical Scheme

The SETS method takes two fractional steps composed of basic and stabilization steps.

2.1 Difference Equations in the Basic Step

The basic step use the conventional I. C. E. method. therefore, the finite difference equations are the same as THERMIT[4].

· Mass conservation :

$$\frac{1}{\Delta t}\left[(\tilde{\alpha}_k\tilde{\rho}_k)_i^{n+1}-(\alpha_k\rho_k)_i^n\right]+\nabla_i\left[(\alpha_k\rho_k)^n V_k^{n+1}\right]=\Gamma_k^{n+1/2} \tag{1}$$

· Energy conservation :

$$\frac{1}{\Delta t}\left[(\tilde{\alpha}_k\tilde{\rho}_k\tilde{e}_k)_i^{n+1}-(\alpha_k\rho_k e_k)_i^n\right]+\nabla_i\left[(\alpha_k\rho_k e_k)^n V_k^{n+1}\right]$$
$$+P_i\left[\frac{1}{\Delta t}\left(\tilde{\alpha}_k^{n+1}-\alpha_k^n\right)+\nabla_i(\alpha_i^n V_k^{n+1})\right]=Q_{wk}^{n+1}+Q_{ik}^{n+1/2} \tag{2}$$

· Momentum conservation :

$$\frac{1}{\Delta t}\left(\tilde{V}_k^{n+1}-V_k^n\right)+V_k^n\nabla_i V_k^n+\frac{1}{\rho_k}\nabla_i P^{n+1} \tag{3}$$
$$+\frac{f_{wk}}{\alpha_k\rho_k}\tilde{V}_k^{n+1}+\frac{f_i}{\alpha_k\rho_k}(\tilde{V}_k^{n+1}-\tilde{V}_{k-1}^{n+1})=g,$$

where

k=1 : vapor, k=3 : liquid.

A tilde above a variable indicates that it is the result of an intermediate step. An over var is an average between its values at the adjacent cell. The time discretization used in the momentum conservation equation results in linear expression relating each new time phase velocity at a cell face to new time

adjacent pressure. The newton-Rahpson method linearizes the set of nonlinear equations, and constructs a linear matrix equation of pressure field. By solving the matrix for pressure, the other variables can be obtained.

2. 3. Difference Equations in the Stabilization Step

The convective terms is stabilized by treating implicitly in the stabilizing step.

· Mass conservation :

$$\frac{1}{\Delta t}\left[(\alpha_k\rho_k)_i^{n+1}-(\alpha_k\rho_k)_i^n\right]+\nabla_i\left[(\alpha_k\rho_k)^{n+1}V_k^{n+1}\right]=\Gamma_k^{n+1/2}. \tag{4}$$

· Energy conservation :

$$\frac{1}{\Delta t}\left[(\alpha_k\rho_k e_k)_i^{n+1}-(\alpha_k\rho_k e_k)_i^n\right]+\nabla_i\left[(\alpha_k\rho_k e_k)^{n+1}V_k^{n+1}\right]$$
$$+P_i\left[\frac{1}{\Delta t}\left(\alpha_k^{n+1}-\alpha_k^n\right)+\nabla_i(\alpha_i^n V_k^{n+1})\right]=Q_{wk}^{n+1}+Q_{ik}^{n+1/2}. \tag{5}$$

The above equations stabilize error growth by the convective terms in the basic step.

3. Numerical Stability

It is well known that every variable has the stability limit according to the degree of implicitness [5]. For example, pressure in I. C. E. has no stability limit, but temperature has the convective limit. If a stabilization step of a variable is removed, then the stability limit is determined by the variable in the basic step. Therefore, if the momentum stabilization steps are removed from the SETS method, then the unstabilized momentum in the basic step would result in the numerical instability of the SETS method for a large time step size.

In order to see clearly the effect of interfacial drag force on the numerical stability, the mass exchange rate and the wall friction force are neglected. Also, energy equations are dropped in stability analysis because enthalpies in energy equations are completely stabilized in the stabilizing step and, in turn, energy equations have no effect on stability.

Treating the coefficients as constants, let us apply the von Neumann local linear stability analysis, $U_k^n=\zeta^n\exp(ik\triangle z)$, to the mass and momentum conservation equations, Eqs. (1) and (3). for the differences of the basic variables α, p, V_v, V_l, the determinant form for a nontrivial solution is obtained :

$$\det\begin{vmatrix} \rho_v(\zeta-1+\bar{V_v}) & a_v^{-2}\alpha_v(\zeta-1+\bar{V_v}) & \alpha_v\rho_v\zeta q & 0 \\ \rho_l(\zeta-1+\bar{V_l}) & a_l^{-2}\alpha_l(\zeta-1+\bar{V_l}) & 0 & \alpha_l\rho_l\zeta q \\ 0 & \alpha_v\zeta q & \alpha_v\rho_v(\zeta-1+\bar{V_v})+f_i\Delta t\,\zeta & -f_i\Delta t\,\zeta \\ 0 & \alpha_l\zeta q & -f_i\Delta t\,\zeta & \alpha_l\rho_l(\zeta-1+\bar{V_l})+f_i\Delta t\,\zeta \end{vmatrix}=0$$

where

$$\bar{V_v}=\left(\frac{\Delta t}{\Delta z}V_v\right)(1-e^{-i\theta}),$$

$$\bar{V_l}=\left(\frac{\Delta t}{\Delta z}V_l\right)(1-e^{-i\theta}),$$

$$q=2i\left(\frac{\Delta t}{\Delta z}\right)\sin\frac{\theta}{2},\ \text{and}$$

$$\theta=\frac{\pi}{N},\quad N=1\rightarrow J.$$

where J is the number of axial mesh. for simplicity, we assume that

$a_v \rangle V_v$, $a_l \rangle V_l$.

If void fraction is not equal to zero or one, that is, flow is in two phase, the following characteristic equation can be obtained :

$$a_1\zeta^2+a_2\zeta+a_3=0 \tag{7}$$

where

$$a_1 = \alpha_v\alpha_l(\alpha_v\rho_l + \alpha_l\rho_v) + f_i\Delta t,$$

$$a_2 = \alpha_v\alpha_l\left[(2\alpha_l\rho_v + f_i\Delta t/\alpha_v)(\overline{V_v} - 1) + (2\alpha_v\rho_l + + f_i\Delta t/\alpha_l)(\overline{V_l} - 1)\right]$$

$$a_3 = \alpha_v\alpha_l\left[\alpha_l\rho_v(\overline{V_v} - 1)^2 + \alpha_v\rho_l(\overline{V_l} - 1)^2\right]$$

If the characteristics, ζ, is divided as real and imaginary parts such as $\zeta = p + qi$ Eq. (7) results in the following two equations :

· Real part :

$$a(p^2 - q^2) + bp - cq + d_1 = 0. \quad (8)$$

· Imaginary part :

$$2apq + bq + cp + d_2 = 0. \quad (9)$$

where

$$a = \alpha_v\alpha_l\left[\alpha_l\rho_v + \alpha_v\rho_l\right] + f_i\Delta t,$$

$$b = \alpha_v\alpha_l\left[(2\alpha_l\rho_v + f_i\Delta t/\alpha_v)(x_vC_s - 1) + (2\alpha_v\rho_l + f_i\Delta t/\alpha_l)(x_lC_s - 1)\right]$$

$$c = \alpha_v\alpha_l\left[(2\alpha_l\rho_v + f_i\Delta t/\alpha_v)x_vS_s + (2\alpha_v\rho_l + f_i\Delta t/\alpha_l)x_lS_s\right]$$

$$d_1 = \alpha_v\alpha_l\left[\alpha_l\rho_v((x_vC_s - 1)^2 - (x_vS_s)^2) + \alpha_v\rho_l((x_lC_s - 1)^2 - (x_lS_s)^2)\right],$$

$$d_2 = 2\alpha_v\alpha_l\left[\alpha_l\rho_v(x_vC_s - 1)x_vS_s + \alpha_v\rho_l(x_lC_s - 1)x_lS_s\right],$$

$$x_v = (\Delta t/\Delta z)V_v,$$

$$x_l = (\Delta t/\Delta z)V_l,$$

$$C_s = 1 - \cos\theta, \text{ and}$$

$$S_s = \sin\theta.$$

The error growth rate must be less than one to ensure the stability :

$$\zeta\zeta^* < 1. \quad (10)$$

The above imequality gives the following expression :

$$p^2 + q^2 \leq 1. \quad (11)$$

Applying Eqs. (8) and (9) to the inequality (11) yields the stability criterion. However. the nonlinearity of Eqs. (8) and (9) is too high to obtain the values p and q explicitly except the global spectrum ($\theta = \pi$). Lots of numerical tests are performed for every spectrum to know the error growth rate by solving the nonlinear Eqs. (8) and (9) with the arbitrarilly given void fraction and interfacial drag coefficient. the numerical test results show that the stability limit in the case with $\theta = \pi$ is smallest. For example, the error growth rates with the void fraction, 0.35, the interfacial drag coefficient, 1.0×10^5, pressure, 13.8 MPa, and the material convective limit, 0.3 sec, are shown in Figs. 1 and 2 according to the spectrum. As the interfacial coefficient decreases, the error growth rates in the case with $\theta = \pi$ increase, but still they are smaller than that in the case with $\theta \neq \pi$. The error growth rate in the case with $\theta = \pi$ is easily obtainable because coefficients c and d_2 in Eqs. (8) and (9) becomes zero. Therefore, the following simple relation is obtained from Eqs. (8) and (9) :

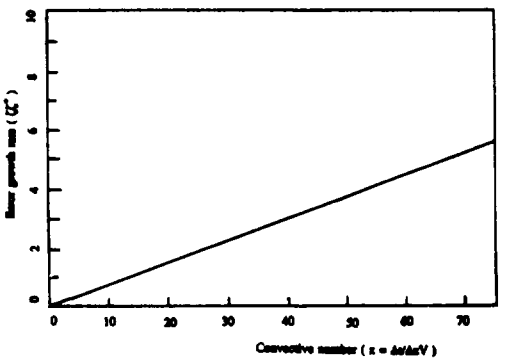

Fig. 1. The error growth rate of SETS-WM in the case with $= \pi$.

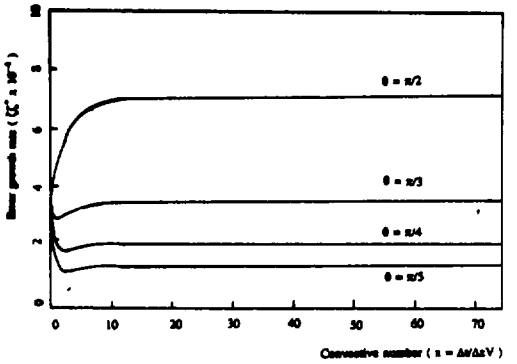

Fig. 2. The error growth rate of SETS-WM in the case with $\theta = \pi$.

$$2apq + bq = 0. \tag{12}$$

$$q^2 = p^2 = (b/a)p + d_1/a. \tag{13}$$

Substituting eqs. (12) and (13) to the stability condition of Eq (11) yields :

$$4\alpha_v\alpha_l\left[\alpha_v\rho_l(\frac{V_l}{\Delta z})^2 + \alpha_l\rho_v(\frac{V_v}{\Delta z})^2\right]\Delta t^2 \le 4\alpha_v\alpha_l \left[\alpha_l\rho_v(\frac{V_v}{\Delta z}) + \alpha_v\rho_l(\frac{V_l}{\Delta z})\right]\Delta t + f_i\Delta t. \tag{14}$$

Rearranging Eq. (14) gives the following stability limit for SETS-WM :

$$\Delta t < \frac{\alpha_l\rho_v(V_v/\Delta z) + \alpha_v\rho_l(V_l/\Delta z)}{\alpha_l\rho_v(V_v/\Delta z)^2 + \alpha_v\rho_l(V_l/\Delta z)^2} + \frac{f_i/(4\alpha_v\alpha_l)}{\alpha_l\rho_v(V_v/\Delta z)^2 + \alpha_v\rho_l(V_l/\Delta z)^2}. \tag{15}$$

where the first and second terms of the limit represents the convective limit and contribution enhanced by the interfacial drag force, respectively.

If void fraction is equal to zero or one,

$$\zeta = 0.$$

In single-phase flow, SETS-WM is unconditionally stable

3.2 Comparison of Error growth Rates between the I. C. E. Method and the SETS-WM Method

The numerical stability test shows that the I. C. E. method fails within few steps if the time step size is larger than the material convective limit. But the SETS-WM method fails after very large steps when the time step size is larger than the limit suggested in Eq. (15). This is due to the fact that the dependance of the error growth rate upon the time step size is different between the I. C. E. method and the SETS-WM method.

The error growth rate of the I. C. E. method is as follows :

$$\zeta\zeta^* = 1 + 2(1 - \cos\theta)(\Delta t/\Delta zV)(\Delta t/\Delta zV - 1). \tag{16}$$

The error growth rate is largest in the case with $\theta = \pi$.

$$R = \sqrt{\zeta\zeta^*} = (2V/\Delta z)\Delta t - 1. \tag{17}$$

However, the interfacial momentum exchange terms make the error growth rate of the SETS-WM method as follows :

$$R = \frac{(2V/\Delta z)\Delta t - 1}{\sqrt{A\Delta t + 1}}, \tag{18}$$

where

$$A = \frac{f_i}{\alpha_v\alpha_l(\alpha_v\rho_l + \alpha_l\rho_v)}$$

Figure 3 shows the diffecence of the error growth rate between the SETS-WM method and the I. C. E. method. the differences are as follows :

1. The error growth rate of the I. C. E. method has linear dependence on the time step size, but that of the SETS-WM method

very mildly depends on the time step size for the sufficiently large interfacial drag coefficient.

2. Every spectrum takes part in the instability of the I. C. E. method but only the spectrum with $\theta = \pi$ takes part in the instability of the SETS-WM method.

The above differences affect the stability characteristics between the I. C. E. method and the SETS-WM method.

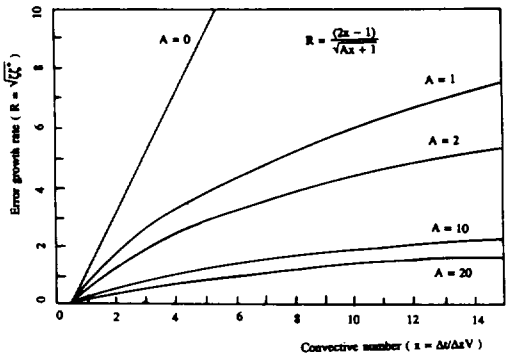

Fig. 3. The effect of the interfacial momentum exchange term on the error growth rate : A=0 : the I. C. e. method

4. Results and Discussions

A number of numerical experiments are performed using the SETS-WM scheme and the results are compared to those of THERMIT which uses the semi-implict scheme, and to those of THERMIT-WM which are made on the basis of THERMIT with the SETS and SETS-WM schemes, respectively.

Problem 1. single Phase Heating

The object of the present simulation is to see the stability of THERMIT-WM with the larger time step size than the convective stability limit in single phase.

The straight vertical pipe of 12 volumes as listed in table 1 is heasted by the internal heat source, 5MW/㎥ in the step at t=0. The inlet boundary conditions of the pipe consist of an influx of subcooled water with a temperature of 550°K at 13.79 Mpa.

Table 1. Data of heating and cooling down pipe

Total length (m)	0.762
Number of Nodes	12
Area per node (㎡)	0.0019635
Inlet velocity (m/s)	0.36
Pressure (MPa)	13.79

THERMIT-WM is run with time step sizes from 0.01 seconds to 10 seconds increasing linearly at each ste. the time step size, 10 seconds, corresponds to the fifty-five times of the material Courant limit and there is no instability as shown in Fig. 4. the exceeding time step size des not affect velocity but temperature at the basic step, and the stabilization of the mass and energy corrects temperatur properly. this simulation demonstrates that THERMIT-WM is unconditionally stable in the single phase case as expected in the linear local von Neumann stability analysis.

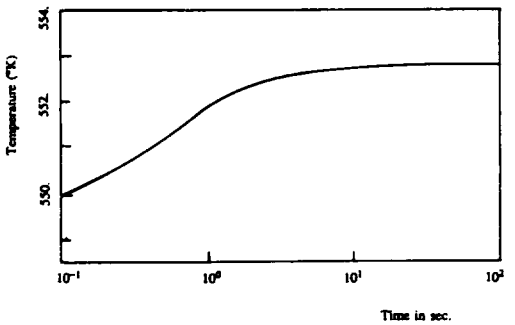

Fig. 4. The temperature change for the problem of the single-phase heating by the SETS-WM method.
[time step size=55 xconvective limit (0.3sec)]

Problem 2. Heating and Cooling

In order to see the stability and the accuracy of THERMIT-WM, a numerical test is performed in the same geometry as problem 1. The inlet boundary conditions of the pipe consist of an influx of the liquid with velocity of 0.36m/sec which is slightly subcooled of 608.6° at 13.79 MPa. This pipe is heated by the internal heat source 15 MW/m^3 from the step at t=0 to the step at t=6 seconds and the internal heat source is removed at 6 seconds.

figure 5 shows the change of the void fraction at the last cell in the pipe. THERMIT-WM and THERMIT obtain the same steady state value after 3 seconds and 9 seconds, with time step size of 0.1 sec, which is two or three times of the convective limit. The required steps for THERMIT, THERMIT-WM, and THERMIT-PF to simulate till 12 second are 382, 117, and 117, respectively. And the computational time of THERMIT, THERMIT-PF, and THERMIT-WM with I BM-370 is 83 seconds, 37.64 seconds,

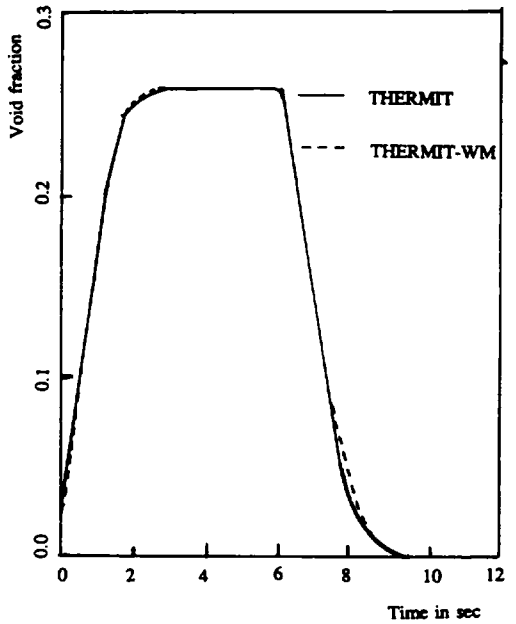

Fig. 5. The estimated void fraction by THERMIT and THERMIT-WM for the problem of the two-phase heating-up and cooling-down.
[time step size=3 xconvective limit (0.3sec)]

and 24.69 seconds, respectively. The relative error of the void fraction between THERMIT-WM and THERMIT is within 5% and the same degree of agreement for velocity and temperature are obtained.

The comparison of the CPU time is done at the steady state obtained after 6 seconds. the node cycles, which is the CPU time per node per number of step, is 0.118 sec, 0.0181 sec, and 0.0279 sec, for THERMIT, THERMIT-WM, and THERMIT-PF with IBM-370, respectively.

Problem 3. Blowdown simulation

The object of this simulation are to show efficiency in the compuatation of THERMIT-WM comparing to that of THERMIT. A

test tank has the geometry as shown in Fig. 6 with 9 cells. there is subcooled liquid with temperature of 520°F and with pressure of 2337.4 psia in the tank. the FLASH-1 test is performed with the break of the 0.0061% of outlet area and the FLASH-2 test is compared with results predicted by the WFLASH code with the 6.1×10^{-4}% of the break area.

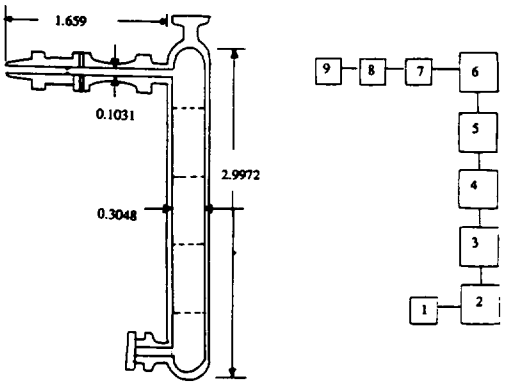

Fig. 6. The apparatus of FLASH-test and nodalization.

In the FLASH-1 test the transient takes 50 seconds to blow down to atmospheric pressure. Its numerical simulation is done up to 20 seconds shown in Fig. 7. THERMIT spent 49.25 second with 320 steps to simulate it. Both of THERMIT-WM and THERMIT-PF, however, take 45 steps during this numerical simulation and take the computational times of 16.41 seconds and 22.13 seconds, respectively.

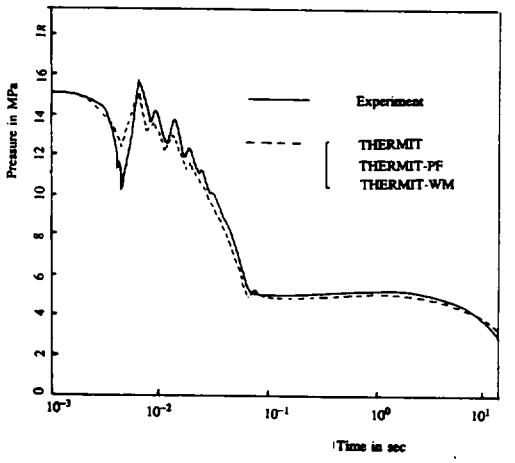

Fig. 7. Pressure transient for the FLASH-1 test.

In the FLASH-2 test the transient is so slow that the pressure could reach around 500 psia after 5 hours. all of the scheme show the same trend as shown in Fig. 8. Up to 100000 real time calculation THERMIT takes 250 steps, and THERMIT-WM and THERMIT-PF takes 41 steps. The calculation times of THERMIT, THERMIT-PF, and THERMIT-WM take 55.74 sec, 12.12 sec, and 9.98 sec, respectively.

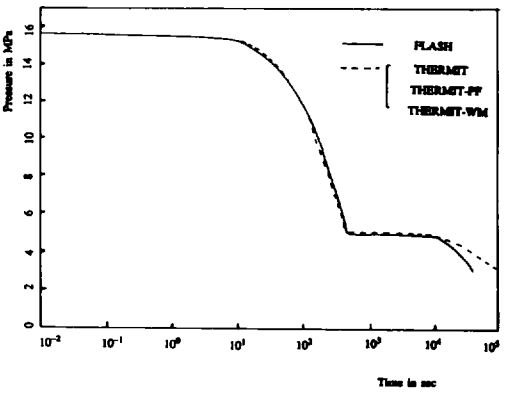

Fig. 8. Pressure transient for the FLASH-2 test.

Table 2 shows the fast running capacity of THERMIT-WM. the difference of results between of the SETS-WM and of the WFLASH are due to the difference of physical model such as the wall friction and the critical flow model.

Conclusions

Based on the stability analysis and the

Table 2. Comparisons between real time and CPU time.

	Blowdown test	Real time time (sec)	CPU (sec)	Time step size (sec)
THERMIT	FLASH-1	20.	49.25	0.052
THERMIT-PF		20.	22.13	0.52
THERMIT-WM		20.	16.41	0.52
THERMIT	FLASH-2	100000.	55.74	640.
THERMIT-PF		100000.	12.12	8000.
THERMIT-WM		100000.	9.98	8000.

*The time step sizes of FLASH-1 test are during real time from 13.7 sec to 14.22 sec.

*The time step sizes of FLASH-2 test are during real time from 54393. 14 sec to 62393. 14 sec.

results of the numerical simulation, the following conclusions can be concluded :

1. Removing the momentum stabilization steps from the SETS method results in the stability limit which is larger than the convective limit.

2. The interfacial momentum exchange terms play a most important role in enlarging the stability limit larger than the convective stability limit.

3. The instability of the SETS-WM method reslts from one-mode spectrum with $\theta = \pi$ and the growing rate of the SETS-WM method is much smaller than that of the I. C. E. method.

4. The SETS-WM method produces unconditional stability in single phase flow.

5. The easy extension of the SETS-WM scheme to the multi-dimensional case can be assured.

Referneces

Hanna, B. N. N. Hobson, and D. J. Richards, August 1985. One Step Semi-Implicit Methods for Solving the Transient Two-fluid Equations that is Non-Courant Limited, American Nu-clear society proceedings of the 1986 National Heat Transfer Conference, Denver, Colorado.

Mahaffy J. H. 1982, A Stability-Enhancing Two-Step Method for Fluid Flow Calculation, J. comput. Phys. 46, 329.

Trapp, J. A. and R. A. Riemke, 1986, A Nearly-Implicit Hydrodynamic Numerical Scheme for Two-Phase Flows, J. Comput. Phys. 66, 62-82.

Kelly J. H. and M. S. Kazimi, 1980. Development of a Two-Fluid, Two-Phase Model for Light Water Reactor Subchannel Analyis, Massachussetts Institute of Technology Ph. D. Thesis.

No H. C. and M. S. Kazimi, 1985, Effects of Virtual Mass on the Mathematical Characteristics and Numerical Stability of the Two-Fluid Model, Nucl. Sci. & Eng., 89, 197-206.

Nomenclature

a = accoustic velocity, coefficient
e = internal energy
f_i = interfacial momentum exchange
p = pressure
Q = rate of heat transfer per unit volume
t = time
T = temperature
$\triangle t$ = time step size
v = velocity
x = convective limit ($x=\triangle t/\triangle zV$)
$\triangle z$ = axial mesh size

α = void fraction
Γ = Vapor generation rate
ρ = density
ζ = characteristic

subscripts

i = interfacial, nodal direction
l = liquid
v = vapor
w = wall

Superscripts

n = time step

國 文 抄 錄

I. C. E (음함수 쿠랑 오일러리안) 기법이 갖는 안정성 제약조건을 증대시키기 위하여 2단계 안정성 증대기법(SETS)이 최근 미국 원자력 규제위원회 최대 정확 전산코트인 TRAC-PF1에서 사용되었는데, 이는 I. C. E 방식의 기본단계에 안정화 단계를 추가한다. 이러한 SETS 방식의 행렬식 크기는 완전 음함수 방식의 것보다 작아서 계산속도를 증대시킨다. 그러나 SETS 방식에 사용되는 모멘텀 안정화 단계는 해석공간의 차원이 증대됨에 따라 행렬식의 크기도 증대된다.

본 연구는 다차원 문제 해석시 SETS 방식의 행렬식의 크기를 줄이기 위해서, 그리고 이상유체의 경계에 작용하는 드랙힘의 안정성에 끼치는 영향을 고찰하기 위해서, SETS 방식에서 모멘텀 안정화 단계를 제거하고(SETS-WM) 수치적 안정성을 해석한다.

1차원 안정성 조건은 폰 노이만 안정성 해석에 의해 구해진다.

경계면 드랙힘은 안정성 증대에 큰 기여를 하는 것이 발견된다.

SETS-WM 방식은 M. I. T 공대에서 개발된 THERIT 코드를 근간으로 하여 개발하였고, 많은 수치계산 결과 본 연구에서 제시한 계산 시간크기 내에서 수치적 안정성 파괴없이 계산됨이 발견되었다.